Anomalous dimensions at four loops in ABJM and ABJ

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J. Minahan, O. Ohlsson Sax, C. S., work in progress

Outline

Introduction and motivation

$SU(2) \times SU(2)$ subsector

Extraction of $h(\lambda)$

Field theory calculation: 2-loop warm-up

Field theory calculation: 4-loops

From ABJM to ABJ
- Symmetries

What made us suffer

AdS_4/CFT_3 (ABJM) correspondence

[Aharony, Bergman, Jafferis, Maldacena]

Type II A ST $AdS_4 \times \mathbb{CP}_3$ $\Leftrightarrow$ 3-dim. $\mathcal{N} = 6$ CS theory

energy E ← $\lambda \gg 1$ — $\lambda \ll 1$ → anom. dim. γ

(semicl.) strings

comp. operators

$\mathcal{O}_J \propto \mathrm{tr}\, \phi_{n_1} \dots \phi_{n_J}$

integrable systems

(asymptot.) Bethe ansätze

[Minahan, Zarembo]
[Bak, Rey]
[Gromov, Vieira]

magnon dispersion relation

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$$E = \frac{1}{2}\left(\sqrt{1 + 16 h(\lambda)^2 \sin^2 \frac{p}{2}} - 1\right)$$

BMN limit
giant magnons

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bifundamentals $(N, \bar{N})$ and $(\bar{N}, N)$ of $U(N) \times \overline{U(N)}$ gauge group
in ABJ: $(M, \bar{N})$ and $(\bar{M}, N)$ of $U(M) \times \overline{U(N)}$ [Aharony, Bergman, Jafferis]

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gauge invariant composite operator

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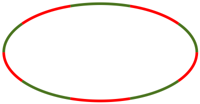

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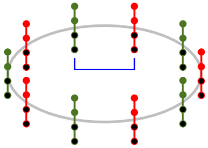

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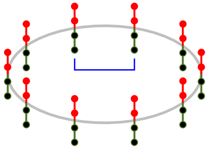

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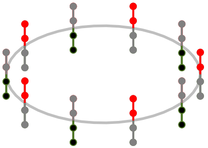

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$(2, 2)$ of $SU(2) \times SU(2)$
$SU(2) \times SU(2)$ subsector, free of flavour traces

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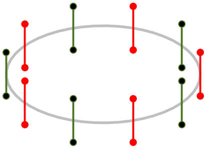

Bethe ansatz

[Minahan, Zarembo]
[Bak, Rey]
[Gromov, Vieira]

$SU(2) \times SU(2)$ sector:
two copies of the $XXX_{\frac{1}{2}}$ Heisenberg spin chain
only coupled via momentum conservation
and BES dressing phase [Beisert, Eden, Staudacher]

$$\sum_{j=1}^{M_2} p_{2,j} + \sum_{j=1}^{M_{\dot{2}}} p_{\dot{2},j} = 0\,,$$

$$e^{ip_{2,j}L} = \prod_{k\neq j}^{M_2} S(u_{2,k}, u_{2,j}) \prod_{k=1}^{M_{\dot{2}}} \sigma(u_{2,k}, u_{\dot{2},j})$$

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dispersion relation: $E = E_2 + E_{\dot{2}}$

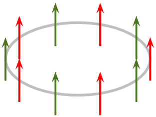
$$E_2 = \sum_{j=1}^{M_2} \frac{1}{2}\left(\sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p_j}{2}} - 1\right)$$

$$E_{\dot{2}} = \sum_{j=1}^{M_{\dot{2}}} \frac{1}{2}\left(\sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p_j}{2}} - 1\right)$$

States in the $SU(2) \times SU(2)$ subsector

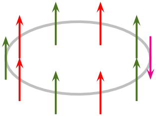
ground state, $E = 0$

$$\mathcal{O}_{2L,(0,0)} = \text{tr}((A_1 B^\dagger_{\dot 1})^L) =$$



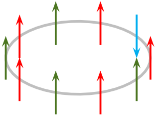
single magnon states, $E = 0$

$$\mathcal{O}_{2L,(1,0)} = \text{tr}(A_2 B^\dagger_{\dot 1}(A_1 B^\dagger_{\dot 1})^{L-1}) =$$



$, \quad p_2 = 0$

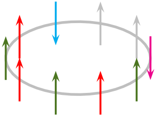
$$\mathcal{O}_{2L,(0,1)} = \text{tr}(A_1 B^\dagger_{\dot 2}(A_1 B^\dagger_{\dot 1})^{L-1}) =$$



$, \quad p_{\dot 2} = 0$

two magnon states, $p_2 = -p_{\dot 2} = \frac{2\pi n}{L}$, $E = \sqrt{1 + 16 h(\lambda)^2 \sin^2 \frac{p_2}{2}} - 1$

$$\mathcal{O}_{2L,(1,1)} = \text{tr}(A_2 B^\dagger_{\dot 1}(A_1 B^\dagger_{\dot 1})^k A_1 B^\dagger_{\dot 1}(A_1 B^\dagger_{\dot 1})^{L-k-2}) =$$



Extraction of $h(\lambda)$

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compare with the λ^4 coefficient of E_2 with $h(\lambda)^2 = \lambda^2 + \lambda^4 h_4$

$$E_2 = \frac{1}{2}\left(\sqrt{1 + 16 h(\lambda)^2 \sin^2 \tfrac{p}{2}} - 1\right) \to 2h_4 - 6 - 2(h_4 - 4) \cos p - 2 \cos 2p$$

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$\Rightarrow$ range three interactions determine $h_4 = -\frac{1}{2}\delta_{4,3} + 4$
$\Rightarrow$ range five interactions $\delta_{4,5} = -2$

Scheme independence of $h(\lambda)$

Bethe ansatz only depends on $h(\lambda)$, ABJM theory depends on $\lambda = \lambda(\mu)$

dilatation operator from $\mathcal{O}_a = Z_a{}^b \mathcal{O}_b^{\text{bare}}$

$$D(\lambda) = \mu \frac{d}{d\mu} \ln Z(\mu, \lambda)$$

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scheme change:

$$\lambda \to \lambda'(\lambda) = \lambda + a\lambda^3 + \dots$$

$$Z'(\mu, \lambda') = U(\lambda) Z(\mu, \lambda)(\mathbb{1}\lambda + A\lambda^2 + \dots) U(\lambda)^{-1}$$

in a conformal QFT: $\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \lambda = \beta(\lambda) = 0$

$$D'(\lambda') = U(\lambda) D(\lambda) U(\lambda)^{-1} + \beta(\lambda) \text{ rest} = U(\lambda) D(\lambda) U(\lambda)^{-1}$$

$\Rightarrow$ eigenvalues $\gamma(\lambda)$ of $D(\lambda)$ scheme independent

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$h(\lambda)$ fixed by matching $E(h(\lambda)) = \gamma(\lambda)$
$\Rightarrow$ $h(\lambda)$ scheme independent
coefficients of perturbative expansions w.r.t. λ and λ' can be different

$$h'(\lambda')^2 = \lambda'^2 + h_4'\lambda'^4 + \dots = \lambda^2 + h_4\lambda^4 + \dots = h(\lambda)^2 , \qquad h_4' = h_4 - 2a$$

Regularization and ϵ-tensors

Dimensional reduction:

consistent up to $\begin{cases} \text{3 loops:} & \text{pure CS} \\ \text{2 loops:} & \text{CS + matter} \end{cases}$

[Chen, Semenoff, Wu]
[Chen, Semenoff, Wu]
[Alves, Gomes, Pinheiro, da Silva]

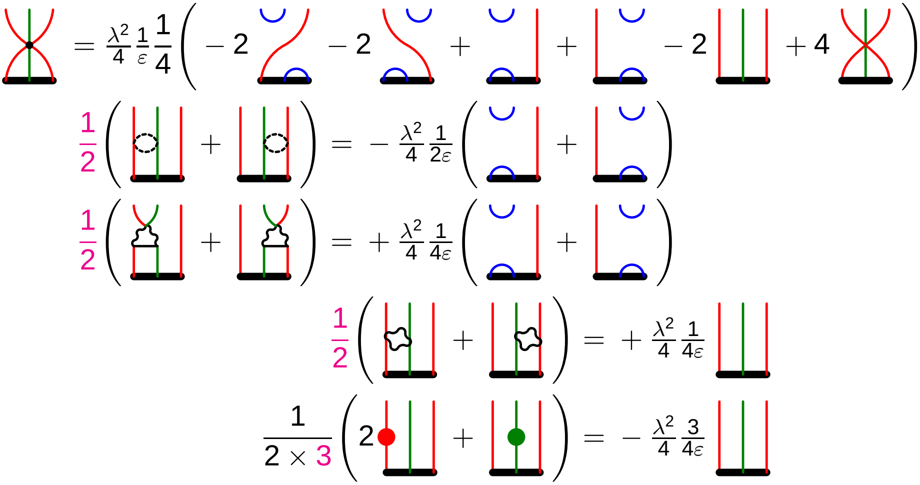
observation: in all appearing two- and four-loop integrals:

$$\text{number}(\epsilon_{\mu\nu\rho}) \quad + \quad \text{number}(\underbrace{\text{tr}(\gamma^{\mu_1} \dots \gamma^{\mu_{2n-1}})}_{\sim\epsilon^{\mu_i \mu_j \mu_k}}) \quad = \quad 2\,\mathbb{N}$$

expand product of even number of ϵ-tensors $\rightarrow$ scalar products
reduce dimensions in the integrals to $D = 3 - 2\varepsilon$

Field theory calculation: 2-loop warm-up

with symmetry factors to avoid overcounting:



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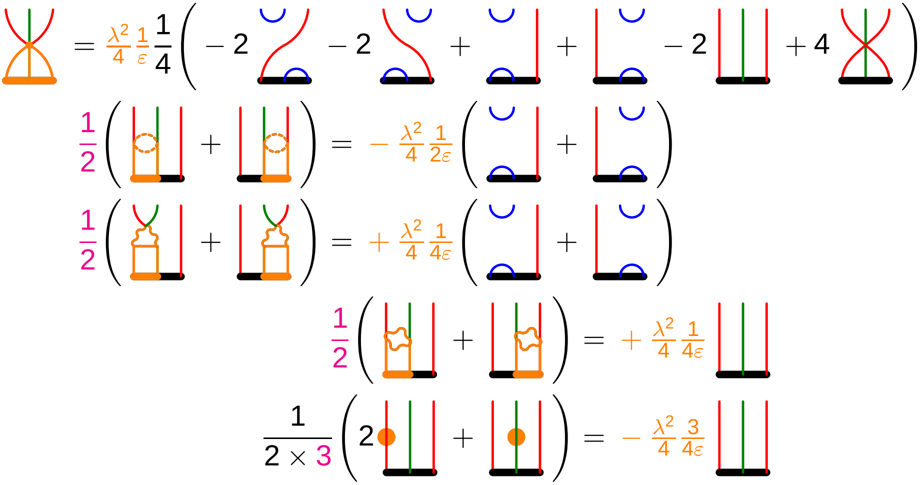
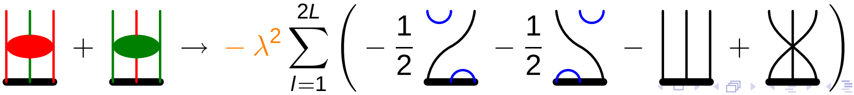


dilatation operator: above sum multiplied by -4ε



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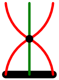
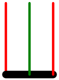
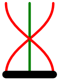
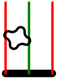
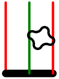
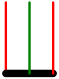
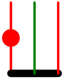
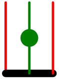
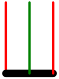
$SU(2) \times SU(2)$ sector:

 $= \frac{\lambda^2}{4}\frac{1}{\varepsilon}\frac{1}{4}\Bigg($ $- 2$  $+ 4$  $\Bigg)$

$\frac{1}{2}\Bigg($  $+$  $\Bigg) = + \frac{\lambda^2}{4}\frac{1}{4\varepsilon}$ 

$\frac{1}{2 \times 3}\Bigg(2$  $+$  $\Bigg) = - \frac{\lambda^2}{4}\frac{3}{4\varepsilon}$ 

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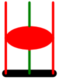
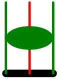
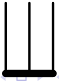
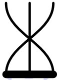
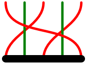
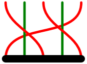
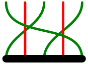
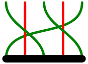
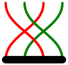
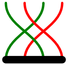
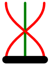
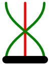
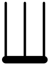
 $+$  $\rightarrow \; - \lambda^2 \sum_{l=1}^{2L} \Bigg($ $-$  $+$  $\Bigg)$

Structure of the four-loop dilatation operator

permutation structures

$$\{a_1, a_2, \ldots, a_m\} = \sum_{i=1}^{L} \mathrm{P}_{2i+a_1\; 2i+a_1+2}\, \mathrm{P}_{2i+a_2\; 2i+a_2+2} \cdots \mathrm{P}_{2i+a_m\; 2i+a_m+2}$$

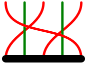
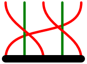
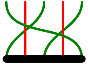
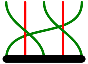
 $= \{1, 3\}$,  $= \{3, 1\}$,  $= \{2, 4\}$,  $= \{4, 2\}$

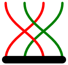
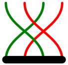
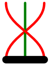
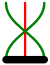
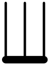
 $= \{1, 2\} =$  $= \{2, 1\}$,  $= \{1\}$,  $= \{2\}$,  $= \{\}$

Structure of the four-loop dilatation operator

permutation structures

$$\{a_1, a_2, \ldots, a_m\} = \sum_{i=1}^{L} \mathrm{P}_{2i+a_1\, 2i+a_1+2}\, \mathrm{P}_{2i+a_2\, 2i+a_2+2} \cdots \mathrm{P}_{2i+a_m\, 2i+a_m+2}$$

 $= \{1,3\}$,  $= \{3,1\}$,  $= \{2,4\}$,  $= \{4,2\}$

 $= \{1,2\} =$  $= \{2,1\}$,  $= \{1\}$,  $= \{2\}$,  $= \{\}$

ansatz: $D_4 = D_4 + D_4 + D_4$

$$D_4 = c_5(\{1,3\} + \{3,1\}) + c_3\{1\} + c_0\{\}$$
$$D_4 = c_5(\{2,4\} + \{4,2\}) + c_3\{2\} + c_0\{\} ,$$

$$D_4 = 2c_4\{1,2\}$$

fixed by $D_4\mathcal{O}_{2L,0} = 0$

$c_0 = -2c_5 - c_4 - c_3$

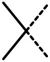
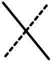
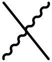
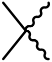
integrability:

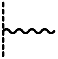
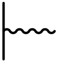
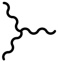
$c_4 = 0$

we only have to calculate structures with permutations

Vertex combinations at four loops

$V_{Y^6} =$ 

$V_{\psi^2 Y^2} =$  , $V_{\psi Y \psi Y} =$  , $V_{AYAY} =$  , $V_{A^2 Y^2} =$ 

$V_{\psi^2 A} =$  , $V_{Y^2 A} =$  , $V_{A^3} =$ 

$$(V_{Y^6})^2 \, ,$$

$$V_{Y^6}(V_{\psi^2 Y^2})^2 \, , V_{Y^6}(V_{A^2 Y^2})^2 \, ,$$

$$V_{Y^6} V_{\psi^2 Y^2} V_{\psi^2 A} V_{Y^2 A} \, , V_{Y^6} V_{AYAY} (V_{Y^2 A})^2 \, , V_{Y^6} V_{A^2 Y^2} (V_{Y^2 A})^2 \, , V_{Y^6} V_{A^2 Y^2} V_{Y^2 A} V_{A^3} \, ,$$

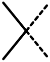
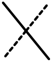
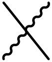
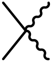
$$V_{Y^6}(V_{\psi^2 A})^2 (V_{Y^2 A})^2 \, , V_{Y^6}(V_{Y^2 A})^4 \, , V_{Y^6}(V_{Y^2 A})^3 V_{A^3} \, , V_{Y^6}(V_{Y^2 A})^2 (V_{A^3})^2 \, ,$$

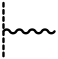
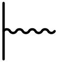
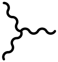
$$(V_{\psi Y \psi Y})^2 (V_{\psi^2 Y^2})^2 \, , (V_{\psi^2 Y^2})^4 \, ,$$

$$(V_{\psi^2 Y^2})^3 (V_{\psi^2 A})^2 \, , (V_{\psi^2 Y^2})^3 V_{\psi^2 A} V_{Y^2 A} \, , (V_{\psi^2 Y^2})^3 (V_{Y^2 A})^2 \, .$$

Vertex combinations at four loops

$V_{Y^6} =$ 

$V_{\psi^2 Y^2} =$  , $V_{\psi Y \psi Y} =$  , $V_{AYAY} =$  , $V_{A^2 Y^2} =$ 

$V_{\psi^2 A} =$  , $V_{Y^2 A} =$  , $V_{A^3} =$ 

range five, four, three

$$(V_{Y^6})^2 \,,$$

$$V_{Y^6}(V_{\psi^2 Y^2})^2 \,, V_{Y^6}(V_{A^2 Y^2})^2 \,,$$

$$V_{Y^6} V_{\psi^2 Y^2} V_{\psi^2 A} V_{Y^2 A} \,, V_{Y^6} V_{AYAY} (V_{Y^2 A})^2 \,, V_{Y^6} V_{A^2 Y^2} (V_{Y^2 A})^2 \,, V_{Y^6} V_{A^2 Y^2} V_{Y^2 A} V_{A^3} \,,$$

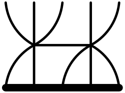
$$V_{Y^6}(V_{\psi^2 A})^2 (V_{Y^2 A})^2 \,, V_{Y^6}(V_{Y^2 A})^4 \,, V_{Y^6}(V_{Y^2 A})^3 V_{A^3} \,, V_{Y^6}(V_{Y^2 A})^2 (V_{A^3})^2 \,,$$

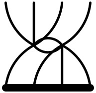
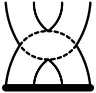
$$(V_{\psi Y \psi Y})^2 (V_{\psi^2 Y^2})^2 \,, (V_{\psi^2 Y^2})^4 \,,$$

$$(V_{\psi^2 Y^2})^3 (V_{\psi^2 A})^2 \,, (V_{\psi^2 Y^2})^3 V_{\psi^2 A} V_{Y^2 A} \,, (V_{\psi^2 Y^2})^3 (V_{Y^2 A})^2 \,.$$

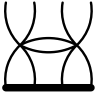
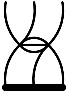
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+$ refl. $\to$ $(\{1,3\}+\{3,1\}-2\{1\})$,

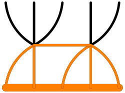
 $+$ refl. $+$  $\to$ $2(\{1,2\}-\{1\})$

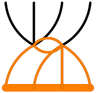
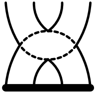
[Bak, Min, Rey]

 $\to$ $\frac{1}{2}\{1\}$,  $+$ refl. $\to$ $2(-\{1\})$

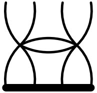
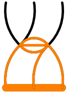
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+$ refl. $\rightarrow \quad (\{1,3\} + \{3,1\} - 2\{1\})$,

 $+$ refl. $+$  $\rightarrow \quad 2(\{1,2\} - \{1\})$

[Bak, Min, Rey]

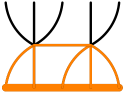
 $\rightarrow \quad \frac{1}{2}\{1\}$,  $+$ refl. $\rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

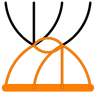
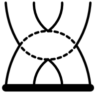
integrals:

 $=$

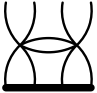
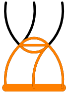
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

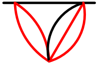
 $+\,\text{refl.} \rightarrow \quad (\{1,3\}+\{3,1\}-2\{1\})\ ,$

 $+\,\text{refl.}\,+$  $\rightarrow \quad 2(\{1,2\}-\{1\})$

(both lines above bracketed:) [Bak, Min, Rey]

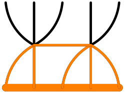
 $\rightarrow \quad \frac{1}{2}\{1\}\ ,$  $+\,\text{refl.} \rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

integrals:

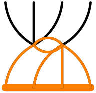
 $= G(1,1)^2$

Range five and four, complete $(V_{Y^6})^2$

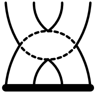
non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles



$+$ refl. $\rightarrow$ $(\{1,3\}+\{3,1\}-2\{1\})$,

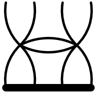


$+$ refl. $+$

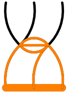


$\rightarrow$ $2(\{1,2\}-\{1\})$

[Bak, Min, Rey]

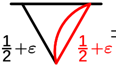


$\rightarrow$ $\frac{1}{2}\{1\}$,



$+$ refl. $\rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

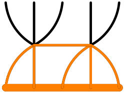
integrals:

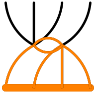
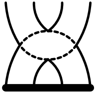


$= G(1,1)^2 G(\frac{1}{2}+\varepsilon,1)$

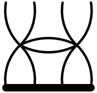
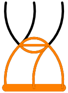
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

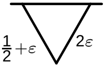
 $+$ refl. $\rightarrow$ $(\{1,3\}+\{3,1\}-2\{1\})$,

 $+$ refl. $+$  $\rightarrow$ $2(\{1,2\}-\{1\})$

[Bak, Min, Rey]

 $\rightarrow$ $\frac{1}{2}\{1\}$,  $+$ refl. $\rightarrow$ $\frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

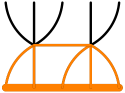
integrals:

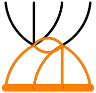
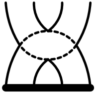


$$= G(1,1)^2 G(\tfrac{1}{2}+\varepsilon,1)$$

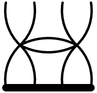
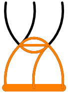
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+\,\text{refl.} \rightarrow \quad (\{1,3\}+\{3,1\}-2\{1\})\,,$

 $+\,\text{refl.}\,+$  $\rightarrow \quad 2(\{1,2\}-\{1\})$

[Bak, Min, Rey]

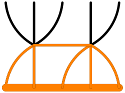
 $\rightarrow \quad \frac{1}{2}\{1\}\,,$  $+\,\text{refl.} \rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

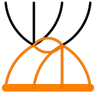
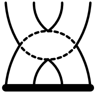
integrals:

$$= G(1,1)^2 G(\tfrac{1}{2}+\varepsilon,1)\,G(\tfrac{1}{2}+3\varepsilon,1)$$

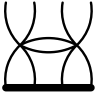
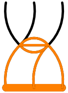
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+\,\text{refl.} \to \frac{\lambda^4}{16}\frac{2}{\varepsilon}(\{1,3\}+\{3,1\}-2\{1\})$,

 $+\,\text{refl.}\,+$  $\to \frac{\lambda^4}{16}\left(\frac{2}{\varepsilon}\qquad\right)2(\{1,2\}-\{1\})$

[Bak, Min, Rey]

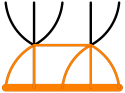
 $\to \qquad \frac{1}{2}\{1\}$,  $+\,\text{refl.} \to \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

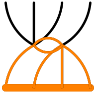
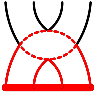
integrals:

 $= G(1,1)^2 G(\tfrac{1}{2}+\varepsilon,1)G(\tfrac{1}{2}+3\varepsilon,1) \to \frac{\lambda^4}{16}\left(-\frac{1}{2\varepsilon^2}+\frac{2}{\varepsilon}\right)$

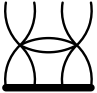
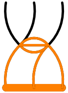
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+\,\text{refl.} \rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}(\{1,3\}+\{3,1\}-2\{1\})\,,$

 $+\,\text{refl.}\,+$  $\rightarrow \frac{\lambda^4}{16}\left(\frac{2}{\varepsilon}-\frac{2}{\varepsilon}\right)2(\{1,2\}-\{1\})$

[Bak, Min, Rey]

 $\rightarrow \quad \frac{1}{2}\{1\}\,,$  $+\,\text{refl.} \rightarrow \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

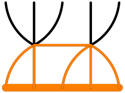
integrals:

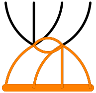
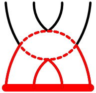
 $= G(1,1)^2 G(\tfrac{1}{2}+\varepsilon,1)G(\tfrac{1}{2}+3\varepsilon,1) \rightarrow \frac{\lambda^4}{16}\left(-\frac{1}{2\varepsilon^2}+\frac{2}{\varepsilon}\right)$

 $= G(1,1)G(\tfrac{1}{2}+3\varepsilon,1)$  $_{\frac{1}{2}+\varepsilon} \rightarrow \frac{\lambda^4}{16}\left(-\frac{2}{\varepsilon}\right)$

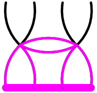
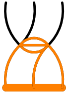
Range five and four, complete $(V_{Y^6})^2$

non-trace non-identity contributions, only $\frac{1}{\varepsilon}$ poles

 $+\,\text{refl.} \to \frac{\lambda^4}{16}\frac{2}{\varepsilon}(\{1,3\}+\{3,1\}-2\{1\})$,

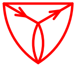
 $+\,\text{refl.}\,+$  $\to \frac{\lambda^4}{16}\left(\frac{2}{\varepsilon}-\frac{2}{\varepsilon}\right)2(\{1,2\}-\{1\})$

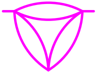
[Bak, Min, Rey]

 $\to \frac{\lambda^4}{16}\frac{1}{4\varepsilon}\frac{1}{2}\{1\}$,  $+\,\text{refl.} \to \frac{\lambda^4}{16}\frac{2}{\varepsilon}2(-\{1\})$

integrals:

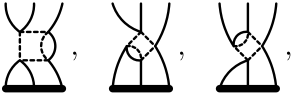
 $= G(1,1)^2G(\tfrac{1}{2}+\varepsilon,1)G(\tfrac{1}{2}+3\varepsilon,1) \to \frac{\lambda^4}{16}\left(-\frac{1}{2\varepsilon^2}+\frac{2}{\varepsilon}\right)$

 $= G(1,1)G(\tfrac{1}{2}+3\varepsilon,1)$  $_{\frac{1}{2}+\varepsilon} \to \frac{\lambda^4}{16}\left(-\frac{2}{\varepsilon}\right)$

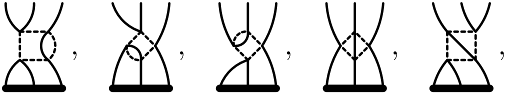
 $= G(1,1)^3G(\tfrac{1}{2}+\varepsilon,1+2\varepsilon) \to \frac{\lambda^4}{16}\frac{3}{\varepsilon}\zeta(2)$

Range three interactions

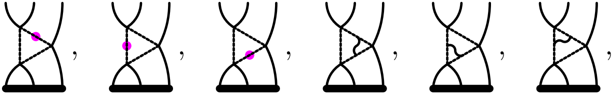
$(V_{\psi^2 Y^2})^4$:



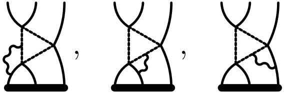
$(V_{\psi Y \psi Y})^2 (V_{\psi^2 Y^2})^2$:

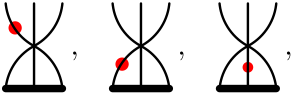


$(V_{\psi^2 Y^2})^3 (V_{\psi^2 A})^2$:

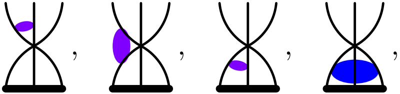


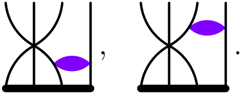
$(V_{\psi^2 Y^2})^3 V_{\psi^2 A} V_{Y^2 A}$:





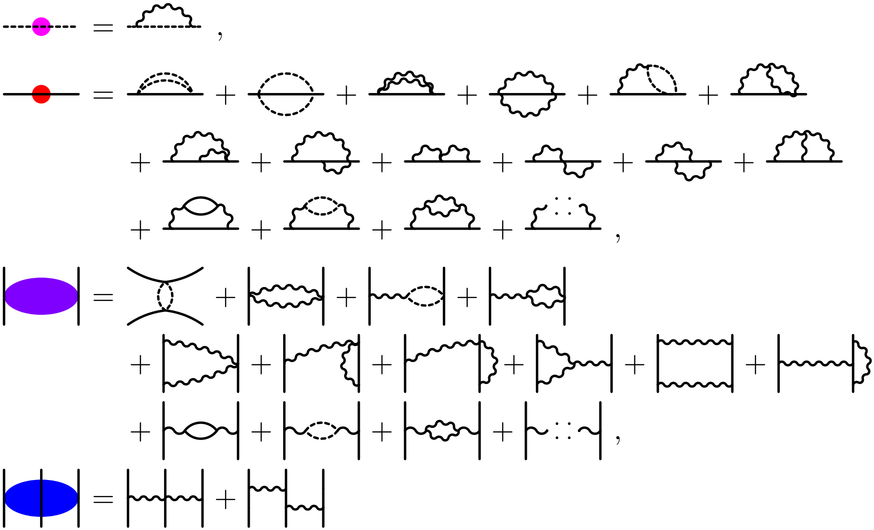
$(V_{Y^6})X$:



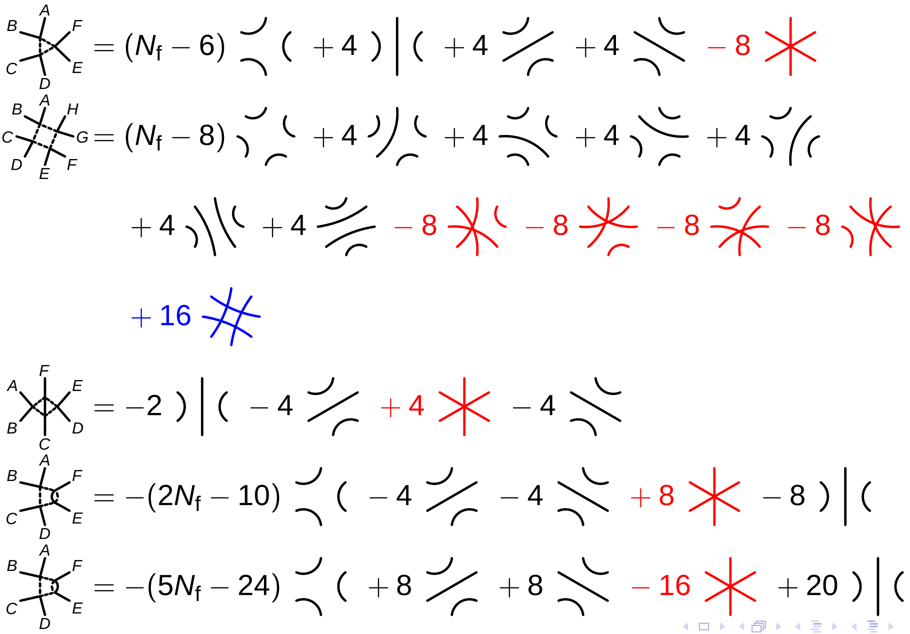


Substructures

Up to reflections:



Flavour structures

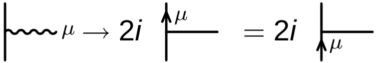


Effective Feynman rules

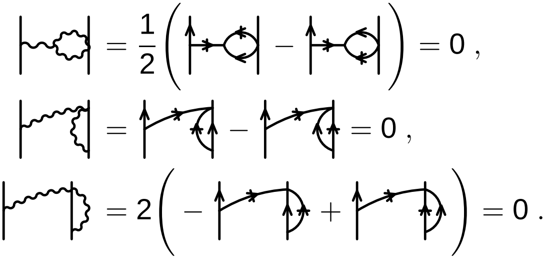
gauge boson propagators have numerator with ϵ-tensor

$$\langle A_\alpha(p) A_\gamma(p)\rangle = -\langle \hat{A}_\alpha(p)\hat{A}_\gamma(p)\rangle = -\frac{2\pi}{k}\frac{1}{p^2}\epsilon_{\alpha\beta\gamma}p^\beta$$

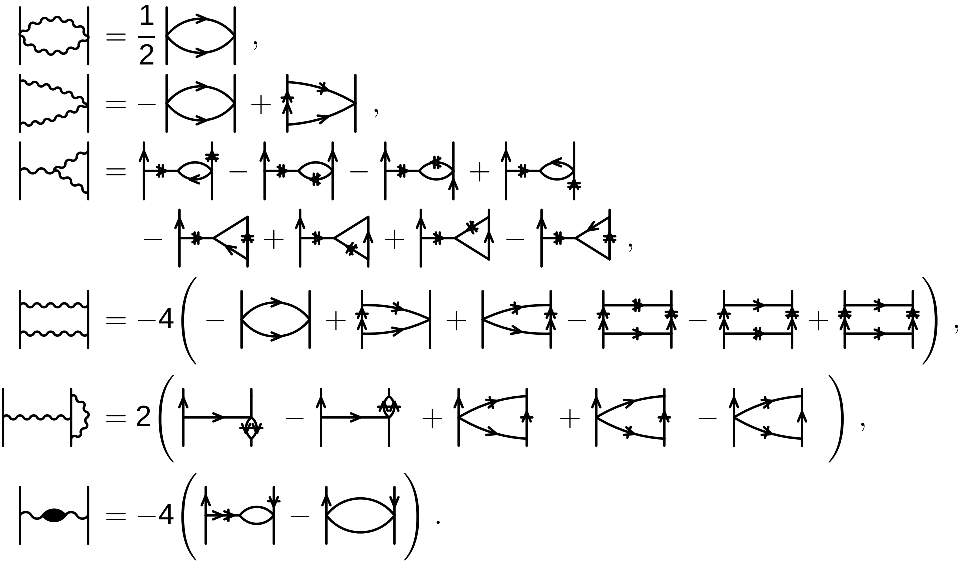
$\Rightarrow$ effective Feynman rules



allow for manipulations directly on the graphs



Effective Feynman rules



From ABJM to ABJ

gauge groups: $U(N) \times \overline{U(N)} \to U(M) \times \overline{U(N)}$

't Hooft coupling constants: $\lambda = \frac{M}{k}$ and $\bar{\lambda} = \frac{N}{k}$

two-loops: $\lambda^2 \to \lambda\bar{\lambda}$ [Bak, Gang, Rey]

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two loops with a common matter propagator: $\to \lambda\bar{\lambda}$

trafo rule:

loop with a A or $\hat{A}$ gauge boson propagator: $\to \begin{cases} \lambda & \langle AA \rangle \\ \bar{\lambda} & \langle \hat{A}\hat{A} \rangle \end{cases}$

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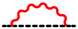
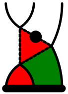
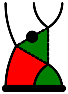
finite one-loop fermion wave function renormalization (parity breaking):

 $=$  $+$  $= i(M-N)$ 

$\Rightarrow$ log. divergent four-loop fermion triangle graphs ($V_{\psi^2 Y^2}$ parity breaking):

 $= (M-N)M^2N\frac{1}{4}$ 

 $= -(M-N)MN^2\frac{1}{4}$ 

$\rightarrow \frac{1}{4}(M-N)^2 MN \frac{1}{16k}\left(-\frac{1}{4\varepsilon^2}\right)$

$\Rightarrow$ parity conserving interaction

Symmetries

transformation of a Feynman graph by reflection and shifts *odd* $\leftrightarrow$ *even*

$$c(M,N)\{a_1,\ldots,a_m\} \to (-1)^{P_{A^2}+V_{AY^2}+V_{A\psi^2}+V_{Y^2\psi^2}} c(N,M)\{a_m,\ldots,a_1\}$$

$$c(M,N)\{a_1,\ldots,a_m\} \to (-1)^{P_{A^2}+V_{A^3}+V_{A\psi^2}+V_{Y^2\psi^2}} c(N,M)\{a_1+1,\ldots,a_m+1\}$$

P_x, V_x: number of propagators, vertices of type x

Symmetries

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P_x, V_x: number of propagators, vertices of type x

four-loop graphs do not acquire signs

$$m \le 1 : c(M,N) = c(N,M)$$

$$m \ge 2 : \begin{array}{ll} \text{reflection:} & c(M,N) = c(N,M) \\ \text{shift:} & c(M,N) = c(N,M) \end{array}$$

Symmetries

transformation of a Feynman graph by reflection and shifts *odd* $\leftrightarrow$ *even*

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four-loop graphs do not acquire signs

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symmetries of the dilatation operator: $D_4 = D_4 + D_4$

$$D_4 = c_5(\{1,3\}+\{3,1\}) + c_3\{1\} + c_0\{\}$$

$$D_4 = c_5(\{2,4\}+\{4,2\}) + c_3\{2\} + c_0\{\}$$

$$c_i(M,N) = c_i(N,M)$$

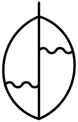
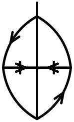
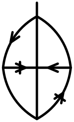
$$\Rightarrow h(\lambda,\bar{\lambda}) = h(\bar{\lambda},\lambda)$$

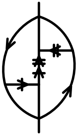
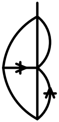
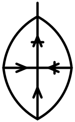
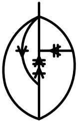
What made us suffer

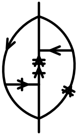
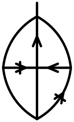
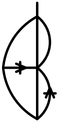
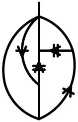
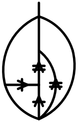
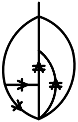
gauge boson propagators have numerator with ϵ-tensor:

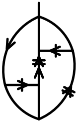
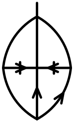
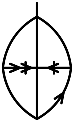
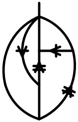
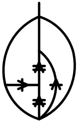
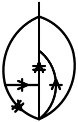
$\Rightarrow$ contracted momenta can be far apart in the graph of the integral: completion of squares of momenta hard to apply

$\Rightarrow$ diagrams with gauge fields decompose into many scalar diagrams

$I_{4gl} =$  $= 4\Big($  $-$  $- I_{4222l1} + 2I_{4222l2} - I_{4222l3}\Big)$,

$I_{4222l1} =$  $= \frac{1}{2}\Big($  $+ 2$  $-$  $+ 2$  $\Big)$,

$I_{4222l2} =$  $= \frac{1}{2}\Big($  $-$  $+$  $+$  $-$  $\Big)$,

$I_{4222l3} =$  $= \frac{1}{2}\Big($  $+$  $+$  $+$  $-$  $\Big)$.

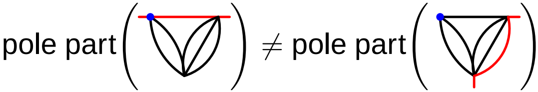
What made us suffer

gauge boson propagators have numerator with ϵ-tensor:

$\Rightarrow$ contracted momenta can be far apart in the graph of the integral: completion of squares of momenta hard to apply

$\Rightarrow$ diagrams with gauge fields decompose into many scalar diagrams

simple cubic vertices in $D = 3$ are not infrared safe:



$\Rightarrow$ cubic vertices become quartic vertices due to external momentum
integration by parts requires cubic vertices
completion of squares hard to apply

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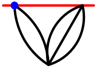
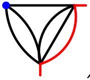
pole part $\Bigg($  $\Bigg) \neq$ pole part $\Bigg($  $\Bigg)$

$\Rightarrow$ cubic vertices become quartic vertices due to external momentum
integration by parts requires cubic vertices
completion of squares hard to apply

integrals with odd number of loops give half-integer propagator weights

$\Rightarrow$ uniqueness and integration by parts methods hard to apply

Status of the calculation

- all diagrams have been calculated
- $\sim$ 160 two-, three- and four-loop integrals have been computed: using integration by parts, conformal inversion, sewing and cutting, Gegenbauer polynomial x-space techniques
- several relations between integrals have been checked
- consistency with known two-loop and maximal range four-loop results
- cancellation of $\frac{1}{\varepsilon^2}$ poles in $\ln Z$ to be checked
- sign of the fermion triangle graph to be fixed by computing the two-loop vertex renormalization of V_{Y^6}
- everything to be rechecked and written up
- proposal of simple rational function [Gromov, Vieira] almost certainly not correct for $h(\lambda)^2 = \lambda^2 + \lambda^4 h_4$

$$h_4 = a + b\,\zeta(2) < 0\,, \qquad a, b \in \mathbb{Z}$$

  $\zeta(2)$ appears already in simple four-loop integrals
- no parity breaking effects $h(\lambda, \bar{\lambda}) = h(\bar{\lambda}, \lambda)$