The Holographic Dual of

$AdS_3 \times \mathbf{S}^3 \times \mathbf{S}^3 \times \mathbf{S}^1$

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Based on arXiv:1402.5135

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All About AdS3, Zurich, 2015

Supergravity Solution

$$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$$

Supported by fluxes $Q_5^\pm$ and Q_1

Elitzur, Feinerman, Giveon and Tsabar (1998)
de Boer and Skenderis (1999)
Gukov, Martinec, Moore and Strominger (2004)

Large N=4 Superalgebra

R-symmetry:

$$SO(4)^- \times SO(4)^+ \cong SU(2)^-_L \times SU(2)^-_R \times SU(2)^+_L \times SU(2)^+_R$$

- There are two *SU(2)* current algebras.
- There is also a *U(1)* coming from S^1 factor of the geometry.

Elitzur, Feinerman, Giveon and Tsabar (1998)
de Boer and Skenderis (1999)
Gukov, Martinec, Moore and Strominger (2004)

Central Charge

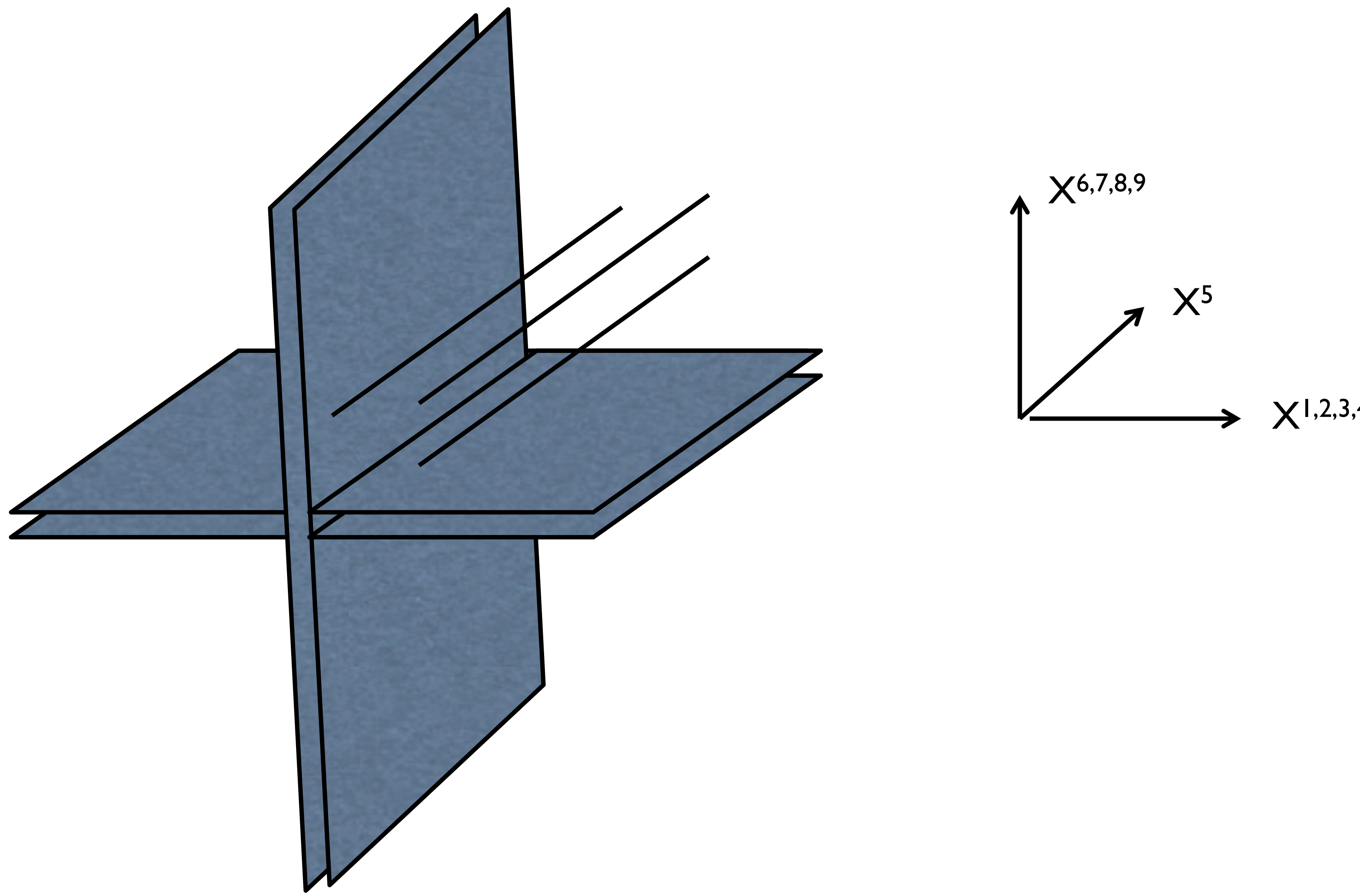
$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$ supported by fluxes $Q_5^\pm$ and Q_1

$$\boxed{c = 6Q_1 \frac{Q_5^+ Q_5^-}{Q_5^+ + Q_5^-}}$$

Sevrin, Troost, Van Proeyen (1988)

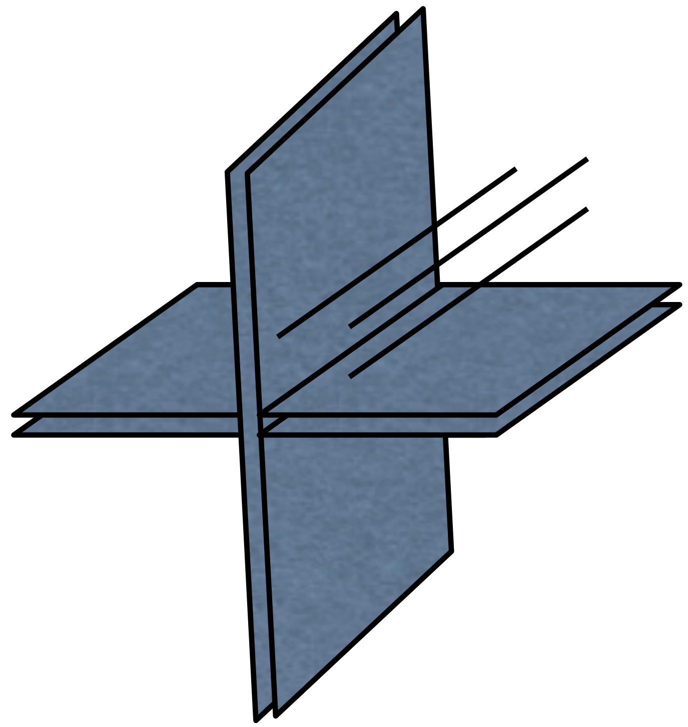
How to Build the Boundary Field Theory

A Tantalising D-Brane Configuration



Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

Taking the Near Horizon Limit



Q_5^+ D5-branes: 012345
Q_5^- D5$^\prime$-branes: 056789
Q_1 D1-branes: 05

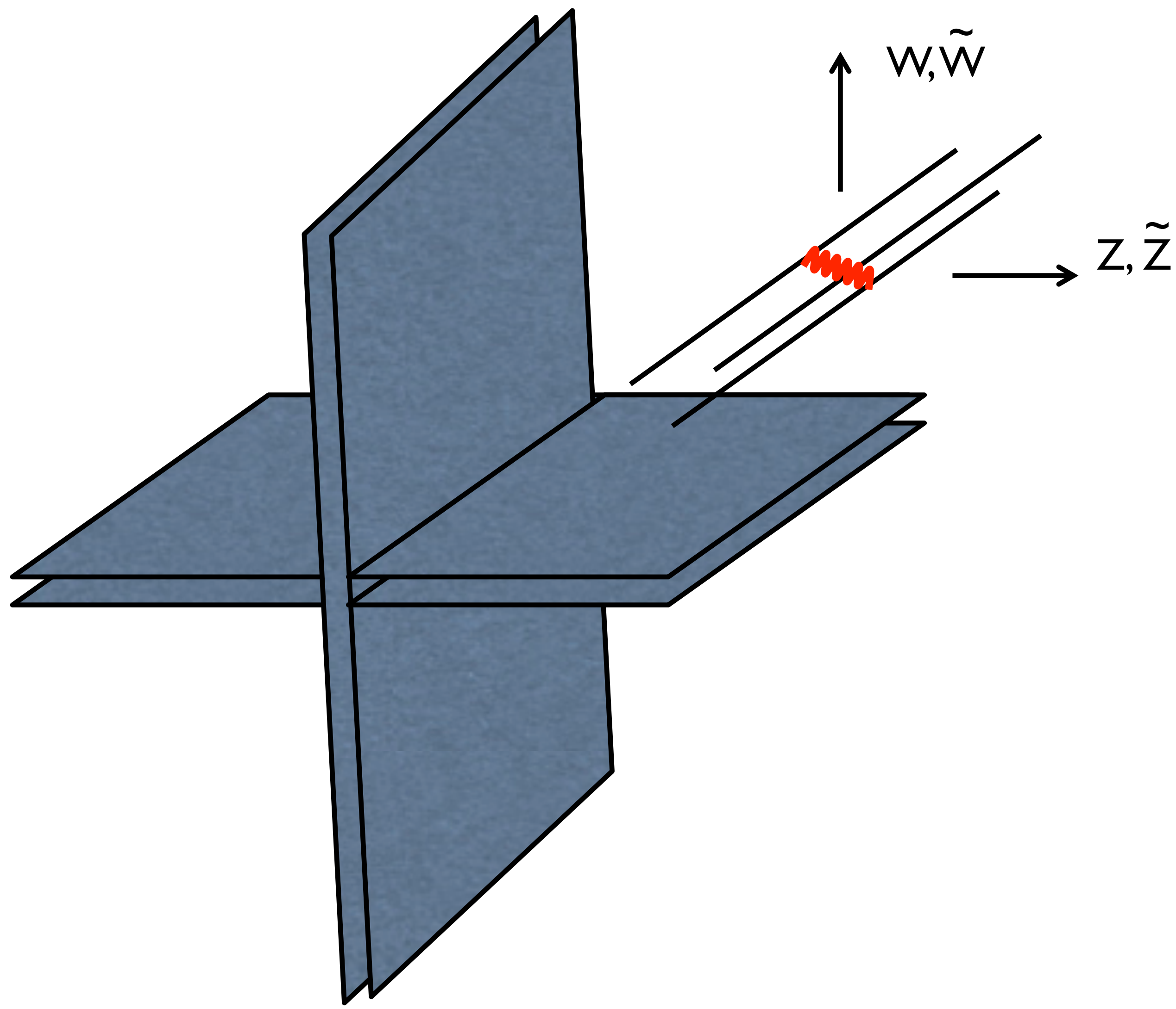
Smear D1-branes along 1234 and 6789. The near horizon limit is:

$$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{R}$$

How should we interpret this?!

Basic Idea: Study this D-brane configuration anyway!

The Low-Energy Dynamics of D-Branes

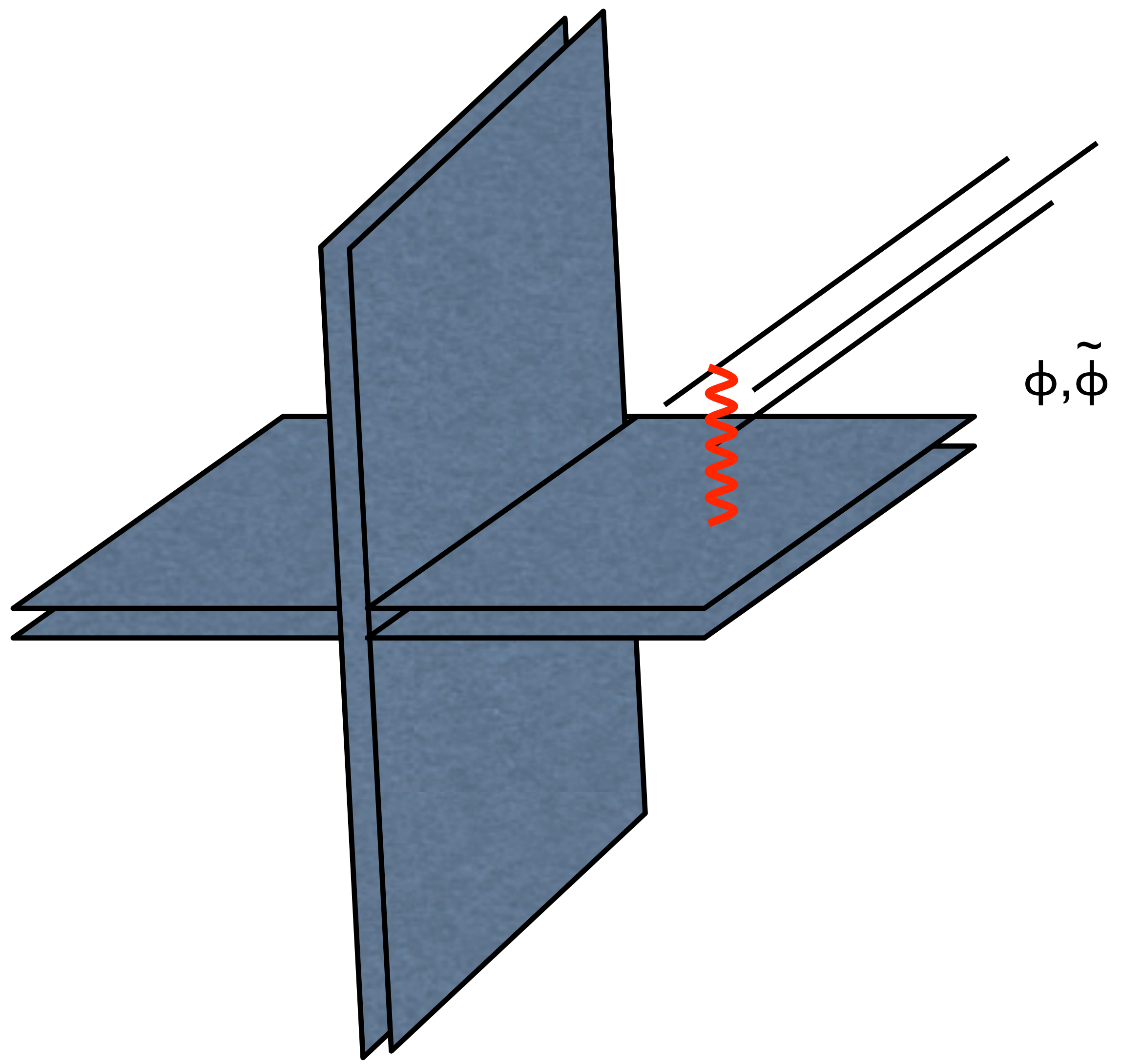


Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

D1-D1 strings: $U(Q_1)$ vector multiplet

- $N=(8,8)$ supersymmetry
- Gauge field and four complex, adjoint scalars

The Low-Energy Dynamics of D-Branes

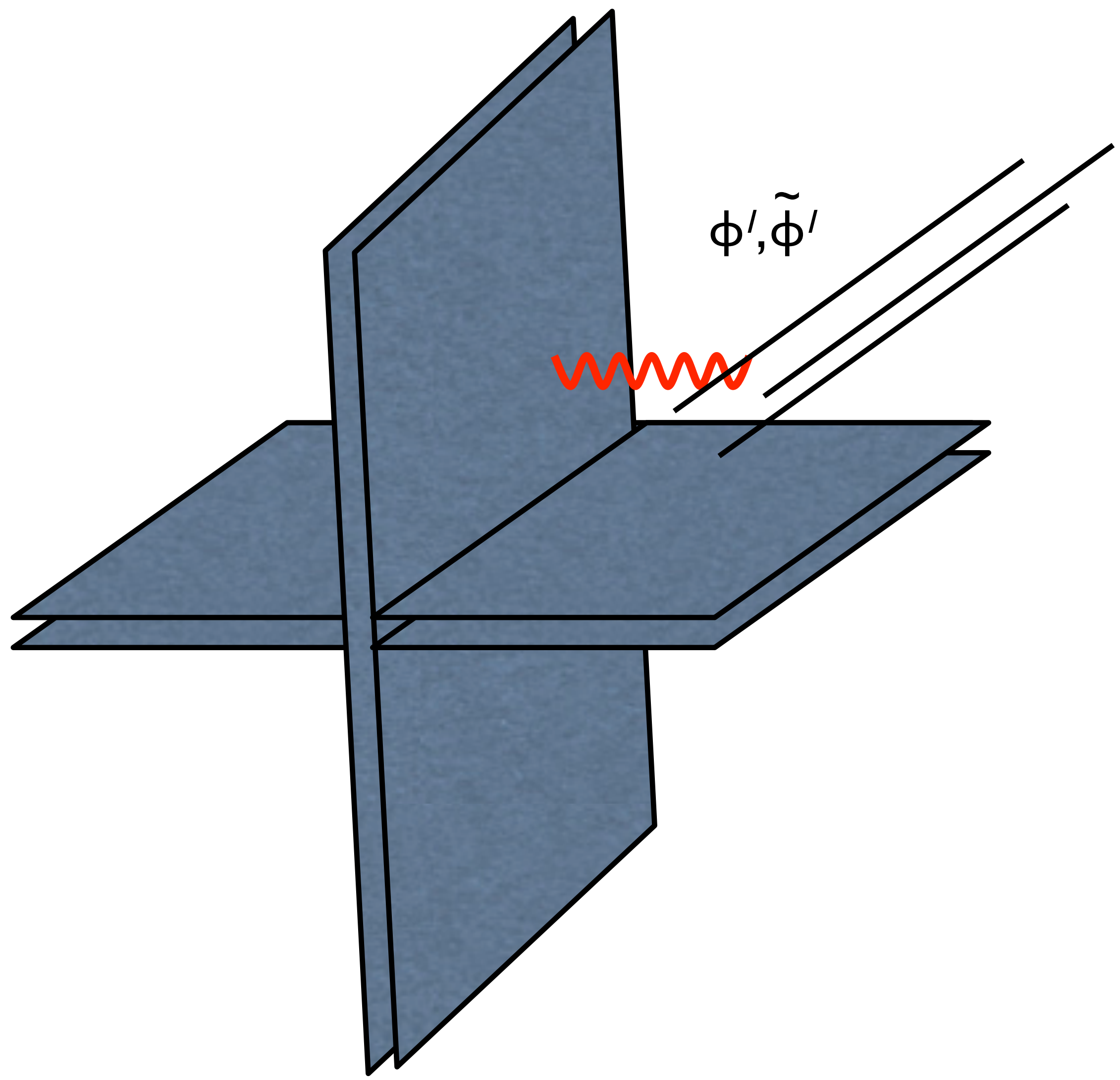


Q_5^+ D5-branes: 012345
Q_5^- D5$^\prime$-branes: 056789
Q_1 D1-branes: 05

D1-D5 strings: Q_5^+ fundamental hypermultiplets

- *N=(4,4)* supersymmetry
- two complex fundamental scalars

The Low-Energy Dynamics of D-Branes

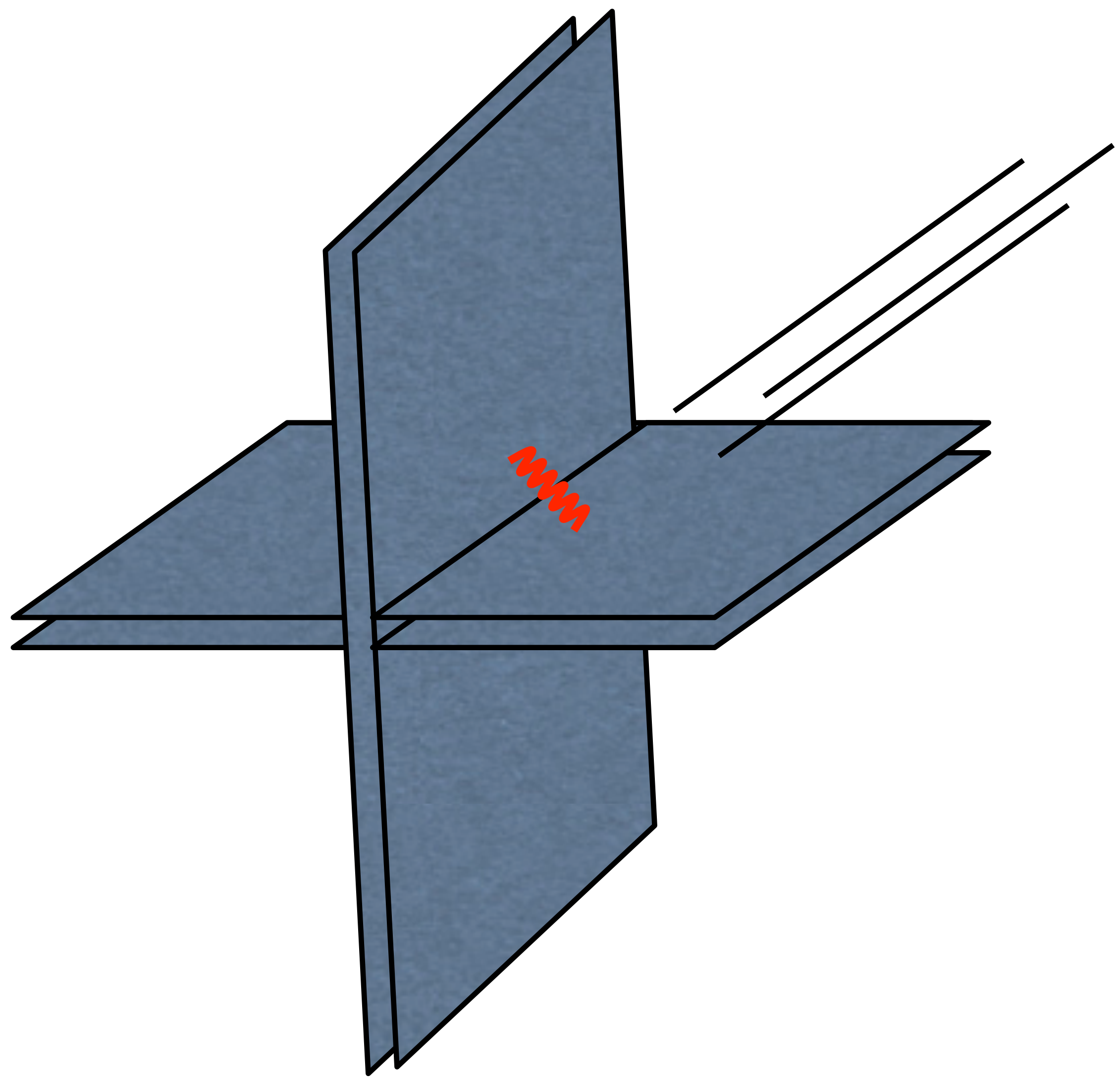


Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

D1-D5$'$ strings: Q_5^- fundamental (twisted) hypermultiplets

- *N=(4,4)* supersymmetry (but a different one!)
- two complex fundamental scalars

The Low-Energy Dynamics of D-Branes

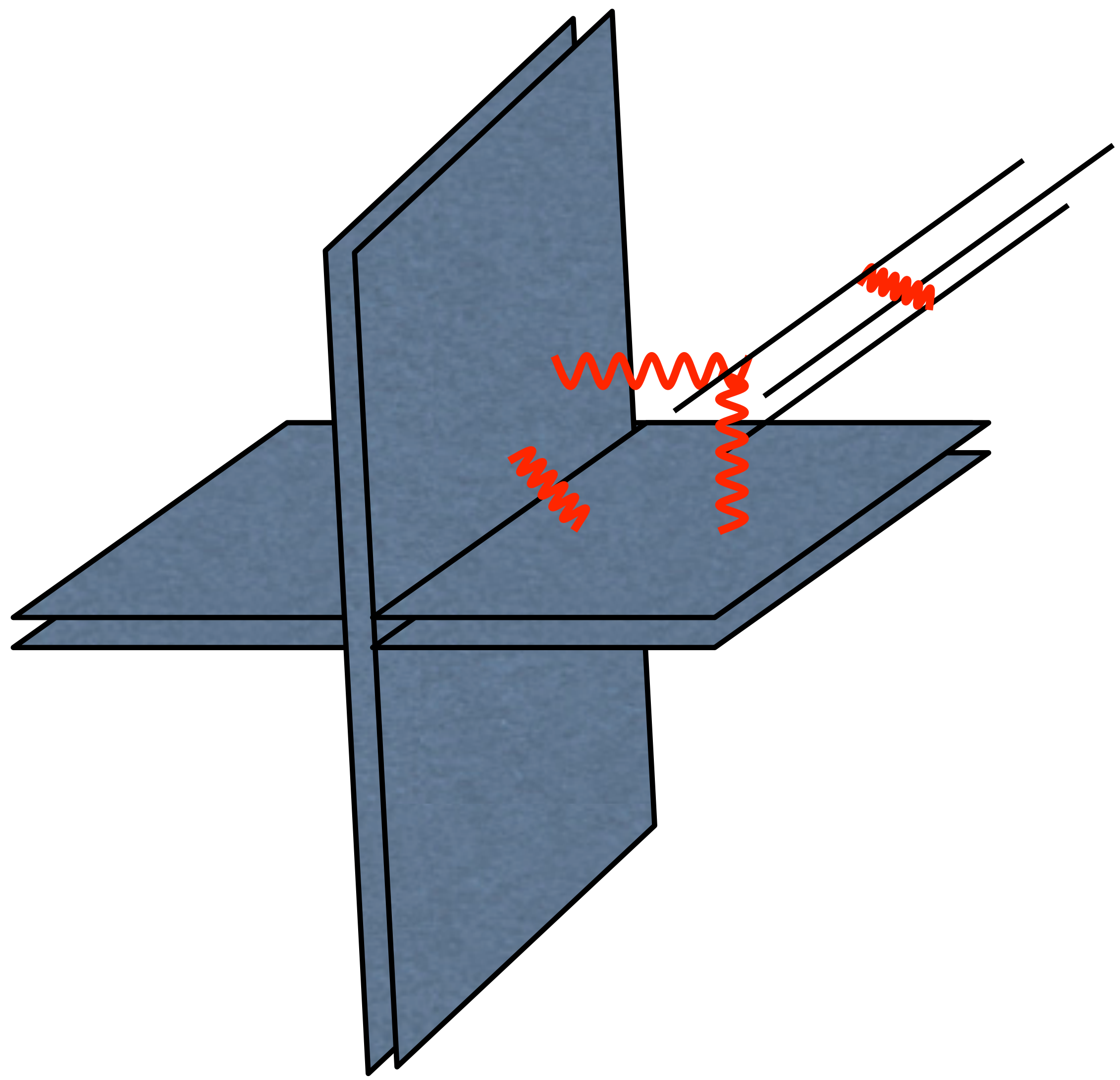


Q_5^+ D5-branes: 012345
Q_5^- D5$^\prime$-branes: 056789
Q_1 D1-branes: 05

D5-D5$^\prime$ strings: $Q_5^+ Q_5^-$ Fermi multiplets

- $N=(0,8)$ supersymmetry
- only left-moving fermions

The Low-Energy Dynamics of D-Branes



- All branes together preserve *N=(0,4)* supersymmetry
- The only question: how do chiral fermions couple to the other fields?
    - Surprising answer: the coupling is fixed by supersymmetry

A Brief Explanation of Supersymmetry

Use *N=(0,2)* superfields. Scalar potential terms are built using Fermi multiplets.

$$\Psi = \psi_- - \theta^+ G - i\theta^+\bar{\theta}^+(D_0 + D_1)\psi_- - \bar{\theta}^+ E(\phi_i) + \theta^+\bar{\theta}^+ \frac{\partial E}{\partial \phi^i}\psi_{+i}$$

<u>E-Terms</u> $$\bar{\mathcal{D}}_+ \Psi_a = E_a(\Phi_i)$$

Holomorphic functions of chiral superfields

<u>J-Terms</u> $$S_J = \int d^2x\, d\theta^+ \sum_a \Psi_a J^a(\Phi_i) \; + \; \text{h.c.}$$

The scalar potential is: $$V = \sum_a |E_a|^2 + |J_a|^2 + D^2$$

But there is only *N=(0,2)* supersymmetry if: $$E \cdot J \equiv \sum_a E_a J^a = 0.$$

The Coupling of the Chiral Fermions

The D5-D5$^\prime$ strings are needed to ensure that *E.J=0*

$$E_\chi = -\frac{1}{2}\tilde{\Phi}\Phi' \qquad E_{\tilde{\chi}} = -\frac{1}{2}\tilde{\Phi}'\Phi$$

$$J_\chi = \frac{1}{2}\tilde{\Phi}'\Phi \qquad J_{\tilde{\chi}} = \frac{1}{2}\tilde{\Phi}\Phi'$$

$N=(0,4)$ $U(Q_1)$ Gauge Theory

- Q_5^+ fundamental hypermultiplets
- Q_5^- fundamental twisted hypermultiplets
- $Q_5^+Q_5^-$ neutral Fermi multiplets

Flavour symmetry: $SU(Q_5^+) \times SU(Q_5^-)$

Also: $SO(4)^- \times SO(4)^+ \cong SU(2)_L^- \times SU(2)_R^- \times SU(2)_L^+ \times SU(2)_R^+$

$(Z,\widetilde{Z})$ in (2,2)

$(Y,\widetilde{Y})$ in (2,2)

These are $N=(0,4)$ R-symmetries

Finally, the theory has a global flavour symmetry which rotates hypers, twisted hypers and Fermi multiplets

The Scalar Potential

To write it in a way in which the symmetries are manifest, define:

$$\omega = \begin{pmatrix} \phi \\ \tilde{\phi}^\dagger \end{pmatrix} \qquad \omega' = \begin{pmatrix} \phi' \\ \tilde{\phi}'^\dagger \end{pmatrix}$$

and

$$\vec{D}_Z = \vec{\eta}_{ij} Z^i Z^j + \omega^\dagger \vec{\sigma} \omega \qquad \vec{D}_Y = \vec{\eta}_{ij} Y^i Y^j + \omega'^\dagger \vec{\sigma} \omega'$$

In **3** of $SU(2)_R^+$

In **3** of $SU(2)_R^-$

Self-dual
't Hooft matrices

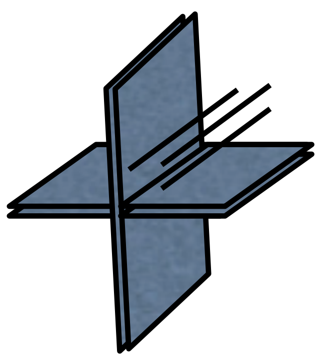
Pauli matrices

$$V = \mathrm{Tr}\,(\vec{D}_Z \cdot \vec{D}_Z + \vec{D}_Y \cdot \vec{D}_Y) + \omega^\dagger Y^i Y^i \omega + \omega'^\dagger Z^i Z^i \omega' + \mathrm{Tr}\,[Y^i, Z^j]^2 + \mathrm{Tr}\,\left(\omega^\dagger \cdot \omega\, \omega'^\dagger \cdot \omega'\right)$$

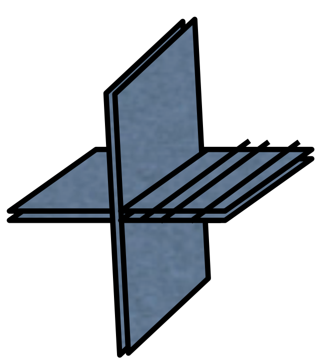
$$\text{last term} = \sum_{a=1}^{Q_5^+} \sum_{b=1}^{Q_5^-} \Big((\phi_a^\dagger \phi_b')(\phi_b'^\dagger \phi_a) + (\tilde{\phi}_a \tilde{\phi}_b'^\dagger)(\tilde{\phi}_b' \tilde{\phi}_a^\dagger) + (\phi_a^\dagger \tilde{\phi}_b'^\dagger)(\tilde{\phi}_b' \phi_a) + (\tilde{\phi}_a \phi_b')(\phi_b'^\dagger \tilde{\phi}_a^\dagger) \Big)$$

Vacuum Moduli Space

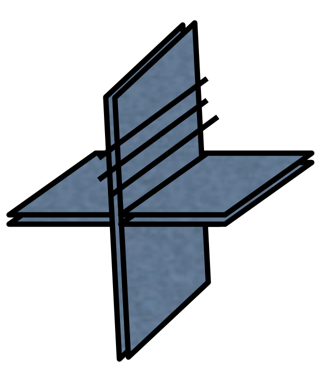
$$V = \mathrm{Tr}\,(\vec{D}_Z\cdot\vec{D}_Z + \vec{D}_Y\cdot\vec{D}_Y) + \omega^\dagger Y^i Y^i \omega + \omega'^\dagger Z^i Z^i \omega' + \mathrm{Tr}\,[Y^i, Z^j]^2 + \mathrm{Tr}\,\left(\omega^\dagger\cdot\omega\,\omega'^\dagger\cdot\omega'\right)$$



- $\omega = \omega' = 0$ with Z^i and Y^i mutually commuting.



- $\vec{D}_Z = 0$ and $Y^i = \omega' = 0$



- $\vec{D}_Y = 0$ and $Z^i = \omega = 0$

But in fact, we'll be interested in modes which appear to localise at the origin…

Flowing to the Infra-Red

Computing the Central Charge of *N=(0,2)* Theories

The OPE of the right-moving R-current includes the term

$$R(x)R(y) \sim \frac{3c_R}{(x^- - y^-)^2} + \ldots$$

But this is the anomaly. Which means that the central charge can be computed in the ultra-violet

$$\boxed{c_R = 3\mathrm{Tr}\, R^2}$$

Sum over right-moving fermions, minus left-moving fermions

Witten (1993)

Computing the Central Charge of *N=(0,2)* Theories

$$c_R = 3\mathrm{Tr}\, R^2$$

There's one small catch: you have to identify the right R-current in the UV. The requirement is:

$$\mathrm{Tr}\, FR = 0$$

Silverstein and Witten (1993)

Any *U(1)* global symmetry *F*

This can be repackaged as "c-extremization"

Benini and Bobev (2012)
Adams, Tong and Wecht
(Stupidly unpublished)

Computing the Central Charge of *N=(0,4)* Theories

Right-moving R-current is: $$SU(2)^-_R \times SU(2)^+_R$$

- No mixing for non-Abelian symmetries
- But *N=(0,2)* R-current is some combination of

$$R^\pm \subset su(2)^\pm_R$$

Both are good *N=(0,2)* R-currents. But there is one combination that is an *N=(0,2)* flavour symmetry

$$U = R^+ - R^-$$

We must have

$$\mathrm{Tr}\, UR = 0$$

$$\begin{aligned} \mathrm{Tr}\, UR^- &= -2Q_1Q_5^- \\ \mathrm{Tr}\, UR^+ &= +2Q_1Q_5^+ \end{aligned} \quad \Longrightarrow \quad R = \frac{Q_5^+}{Q_5^+ + Q_5^-}\, R^- + \frac{Q_5^-}{Q_5^+ + Q_5^-}\, R^+$$

Computing the Central Charge of *N=(0,4)* Theories

$$c_R = 3\mathrm{Tr}\, R^2 \quad \Longrightarrow \quad \boxed{c = 6Q_1 \frac{Q_5^+ Q_5^-}{Q_5^+ + Q_5^-}}$$

In agreement with the result from supergravity

Other Anomalies

- Each SU(2) symmetry in the gauge theory has an anomaly.
- These agree with the levels of the large *N=4* algebra

$$\mathrm{Tr}\,(R^+)^2 = \mathrm{Tr}\,(L^+)^2 = Q_1 Q_5^+$$

$SU(2)_R^+$ and $SU(2)_L^+$

$$\mathrm{Tr}\,(R^-)^2 = \mathrm{Tr}\,(L^-)^2 = Q_1 Q_5^-$$

$SU(2)_R^-$ and $SU(2)_L^-$

Other Anomalies

But….

- There is no symmetry corresponding to the S^1 action of the geometry.
- The gauge theory has more degrees of freedom.
  - These show up in the $SU(Q_5^+)$ and $SU(Q_5^-)$ flavour symmetries

$$\mathrm{Tr}\,(F^+)^2 = Q_5^- \qquad \mathrm{Tr}\,(F^-)^2 = Q_5^+$$

and the left-moving central charge

$$c_L = c_R + 2Q_5^+ Q_5^-$$

Proposal

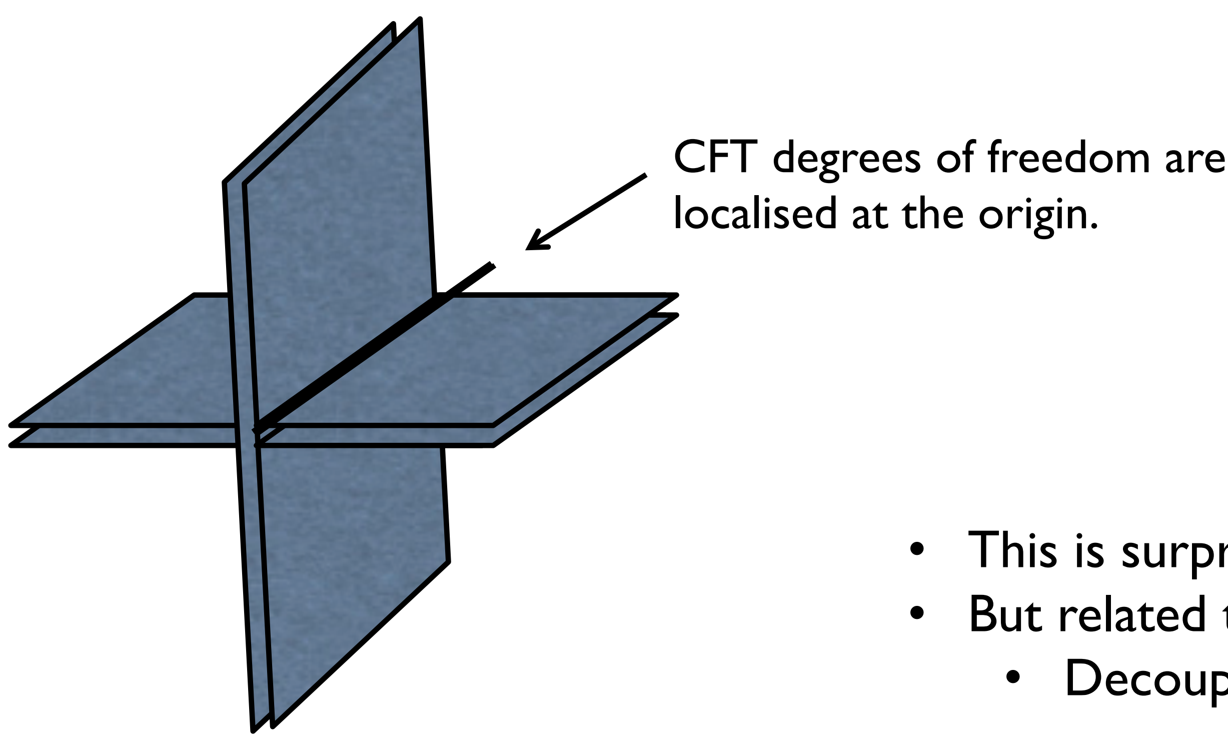
N=(0,4) Gauge Theory $\Rightarrow$ Large *N=4* CFT dual to $AdS_3 x S^3 x S^3 x S^1$

+ decoupled left-moving fermions

Where does this CFT live?

Right-moving R-symmetries cannot act on scalars in asymptotic (semi-classical) parts of moduli space. But….

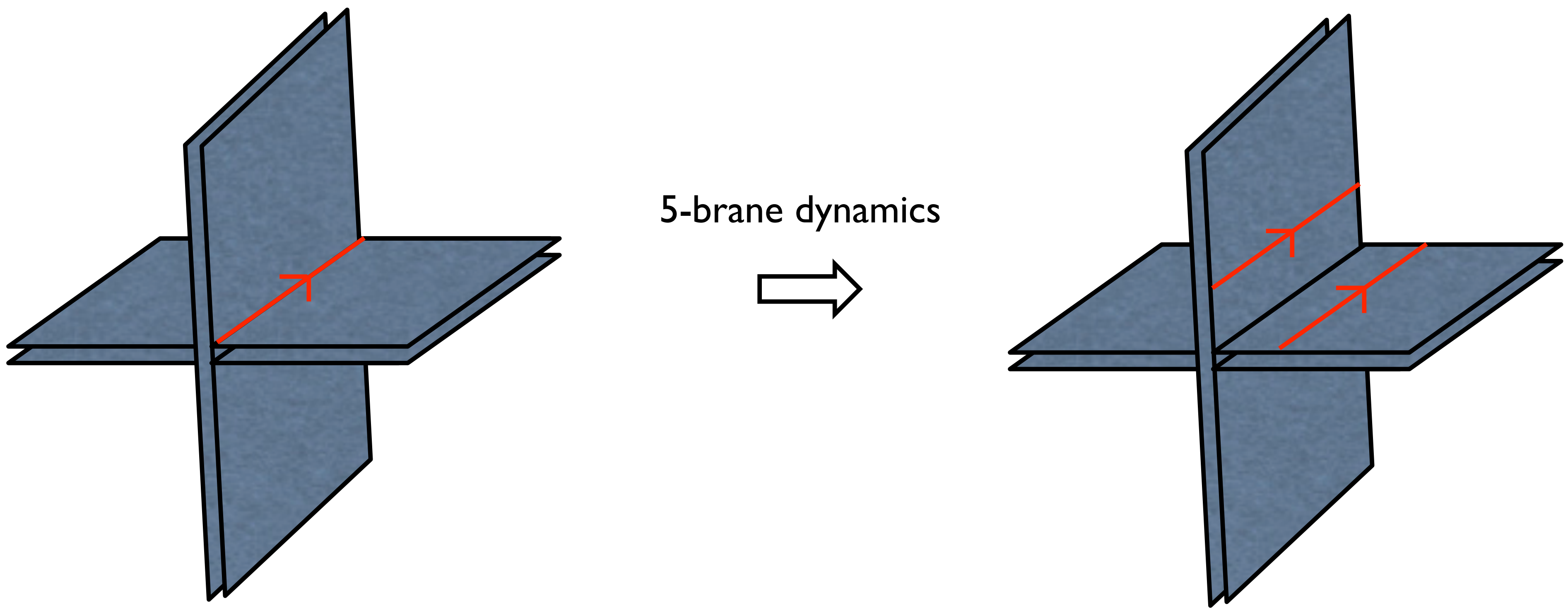
$$R[Y] = R[\tilde{Y}] = \frac{Q_5^-}{Q_5^+ + Q_5^-} \qquad R[Z] = R[\tilde{Z}] = \frac{Q_5^+}{Q_5^+ + Q_5^-}$$



- This is surprising in d=1+1 (Mermin-Wagner theorem)
- But related things happen in *N=(4,4)*
  - Decoupling of Higgs and Coulomb branches

What happened to the Chiral Modes?

An interesting story was found in the absence of D1-branes. This configuration is called the *I-brane*



- Chiral modes are pushed away from the intersection.
- The intersection has a mass gap
- It also has d=2+1 dimensional symmetry!

Itzhaki, Kutasov and Seiberg (2005)

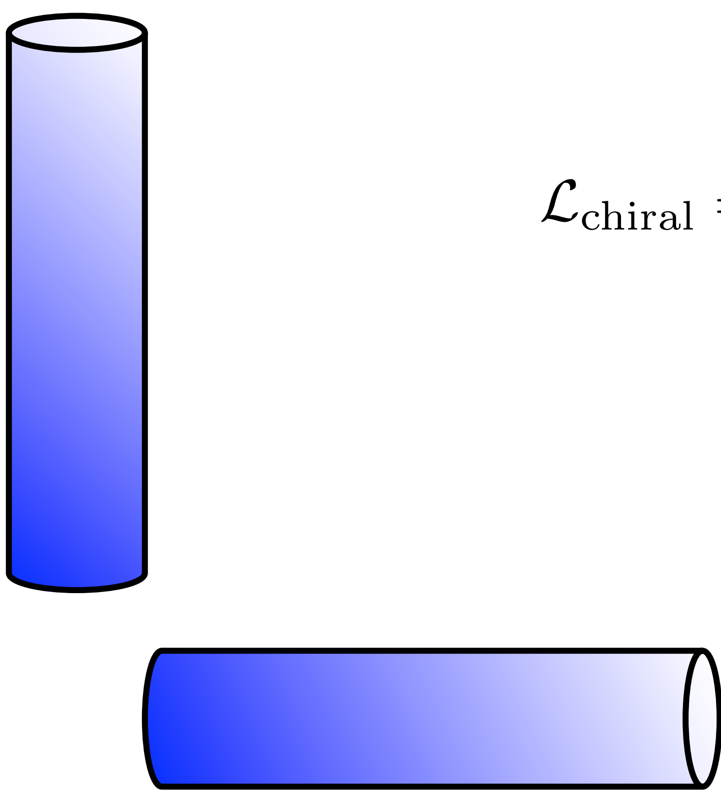
Decoupling in d=1+1 U(1) Theory

An (old) idea: Integrate out hyermultiplets. Focus on D1-D1 fields Y and Z

$$
\begin{aligned}
ds^2 &= \left(\frac{1}{g^2} + \frac{Q_5^+}{y^2}\right)\left(dy^2 + y^2 (d\Omega_3^+)^2\right) + \left(\frac{1}{g^2} + \frac{Q_5^-}{z^2}\right)\left(dz^2 + y^2 (d\Omega_3^-)^2\right) \\
&\longrightarrow \; Q_5^+ \left(\frac{dy^2}{y^2} + (d\Omega_3^+)^2\right) + Q_5^- \left(\frac{dz^2}{z^2} + (d\Omega_3^-)^2\right)
\end{aligned}
$$

Aharony and Berkooz (1999)

The chiral fermions survive this integrating out.



$$
\mathcal{L}_{\text{chiral}} = \left(1 + \frac{\log(y^2/z^2)}{y^2 - z^2}\right) \bar{\chi}_- \partial_+ \chi_- + \ldots
$$

$$
\frac{\log(y^2/z^2)}{y^2 - z^2} \rightarrow \begin{cases} \log y & y \rightarrow 0 \\ \frac{1}{y^2} & y = z \rightarrow 0 \end{cases}
$$

We can show that the right number of chiral fermions decouple. But…

Work with Ofer and Kenny Wong

Summary

$\mathcal{N}=(0,4)$ Gauge Theory $\Rightarrow$ Large $\mathcal{N}=4$ CFT dual to $AdS_3 \times S^3 \times S^3 \times S^1$

+ decoupled left-moving fermions

- What works:
  - central charge
  - level of SU(2) currents
  - decoupling of chiral fermions for U(1) theory
- What doesn't (yet):
  - decoupling of chiral fermions in general
    - (and non-chiral interactions)
  - emergence of U(1) symmetry associated to S^1
    - (and associated marginal operator)

Thank you for your attention

The Algebra

$$
\begin{aligned}
G^a(z)G^b(w) &= \frac{2c}{3}\frac{\delta^{ab}}{(z-w)^3} - \frac{8\gamma\alpha^{+,i}_{ab}A^{+,i}(w) + 8(1-\gamma)\alpha^{-,i}_{ab}A^{-,i}(w)}{(z-w)^2} - \\
&\quad - \frac{4\gamma\alpha^{+,i}_{ab}\partial A^{+,i}(w) + 4(1-\gamma)\alpha^{-,i}_{ab}\partial A^{-,i}(w)}{z-w} + \frac{2\delta^{ab}L(w)}{z-w} + \ldots, \\
A^{\pm,i}(z)A^{\pm,j}(w) &= -\frac{k^{\pm}\delta^{ij}}{2(z-w)^2} + \frac{\epsilon^{ijk}A^{\pm,k}(w)}{z-w} + \ldots, \\
Q^a(z)Q^b(w) &= -\frac{(k^+ + k^-)\delta^{ab}}{2(z-w)} + \ldots, \\
U(z)U(w) &= -\frac{k^+ + k^-}{2(z-w)^2} + \ldots, \\
A^{\pm,i}(z)G^a(w) &= \mp\frac{2k^{\pm}\alpha^{\pm,i}_{ab}Q^b(w)}{(k^+ + k^-)(z-w)^2} + \frac{\alpha^{\pm,i}_{ab}G^b(w)}{z-w} + \ldots, \\
A^{\pm,i}(z)Q^a(w) &= \frac{\alpha^{\pm,i}_{ab}Q^b(w)}{z-w} + \ldots, \\
Q^a(z)G^b(w) &= \frac{2\alpha^{+,i}_{ab}A^{+,i}(w) - 2\alpha^{-,i}_{ab}A^{-,i}(w)}{z-w} + \frac{\delta^{ab}U(w)}{z-w} + \ldots, \\
U(z)G^a(w) &= \frac{Q^a(w)}{(z-w)^2} + \ldots.
\end{aligned}
$$

BPS Bound

$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$ supported by fluxes $Q_5^{\pm}$ and Q_1

$$\boxed{h = \frac{Q_5^-}{Q_5^- + Q_5^+} l^+ + \frac{Q_5^+}{Q_5^- + Q_5^+} l^- + \frac{1}{Q_1(Q_5^+ + Q_5^-)} \left[(l^+ - l^-)^2 + u^2 \right]}$$

Representation under *SU(2)*$^{+/-}$

Charge under *U(1)*

- The bound is non-linear.
  - There is no chiral ring!
- The non-linearities are "1/N" suppressed.
  - They are not seen in supergravity