Integrability in Gauge and String Theory: a Preview

why this conference?
why integrability?
status and outstanding issues
General aims:
• understand quantum gauge theories at any coupling
  [applications to both perturbative and non-perturbative issues]
• understand string theories in non-trivial backgrounds
  [e.g. RR ones for flux compactifications]

AdS/CFT duality:
• relates the two questions suggesting solving them together rather than separately is best strategy
• relates simplest most symmetric theories
use of symmetries on both sides to make progress

Integrability:
Existence of powerful hidden symmetries allowing to solve problem “in principle”
Strategy:
solve simplest most symmetric ("harmonic oscillator") case
then hope to treat other cases "in perturbation theory"

"Harmonic oscillator" (or "Ising", or "WZW"):
planar $\mathcal{N} = 4$ SYM theory = free superstring in $AdS_5 \times S^5$
most symmetric 4-d gauge th. = most symmetric 10-d string th.

$\mathcal{N} = 4$ SYM:
• maximal supersymmetry; conformal invariance;
• integrability? its precise meaning? in which observables?
could be expected in anomalous dimensions
[1-loop gluonic sector – known emergence of XXX spin chain]
• in fact, $\infty$ of hidden symmetries should play broader role:
  "inherited" via AdS/CFT from 2-d integrable QFT –
string $\sigma$-model: use 2-d int. QFT to solve 4-d CFT
Superstring in $AdS_5 \times S^5$:

- integrable in “canonical” sense: sigma-model on symmetric space
  classical equations admit infinite number of conserved charges
  closely related (via Pohlmeyer reduction) to
  (super) sine-Gordon and non-abelian Toda eqs
  e.g. special motions of strings are described by
  the integrable 1-d mechanical systems (Neumann, etc.)

- integrability extends to quantum level:
  evidence directly on string-theory side to 2 loops
  and also indirectly via AdS/CFT “bootstrap” reasoning

Quantum integrability: should control

- spectrum of string energies on $R \times S^1$
  [anom. dim’s of 2-d primary operators = vertex operators on $R^{1,1}$]
- correlation functions of vertex operators (to which extent?)*
  [closed-string scattering amplitudes]
  * cf. flat space; string field theory is not “integrable”
What about open-string sector?
Wilson loops (= disc partition functions)?
definition of “gluon scattering amplitudes”
beyond leading strong-coupling order?

Integrability = hidden infinite dimensional symmetry
– if valid in quantum string theory –
i.e. at any value of string tension $\frac{\sqrt{\lambda}}{2\pi} - \text{any } \lambda = g_{YM}^2 N_c$
should be “visible” also – via AdS/CFT – in
perturbative SYM theory
Integrability should then control:
• spectrum of dimensions of gauge-inv. single tr primary operators
[or spectrum of gauge-theory energies on $R \times S^3$]
• correlation functions of these operators (to which extent?)
What about scattering amplitudes and Wilson loops?
Amplitudes – IR divergent; Cusped WL’s – UV divergent
Hidden (Yangian) symmetries broken at loop level in a “useful” way?

Are there “better” observables? (from integrability point of view)
Cross-sections? Effective actions?
Relation to correlation functions of gauge-inv. ops.?
Hints from string theory?
Recent remarkable progress:

Spectrum of states

I. Spectrum of “long” operators = “semiclassical” string states
determined by Asymptotic Bethe Ansatz (2002-2007)
• its final (BES) form found after intricate superposition
  of information from perturbative gauge theory (spin chain, BA,...)
  and perturbative string theory (classical and 1-loop phase,...),
  use of symmetries (S-matrix), and assumption of exact integrability
• consequences checked against all available gauge and string data

Key example: cusp anomalous dimension $\text{Tr}(\Phi D^S \Phi)$

\[
\begin{align*}
  f(\lambda \ll 1) &= \frac{\lambda}{2\pi^2} \left[ 1 - \frac{\lambda}{48} + \frac{11\lambda^2}{2^8 \cdot 45} - \left( \frac{73}{630} + \frac{4(\zeta(3))^2}{\pi^6} \right) \frac{\lambda^3}{2^7} + ... \right] \\
  f(\lambda \gg 1) &= \frac{\sqrt{\lambda}}{\pi} \left[ 1 - \frac{3 \ln 2}{\sqrt{\lambda}} - \frac{K}{(\sqrt{\lambda})^2} - ... \right]
\end{align*}
\]

Extensions to subleading terms in large $S$ expansion

[Related talks: Dorey, Freyhult]
II. Spectrum of “short” operators = all quantum string states
Thermodynamic Bethe Ansatz (2005-2009)
• reconstructed from ABA using solely methods/intuition of 2-d integrable QFT, i.e. string-theory side
( how to incorporate wrapping terms directly on gauge-theory side?)
• highly non-trivial construction – lack of 2-d Lorentz invariance in the standard “BMN-vacuum-adapted” l.c. gauge
• in few cases ABA “improved” by Luscher corrections is enough: 4- and 5-loop Konishi dimension, 4-loop minimal twist op. dimension
• crucial to check predictions against perturbative gauge and string data

Key example: anomalous dimension of Konishi operator

\[
\begin{align*}
\gamma(\lambda \ll 1) &= \frac{12\lambda}{(4\pi)^2} \left[ 1 - \frac{4\lambda}{(4\pi)^2} + \frac{28\lambda^2}{(4\pi)^4} 
- [208 - 48\zeta(3) + 120\zeta(5)] \frac{\lambda^3}{(4\pi)^6} + \ldots \right] \\
\gamma(\lambda \gg 1) &= 2 \frac{4\lambda}{\sqrt[4]{\lambda}} + b_0 + \frac{b_1}{\sqrt[4]{\lambda}} + \frac{b_2}{(\sqrt[4]{\lambda})^2} + \frac{b_3}{(\sqrt[4]{\lambda})^3} + \ldots
\end{align*}
\]

Related talks: Banjok, Frolov, Gromov, Janik, Kazakov, Roiban; Torrielli
Many open questions:
Analytic form of strong-coupling expansion from TBA/Y-system?
Matching onto string spectrum in near-flat-space expansion?
No level crossing?
Strong-coupling expansion is Borel (non)summable?
Exponential corrections $e^{-a\sqrt{\lambda}}$ like in cusp anomaly case?
...
We are just at the beginning of understanding of structure of spectrum

Deeper issues:
Solve string theory from first principles –
• fundamental variables? preserve 2-d Lorentz invariance?
• prove quantum integrability?
lattice version of “supercoset” sigma model?
Another remarkable recent progress:
Amplitudes, Wilson loops and their symmetries

Weak coupling:
various connections to hidden symmetries and integrability
[talks by Arkani-Hamed, Korchemsky, Lipatov, McLoughlin, Plefka]

Strong coupling:
use of integrability of string theory to determine (via relation to WL’s)
leading contributions to certain gluon scattering amplitudes
[talk by Maldacena]
Extensions and generalizations:

- $\mathcal{N} = 6$ supersymmetric 3-d Chern-Simons-matter theory dual to superstring in $AdS_4 \times CP^3$
  [talks by Nepomechie, Rey, Sieg; Zarembo ]
- integrability methods applied to other
  $d = 4$ supersymmetric gauge theories and other quantities
  [talks by Gaiotto; Shatashvili ]
- Integrability in interesting related systems:
  [talks by Bazhanov; Bobenko; Saleur]
- Mathematical properties of related perturbation-theory coefficients:
  [talk by Vermaseren]

We are going to hear about many new exciting developments,
see closely related interesting posters,
and have stimulating discussions!