5-loop Konishi

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1. **Motivation**
2. **Multiparticle Lüscher corrections**
3. **Magnitudes**
4. **Crosschecks**
5. **4 loops**
   - Konishi
   - Twist two
   - Single impurity
6. **5 loops**
   - Konishi
   - Single impurity
7. **Conclusions**

Motivation

- When computing anomalous dimensions in $\mathcal{N}=4$ SYM theory from two-point functions
$$\langle O(x)O(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$
two classes of Feynman graphs arise:

and

- The first class is contained in the so-called Asymptotic Bethe Ansatz of Beisert and Staudacher
- The second class are 'wrapping interactions' which start to appear at order g^{2L} (these are not contained in the Asymptotic Bethe Ansatz)
- The computation of *all* wrapping graphs is (one of) the aim(s) of the TBA systems proposed for the light-cone string sigma model in $AdS_5 \times S^5$
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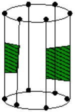

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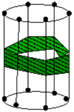

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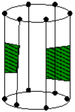
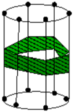
 and 

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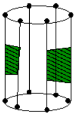

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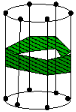

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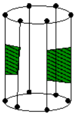

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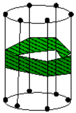

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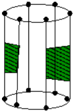

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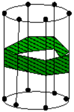

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- The simplest operator nonprotected by supersymetry is the Konishi operator

$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\,Z^2X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\,ZD^2Z + \ldots$$

- The wrapping correction appears first at 4 loops, and, on the string side, can be computed from a single Lüscher correction 'F-term' graph [Bajnok,RJ]

$$\Delta^{(8)}_{w,Konishi} = 324 + 864\,\zeta(3) - 1440\,\zeta(5)$$

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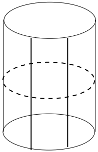

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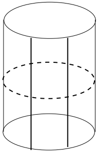

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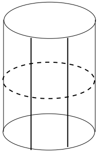

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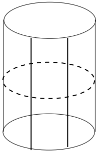

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Why bother with 5 loops???

- Two new features appear
  — an infinite set of coefficients of the BES/BHL dressing phase start to contribute
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Multiparticle Lüscher corrections

- In contrast to relativistic theories the S-matrix cannot be written in a difference form

$$S(p_1, p_2) \neq S(\phi(p_1) - \phi(p_2))$$

- Consequently one cannot trade derivatives w.r.t. p_1 for derivatives w.r.t. p_2 as for relativistic theories where we have

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Construct Thermodynamic Bethe Ansatz (TBA) for a hypothetical theory with a single particle species which scatters with an S-matrix

$$S(p_1, p_2)$$

which does not have any difference property

Program:

1. Construct ground state TBA
2. Get excited state TBA from analytic continuation
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Multiparticle Lüscher corrections

- Consider the theory on a cylinder of size L and height $R \to \infty$
- The partition function will be dominated by the ground state

$$Z(L,R) \underset{R\to\infty}{\sim} e^{-RE_0(L)}$$

- TBA: consider the same partition function in the theory with space and time interchanged ('mirror theory'):

- The same partition function has the interpretation of a the mirror theory on a very large cylinder of size $R \to \infty$ at nonzero temperature $T = 1/L$
- Crucial advantage: virtual corrections to the Asymptotic Bethe Ansatz *of the mirror theory* vanish as $R \to \infty$

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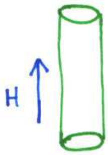

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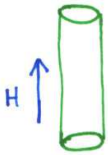

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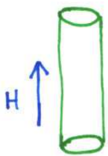

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- Start from the Asymptotic Bethe Ansatz for the mirror theory

$$e^{i\tilde{p}_j R} = \prod_{k:k\neq j} S(\tilde{p}_j, \tilde{p}_k)$$

- Introduce densities of roots $\rho(z)$ and holes $\rho_h(z)$

$$2\pi(\rho(z) + \rho_h(z)) = R\tilde{p}'(z) - \phi * \rho \quad \text{where} \quad \phi \equiv \partial_z \log S(\tilde{p}(z), \cdot)$$

- In order to determine ρ we need a second equation — extremize the free energy

$$F = E - TS \equiv \int \tilde{E}(z)\rho(z)dz - \frac{1}{L} S[\rho, \rho_h]$$

- The above equations completely fix $\rho(z)$.
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Multiparticle Lüscher corrections

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- TBA equation for $\varepsilon(z)$

$$\varepsilon(z) = L\tilde{E}(z) + \int \frac{dw}{2\pi}\, \phi(w,z) \log\left(1 + e^{-\varepsilon(w)}\right)$$

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- During analytical continuation and deforming contours one encounters points where $1 + e^{-\varepsilon(z_i)} = 0$
- Their contribution can be evaluated by residues to give additional source terms in the equations – sign depending on relative orientation of the contour
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$$E = E(z_1) + E(z_2) - \int \frac{dz}{2\pi} \tilde{p}'(z) \log\left(1 + e^{-\varepsilon(z)}\right)$$

so that $\partial_z \log\left(1 + e^{-\varepsilon(z_i)}\right)$ contributes -1.

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- Plug the *leading order* term into the formula for the energy

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- Here we assumed that each physical particle is represented by a *single* root – μ terms appear when several roots correspond to a single particle [Dorey,Tateo; Bazhanov,Lukyanov,Zamolodchikov] see talk by Bajnok
- The F-term integral is not sensitive to the convolution part of TBA equations...
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Multiparticle Lüscher corrections

- Plug the *leading order* term into the quantization condition:

$$i\pi = \varepsilon(z_1) = iLp_1 + i\pi + \log S(z_2, z_1)$$

- This is just the Asymptotic Bethe Ansatz condition
- We have to insert the subleading integral part also! This gives

$$0 = \underbrace{\log\{e^{iLp_1} S(z_2, z_1)\}}_{BY_1} + \underbrace{\int \frac{dw}{2\pi i} (\partial_w S(w, z_1)) S(w, z_2) e^{-L\tilde{E}(w)}}_{\Phi_1}$$

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- Solve as corrections to the Asymptotic Bethe Ansatz values p_i

$$\frac{\partial BY_1}{\partial p_1}\delta p_1 + \frac{\partial BY_1}{\partial p_2}\delta p_2 + \Phi_1 = 0$$

$$\frac{\partial BY_2}{\partial p_1}\delta p_1 + \frac{\partial BY_2}{\partial p_2}\delta p_2 + \Phi_2 = 0$$

- Plug the *leading order* term into the quantization condition:

$$i\pi = \varepsilon(z_1) = iLp_1 + i\pi + \log S(z_2, z_1)$$

- This is just the Asymptotic Bethe Ansatz condition
- We have to insert the subleading integral part also! This gives

$$\begin{aligned} 0 &= \underbrace{\log\{e^{iLp_1} S(z_2, z_1)\}}_{BY_1} + \underbrace{\int \frac{dw}{2\pi i} (\partial_w S(w, z_1)) S(w, z_2) e^{-L\tilde{E}(w)}}_{\Phi_1} \\ 0 &= \underbrace{\log\{e^{iLp_2} S(z_1, z_2)\}}_{BY_2} + \underbrace{\int \frac{dw}{2\pi i} S(w, z_1)(\partial_w S(w, z_2)) e^{-L\tilde{E}(w)}}_{\Phi_2} \end{aligned}$$

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Multiparticle Lüscher corrections

- Final Multiparticle Lüscher corrections

$$E = \underbrace{E(p_1) + E(p_2)}_{\text{ABA}} + \underbrace{E'(p_1)\delta p_1 + E'(p_2)\delta p_2}_{\text{ABA modification}} - \underbrace{\int \frac{dq}{2\pi} e^{-L\tilde{E}} S(z,z_1)S(z,z_2)}_{\text{F-term}}$$

- F-term is sensitive just to the *source terms* of TBA.
- The integrand is essentially given by the transfer matrix
- ABA modification terms depend on the *convolution terms* in TBA equations. Not expressible directly in terms of transfer matrix
- Natural generalization to nondiagonal scattering... [Bajnok,RJ]
  – diagrammatic verification for single particle Lüscher corrections (completely independent from TBA) [Łukowski,RJ]
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$$\log Y_A = source_A + K_{AB} * \log(1 + Y_B)$$

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$$e^{-L\tilde{E}} \sim g^{2L} \qquad \text{for small} \quad g$$

- For the Konishi the F-term appears at order g^8 (4 loops)
- However the ABA modification term first appears at one order higher (5 loops)

$$E'(p_1)\delta p_1 \sim g^2 \cdot g^{2L} \underset{Konishi}{\sim} g^{10}$$

- The S-matrix *dressing factor* in Lüscher corrections behaves like

$$\sigma^2 \sim e^{ig^2 \cdot phase} \sim 1 + g^2 \cdot (\ldots)$$

due to the fact that one particle has mirror kinematics and the other physical kinematics...

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Compute the 5-loop anomalous dimension from string theory using multiparticle Lüscher corrections

How will we know that we get the correct result???

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Crosschecks

- Ultimate crosscheck — direct perturbative computation unfortunately seems *very difficult*
- There are also nontrivial internal consistency crosschecks
- The higher loop integrals in perturbative gauge theory have (here) a rather simple transcendentality structure – a linear combination of (products) of ζ's
- Typical subexpressions from string theory involve much more complicated structures like polygammas etc.
- All these should cancel between the various parts of Lüscher expressions coming from *different* sources - like ABA modification, dressing factor and higher order expansion of F-term integrand
- Another crosscheck – cancellation of μ-terms

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$$\int dq \underbrace{\left(\frac{4g^2}{q^2+Q^2}\right)^4}_{e^{-L\tilde{E}}} \cdot \underbrace{(\partial)S(w,z_1)S(w,z_2)}_{\text{S-matrices}}$$

- The pole at $q = iQ$ comes from the purely kinematical exponential factor
- The product of the S-matrices involves additional poles in q associated to s and t channel poles ($\equiv$ 'dynamical poles')

$$q = i(Q \pm 1) \pm \frac{1}{\sqrt{3}}$$

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General features of the leading 4-loop wrapping corrections:

- No contribution from the modification of the Asymptotic Bethe Ansatz
- No contribution from the dressing phase
- The whole contribution comes from the F-term integral which can be expressed through the transfer matrix:

$$\Delta_w^F = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(\frac{z^-}{z^+} \right)^L \sum_b (-1)^{F_b} \left[S_{Q-1}(q, u_i) S_{Q-1}(q, u_{ii}) \right]_{b(11)}^{b(11)}$$

this can be rewritten using Y-system notation as

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Konishi @ 4 loops

[Bajnok,RJ]

- For the Konishi operator

$$Y_Q(q) = \frac{16384 g^8 Q^2(-1+q^2+Q^2-4u^2)^2}{(q^2+Q^2)^4((q+i(Q+1))^2-4u^2)((q+i(Q-1))^2-4u^2)} \times \frac{1}{((q-i(Q-1))^2-4u^2)((q-i(Q+1))^2-4u^2)} + \ldots$$

with

$$u = \frac{1}{2\sqrt{3}} + \mathcal{O}\left(g^2\right)$$

- Perform the integral by residues...
- Contribution of the dynamical poles cancels out after summation over Q
- The whole result follows just from the kinematical pole:

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Konishi @ 4 loops

[Bajnok,RJ]

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Twist two @ 4 loops

[Bajnok,RJ,Łukowski]

Twist two operators are operators composed of two scalar fields Z and M covariant derivatives D along a light cone direction

$$\mathrm{tr}\, ZD^M Z + \ldots$$

These operators get wrapping corrections at 4 loops

$$\gamma_8(M) = \gamma_8^{Bethe}(M) + \gamma_8^{wrapping}(M)$$

Their anomalous dimensions obey very strong constraints

- Wrapping part should not have a piece proportional to $\log M$ (cusp anomalous dimension should be unmodified)
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- Maximal transcendentality principle of Kotikov, Lipatov. $\gamma_8(M)$ should have transcendentality degree 7
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  where $S_k \equiv S_k(M) = \sum_{n=1}^{M} 1/n^k$, etc.
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Single impurity @ 4 loops

- Suppose we analytically continue $M \to 1$
- We obtain a single impurity with $p = \pi$ (rapidity $u = 0$)
- Such an operator is unphysical in $\mathcal{N} = 4$ SYM, but makes sense in the β deformed theory with $\beta = 1/2$
- The four loop wrapping becomes

$$\Delta^{(8)}_{w,single} = 496\,\zeta(3) - 640\,\zeta(5)$$

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The BES/BHL dressing phase in the Lüscher kinematics

[Bajnok,Hegedus,RJ,Łukowski]

- We have to evaluate $\sigma^2_{BES}(z^\pm, x^\pm)$ where $x^\pm$ is in the physical kinematics while $z^\pm$ is in the mirror one, i.e.

$$x^+ \sim \frac{1}{g} \qquad x^- \sim \frac{1}{g} \quad \text{but} \quad z^+ \sim \frac{1}{g} \qquad z^- \sim g$$

- This scaling upsets the estimates of the weak coupling beaviour of σ^2_{BES}
- In the expression for the phase ($\sigma \sim \exp(i\chi)$)

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- Recall

$$E = \underbrace{E(p_1) + E(p_2)}_{\text{ABA}} + \underbrace{E'(p_1)\delta p_1 + E'(p_2)\delta p_2}_{\text{ABA modification}} - \underbrace{\int \frac{dq}{2\pi} e^{-L\tilde{E}} S(z,z_1)S(z,z_2)}_{\text{F-term}}$$

- The ABA quantization condition will get modified at 5 loops

$$\begin{aligned} \frac{5i}{2}\delta p_1 - \frac{i}{2}\delta p_2 + \Phi &= 0 \\ -\frac{i}{2}\delta p_1 + \frac{5i}{2}\delta p_2 - \Phi &= 0 \end{aligned}$$

where

$$\begin{aligned} i\Phi &= \sum_Q \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(\frac{z^-}{z^+}\right)^L \sum_b (-1)^{F_b} \left[(\partial_q S_{Q-1}(q,u_i))S_{Q-1}(q,u_{ii})\right]_{b(11)}^{b(11)} \\ &\equiv \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \Phi_Q(q,u) \end{aligned}$$

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- This will involve expanding the transfer matrix up to g^{10}. We have to include the following contributions:
  1. Explicit g dependence coming from mirror particle $z^{\pm}$ in the S-matrix elements
  2. We have to sum up subleading terms in the construction of the scalar factor in the *sl(2)* sector for bound states
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$$Y_Q^{(8)}(q,u) \cdot \left[-\frac{32}{1+4u^2} \left(\gamma_E + \frac{1}{2}\psi\left(\frac{1}{2}(-iq-Q)\right) + \frac{1}{2}\psi\left(\frac{1}{2}(iq+Q)\right) \right) \right]$$

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$$\Delta_w^{(10)} = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(\frac{4}{\sqrt{3}} \Phi_Q(q) + Y_Q^{(10,0)}(q) + Y_Q^{(8,2)}(q) \right)$$

- *All* terms have poles at $q = iQ$, and at dynamical poles
- In addition the polygamma ψ functions appearing in the dressing factor analytically continued to the Lüscher kinematics lead to an infinite sequence of poles at $q = i(Q + 2n)$
- After summation over Q, the residues of dynamical poles cancel out! (no μ-terms at weak coupling)

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Our result:

$$\Delta_w^{(10)} = -11340 + 2592\,\zeta(3) - 5184\,\zeta(3)^2 - 11520\,\zeta(5) + 30240\,\zeta(7)$$

This gives for the total anomalous dimension:

$$\begin{aligned}\Delta &= 4 + 12\,g^2 - 48\,g^4 + 336\,g^6 + 96(-26 + 6\,\zeta(3) - 15\,\zeta(5))\,g^8 \\ &\quad -96(-158 - 72\,\zeta(3) + 54\,\zeta(3)^2 + 90\,\zeta(5) - 315\,\zeta(7))\,g^{10}\end{aligned}$$

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Single impurity @ 5 loops

- We performed the same computation for the $L = 2$ single impurity state with $p = \pi$ (should be physical in the $\beta = 1/2$ deformed theory)
- Following our 4 loop observation, we *assumed* that fermionic virtual particles cancel
- Here ABA is *not* modified, but the dressing factor contributes in the same way as for Konishi
- Dynamical poles cancel and the final result has a similar transcendentality structure

$$\Delta^{(10)}_{w,single} = -4096\,\zeta(3) + 5120\,\zeta(5) - 1536\,\zeta(3)^2 + 13440\,\zeta(7)$$

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Conclusions

- We have calculated 5 loop wrapping correction to the Konishi anomalous dimension and single impurity
- μ-terms cancel out
- The result has simple transcendentality structure
- Infinite set of BES/BHL coefficients contribute
- Asymptotic Bethe Ansatz quantization gets modified
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