String Junctions, Abelian Fibrations and Flux-Geometry Duality

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Based on work with R. Donagi and M. B. Schulz, arXiv:0808.ABCD.

Overview

- ✔ IIB $\mathrm{T}^6/\mathbb{Z}_2$ orientifold w. $\mathcal{N}=2$ flux $\equiv$ IIA CY duals with no flux. Goal: Construct the dual manifolds explicitly
  - ✘ Many properties deduced by classical sugra dualities (Schulz [hep-th/0412270])
  - ✘ We have found two explicit constructions:
    - ✓ Monodromy/string-junction description

      analogous to F-theory description of K3, but with T^4 rather than T^2 fibers.
    - ✓ Explicit algebro-geometric construction via relative Jacobian of genus-2 fibered surface.
- ✔ Relation of CYs to one another? Construction of new CYs.

Motivation

Motivations

- ✔ Fate of non-perturbative dualities in the presence of flux.
  Example of open-closed, strong-weak (& RR-NS) duality.

- ✔ IIB $\mathrm{T}^6/\mathbb{Z}_2$ orientifold one of the simplest IIB flux compactifications
  (e.g., Kachru et.al. [hep-th/0201028]).
  May still lead to insight on flux vacua duality in general.

- ✔ IIA CY duals $X_{m,n}$ have $\pi_1 = \mathbb{Z}_n \times \mathbb{Z}_n$ w. $n = 1, 2, 3, 4$.
  $\Rightarrow$ useful for Heterotic phenomenology.
  Few CYs with nontrivial π_1 are known (work in progress by Donagi, Saito.).

- ✔ D3 instantons dualize to WS instantons wrapping $\mathbb{P}^1$ sections.

  $\Rightarrow$ Exact results on D-instantons w. background flux & O-planes.
  (work in progress with Schulz.)

More motivations

- ✔ Studying the moduli space of CY duals in IIA$\Rightarrow$ Can in principle deduce warped KK reduction of the flux compactification in IIB. (e.g., Douglas et.al. [0805.3700])
- ✔ Connection to D(imensional)-duality? (via relative Jacobian of second construction for CYs). (Silverstein; Green et.al.)

Supergravity analysis

Chasing the duality chain

- ✔ Snapshot: 3 T-duals (IIB→IIA) and M-theory lift & drop
  - ✘ Starting point: Warped product $M_4 \times_w \mathrm{T}^6/\mathbb{Z}_2$ w. D3/O3.
    - ✓ Finer structure: $\mathrm{T}^6 = \mathrm{T^2}_{(1)} \times \mathrm{T}^4$, T^4 is a $\mathrm{T^2}_{(2)}$ fiber. over $\mathrm{T^2}_{(3)}$ base w. flat connections.
  - ✘ Step one: T-dualize along $S^1 \subset \mathrm{T^2}_{(1)}$ and $\mathrm{T^2}_{(2)}$, result in: $(M_4 \times \mathrm{T}^3_{\mathrm{fib}}) \times_w \mathrm{T}^3_{\mathrm{base}}/\mathbb{Z}_2$ w. D6/O6 (IIA).
    - ✓ Fate of NS flux: $H_3 \to$ 1st Chern class of dual fibration $\widetilde{\mathrm{T}^2}_{(2)} \subset \mathrm{T}^3_{\mathrm{fib}} \propto n$.
    - ✓ Fate of RR flux: $F_3 \to F_2 = dC_1$ captures the distribution of D6/O6 and curvature ($\propto m$) over $\mathrm{T}^3_{\mathrm{base}}$.
    - ✓ Note: non-trivial dilaton profile, as is generic in T-dualizing.
  - ✘ Step two: Lift to M-theory, result in $M_4 \times {S^1}_{(1)} \times \mathrm{CY3}$ w. $\mathrm{CY3} = ((S^1{}_{10} \rtimes_w \widetilde{\mathrm{T}^2}_{(2)} \ltimes_{w'} S^1{}_{(1)})_{\sim \mathrm{T}^4} \times_{w'} \mathrm{T^2}_{(3)}/\mathbb{Z}_2)$
    - ✓ **Purely** geometric: C_1 identified as $\mathcal{A}_{10}$, D6/O6 → TN/GH. (color conservation 123. $\xrightarrow{\text{step3}} \mathrm{IIA}'$)

Properties of $X_{m,n}$

We can learn the following additional information:

- ✔ Abelian surface (T^4) fibration over $\mathbb{P}^1$,
  has $8+N$ singular fibers of nodal type,
  $N =$ number of D3-branes in $\mathrm{T}^6/\mathbb{Z}_2$.
- ✔ Hodge # of $X_{m,n}$: $h^{11} = h^{21} = N+2, \;\; N + 4mn = 16$.
  Follows from massless spectrum, including open string moduli
  $F_3 \sim 2m, \; H_3 \sim 2n, \; N_{\mathrm{D3}} + \int H \wedge F = \frac{1}{4} N_{\mathrm{O3}}$ in IIB.
- ✔ Generic D_N lattice of sections (mod torsion)
  Follows from N D-branes + O-plane giving rise to $SO(2N)$.
- ✔ Fundamental group and discrete isometries
  $\pi_1 = \mathbb{Z}_n \times \mathbb{Z}_n, \quad$ isometry $= \mathbb{Z}_m \times \mathbb{Z}_m$.
  For flux $m, n \neq 1$, partial higgsing of $U(1)$s in IIB.

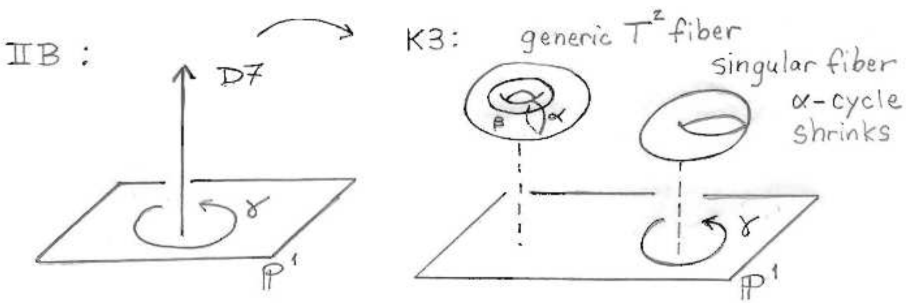
Properties of $X_{m,n}$

- ✔ Approximate metric, harmonic forms, recall isometry along M-th circ. (small parameter = fiber/base $\propto R^{11}$).
- ✔ Polarization: $J_{\text{fiber}} \propto m dy^1 \wedge dy^2 + n dy^3 \wedge dy^4$.
- ✔ Non-vanishing triple intersections:
  $H^2 \cdot A = 2mn, \quad H \cdot \mathcal{E}_I \cdot \mathcal{E}_J = -m\delta_{IJ}$
  Computed using explicit (approximate) harmonic forms.
- ✔ $H \cdot c_2 = 8 + N$, and esp. $\chi(A) = A \cdot c_2 = 0 \longrightarrow$ Abelian surface fibration (Oguiso).

  Follows from $F_1 = \sum_{\alpha=1}^{h^{1,1}(X)} (D_\alpha \cdot c_2) t^\alpha \sim (N+8)\tau_{\text{dil}}$ (Dasgupta et. al.) and $g_s^{\text{IIB}} \longrightarrow J_A$ in IIA CY dual.

Warm-up: IIB on $\mathrm{T}^2/\mathbb{Z}_2$

Recall IIB encoding of elliptic fibration over $\mathbb{P}^1$ (e.g., K3):



IIB: 7-brane

- ✔ $\oint_\gamma F_1 = 1$ unit RR charge $\Rightarrow$ monodromy $\tau_{\text{dil}} \to \tau_{\text{dil}} + 1$
- ✔ (p,q) 7-brane = where (p,q)-string ends, e.g. D7 brane=(1,0) 7-brane.

F-theory: singular elliptic fiber

- ✔ $\tau =$ cplx mod. of T^2 fiber, $\tau \to \tau + 1$ about γ
- ✔ $a\alpha + b\beta$ cycle in T^2: $\binom{a}{b} \to K\binom{a}{b}$, $K = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ monodromy matrix.
- ✔ $p\alpha + q\beta$ (instead of α) cycle shrinks: $K_{[p,q]} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$.
- ✔ Other cplx structure moduli∼7-brane moduli.

Monodromy description

- ✔ Let (p, q) charges $A = (1, 0)$, $B = (1, -1)$ and $C = (1, 1)$.
- ✔ Perturbative description of $\mathrm{T}^2/\mathbb{Z}_2$ orientifold: 16 D7s + 4 O7s. Overall (local) monodromy$\Rightarrow K_{\mathrm{O7}} = -K_A^{-4}$
- ✔ Nonperturbative description: each O7 resolves to BC pair.(Sen) Up to equivalences K_{O7} factorizes uniquely into $(K_{[1,1]}K_{[1,-1]})$.
- ✔ So, F-theory on the manifold K3: Base $\mathbb{P}^1 \cong \mathrm{T}^2/\mathbb{Z}_2$, 24 singular fibers $A^{16}(BC)^4$, with monodromies

$$K_A = \begin{pmatrix} 1 & -1 \\ & 1 \end{pmatrix}, \quad K_B = \begin{pmatrix} & -1 \\ 1 & 2 \end{pmatrix}, \quad K_C = \begin{pmatrix} 2 & -1 \\ 1 & \end{pmatrix}.$$

- ✔ These nonperturbative IIB data define the topology of K3.

Construction I: Monodromy of singular fibers

CY duals $X_{m,n}$ are T^4 fibration over $\mathbb{P}^1$. But Why?

- ✔ Another point of view:
  - ✘ No flux: $(m = n = 0)$

$$\mathrm{T}^6/\mathbb{Z}_2 \text{ orientifold} \quad \leftrightarrow \quad \text{IIA on } \mathrm{K3} \times \mathrm{T}^2$$
$$(\mathrm{K3} = \mathrm{T}^2 \text{ fibration over } \mathbb{P}^1)$$

(both dual to type I or het-SO on T^6).

  - ✘ With $\mathcal{N} = 2$ flux $F_3 \sim 2m, H_3 \sim 2n$:

$$\mathrm{T}^6/\mathbb{Z}_2 \text{ orientifold} \quad \leftrightarrow \quad \text{IIA on CY } X_{m,n}$$
$$(X_{m,n} = \mathrm{T}^4 \text{ fibration over } \mathbb{P}^1)$$

- ✔ Rougly flux induces twists mixing T^2 factor with T^2 fiber of K3.

Monodromy for T^4 fibers

N D3s + O3s of $\mathrm{T}^6/\mathbb{Z}_2$ $\leftrightarrow$ $A^N B_1 C_1 B_2 C_2 B_3 C_3 B_4 C_4$ singular T^4 fibers of $X_{m,n}$.

$$K_A = \left(\begin{array}{cc|cc} 1 & -1 & & \\ & 1 & & \\ \hline & & 1 & \\ & & & 1 \end{array}\right) = (\text{old } K_A) \oplus (\text{identity}) \text{ on } \mathrm{T}^2 \times \mathrm{T}^2,$$

but B_i, C_i differ for $i = 1, 2, 3, 4$. For example,

$$K_{B_1} = \left(\begin{array}{cc|cc} & -1 & & -m \\ 1 & 2 & & m \\ \hline -n & -n & 1 & -m \\ & & & 1 \end{array}\right) = (\text{old } K_B) \oplus (\text{identity}) \text{ on } \mathrm{T}^2 \times \mathrm{T}^2 + m, n \text{ twists.}$$

The monodromies uniquely determine the topology of $X_{m,n}$.

Dual interpretation of RR tadpole

- ✔ On the base of $X_{m,n}$, a $\mathbb{P}^1$, the loop that encloses all singular fibers is contractible (to the point at '∞').

  $\Rightarrow$ Total monodromy must be unity:

$$
\begin{aligned}
1 &= K_{\text{total}} \\
&= K_{C_4} K_{B_4} \dots K_{C_1} K_{B_1} {K_A}^N \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 \\ & 1 & -Q & 0 \\ & & 1 & 0 \\ & & & 1 \end{pmatrix},
\end{aligned}
$$

  where $Q = N - 16 + 4mn$.

- ✔ Purely topological constraint reproduces $\mathrm{T}^6/\mathbb{Z}_2$ D3 charge condition $Q = 0$.

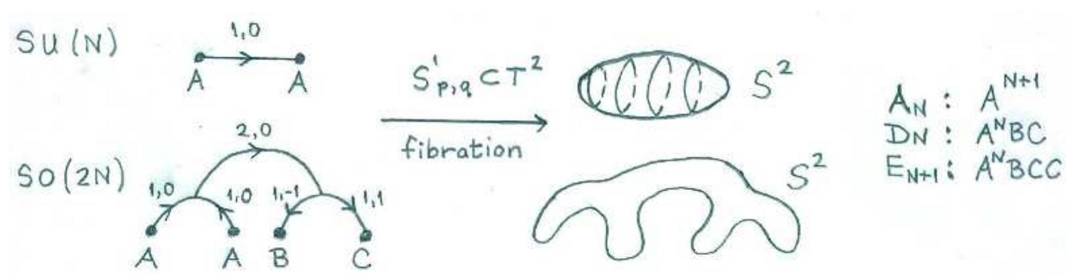
Construction I: (contd.) String-junctions

String junctions & Mordell-Weil lattice

String junctions:

(Sen; Gaberdiel et.al. ;DeWolfe...)

- ✔ are W-bosons of 7-brane gauge theory,
- ✔ encode homology of F-theory elliptic fibration,
- ✔ equivalence classes (charges) form a lattice.



$H_2(S)$ generated by:

- ✔ generic fiber, $\leftarrow H^0(\mathbb{P}^1, R^2\pi_*\mathbb{Z}) \rightarrow$
- ✔ irred. components of singular fibers; (Kodaira)
- ✔ sections. $\leftarrow$ string junctions, $H^1(\mathbb{P}^1, R^1\pi_*\mathbb{Z})$

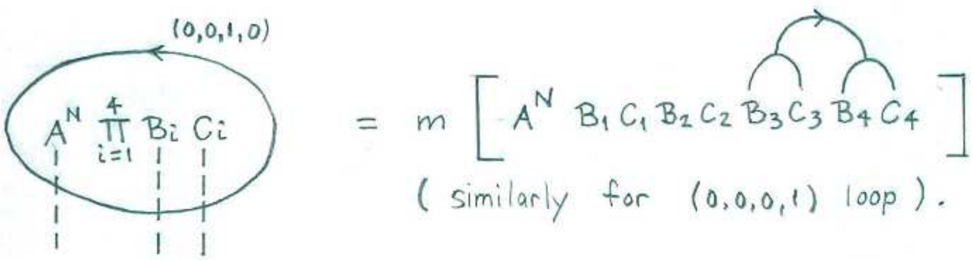
Mordell-Weil lattice of sections = junction lattice/null loops (Fukae et al.).

MW and junction lattice for $X_{m,n}$

- ✔ In CY $X_{m,n}$: a (p,q,r,s) 1-cycle in T^4 fiber shrinks at each A, B_i, C_i on $\mathbb{P}^1$.
- ✔ Obtain 2-cycles in $X_{m,n}$ from $S^1_{[p,q,r,s]} \sim S^1_{10}$ fibration over (p,q,r,s) junction graphs in base $\mathbb{P}^1$. (further leads to θ-divisors.)
- ✔ Again, MW lattice of (rational) sections = junction lattice/null loops.

  $A^N \prod_{i=1}^4 B_i C_i \quad \Rightarrow$ Again D_N from $A^N B_i C_i \quad (A+A = B_i + C_i)$ but NOT E_{N+1} from $A^N B_i C_i C_j \quad (C_i \neq C_j)$.

  D_N = free part of MW lattice.



- ✔ $\mathbb{Z}_m \times \mathbb{Z}_m$ = torsion part of MW lattice = isometry group.

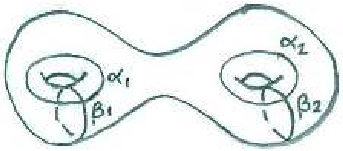
Relations between CYs

- ✔ $N + 4mn = 16$. Complete set of 8 $X_{m,n}$ is $\{X_{1,1}, X_{m,1}, X_{1,n}, X_{2,2}\}$.
- ✔ Relations:
  - ✘ IIB S-duality $H_3 \leftrightarrow F_3$ imples $X_{m,n} \leftrightarrow X_{n,m}$ via fiberwise T-dualizing T^4, $X_{1,1}$,$X_{2,2}$ invariant.
  - ✘ Topologically $X_{m,1}/(\mathbb{Z}_m \times \mathbb{Z}_m) = X_{1,m}$ Discrete isometry $\leftrightarrow$ non-trivial π_1
  - ✘ Similarly $X_{4,1}/(\mathbb{Z}_2 \times \mathbb{Z}_2) = X_{2,2}$ with diagonal $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \mathbb{Z}_4 \times \mathbb{Z}_4$.
- ✔ Is $X_{1,1}$ a good parent for *all* $X_{m,n}$? descending by quotienting: When singular fibers coalesce, additional isometries can develop, adds to MW torsion from "weakly integral" junctions, e.g., a $(1,0)$ string ending on a collapsed A^2 pair: "$(1/2, 0)$ on each." Quotient by new isometry, changes polarization, but only $\pi_1 = \mathbb{Z}_n$.
- ✔ Positive side: leads to new CYs with non-trivial π_1.

Construction II: Relative Jacobian of a surface or "Seeing is Believing."

Relative Jacobian of a surface

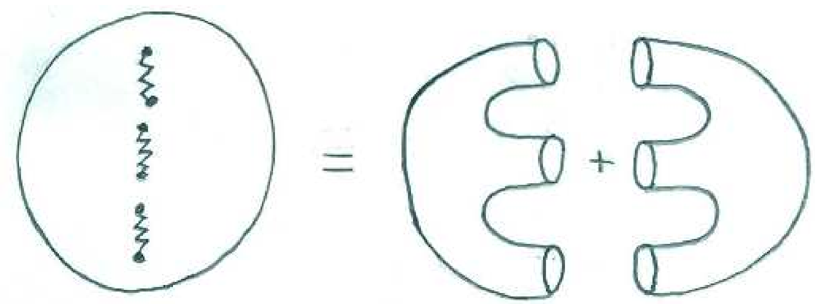
- ✔ Restrict to $m, n = 1, 1$ (principle polarization).
- ✔ Idea: complex surface much easier than 3-fold.
  Economical description for simple singular fibers.
- ✔ To every genus-g curve, can associate a principally polarized Jacobian
  torus T^{2g} with the same H_1 (same space of 1-cycles (p, q, r, s)):

$g = 2:$ 

$$\xrightarrow[\text{“Wilson lines”}]{\text{Abel-Jacobi map}} \mathrm{T}^4.$$

- ✔ So, try to realize CY $X_{1,1}$ as the fiberwise Jacobian, i.e. relative Jacobian
  of a surface S, where S is itself a genus-2 fibration over $\mathbb{P}^1$.

- ✔ A genus-2 curve $=$ double cover of $\mathbb{P}^1$ with 6 branch points.



$$\Rightarrow \quad S \equiv \text{genus-2 fibration over } \mathbb{P}^1_{(1)}$$
$$= \text{branched double cover of } \mathbb{P}^1_{(1)} \times \mathbb{P}^1_{(2)}.$$

- ✔ Degree of branch curve $B \subset S$ is $(d, 6)$
(6 branch pts in generic fiber of $S \to \mathbb{P}^1_{(2)}$, i.e., for genus-2).
Can view as S as 2-fold section $\sqrt{P}$ of $\mathcal{O}(d/2, 3)$, where $B = \{P = 0\}$.

- ✔ For $d = 2$, found a candidate for $X_{1,1}$ from $\text{Jacobian}(S/\mathbb{P}^1)$
A simple construction! Is it what we are looking for?

Identity checks

- ✔ $c_1(X_{1,1}) = 0$, consider $K_{X_{1,1}|\mathbb{P}^1} = K_{\mathbb{P}^1} \otimes \det(N^*_{\mathbb{P}^1})$
  $K_{\mathbb{P}^1} = \mathcal{O}_{\mathbb{P}^1}(-2)$, can show $N^*_{\mathbb{P}^1} = \mathcal{O}_{\mathbb{P}^1}(1) \oplus \mathcal{O}_{\mathbb{P}^1}(1)$.
- ✔ $h^{1,1} = h^{2,1} = 14$, $h^{2,1}$ from cplx deform, $h^{1,1}$ from # of sections...
  Notice also the Euler character χ vanishes.
- ✔ Branch curve $\Rightarrow$ 20 nodal genus-2 fibers $\Rightarrow$ same # of singular T^4 fibers.
- ✔ $c_2 = 20$ elliptic curves (singular loci of fibers are codim. 2).
- ✔ Sections of S
  2nd projection $S \to \mathbb{P}^1_{(2)}$ has genus-0 fibers $C_0 =$ (2$\mathbb{P}^1$ − 2 br pts)
  w. 12 degenerations, where the 2 br pts overlap, and $C_0 \to 2\ \mathbb{P}^1$s
  Pairs ℓ_I, ℓ'_I meeting at a point $(I = 1, \ldots, 12)$.

  $\Rightarrow 2 \times 12$ sections of genus-2 fibration (w. relations $\ell_I + \ell'_I = C_0$). $\mathbb{Z}_2$

More ID checks

- ✔ Sections of $X_{1,1}$
  Given a fixed choice of zero section $\sigma_0 \in \{\ell_I, \ell'_I\}$,
  $\mathrm{MW}(X_{1,1}) \cong \langle \sigma_0, f_2 \rangle^\perp$ (with S intersection pairing).
  $\Rightarrow$ 12 dimensional lattice, w. D_{12}^- matrix.
- ✔ Intersections:
  - ✘ $\ell_I \subset S \quad \mapsto \quad$ "theta surface" $\Theta_I \subset X_{1,1}$.
  - ✘ Identify

    $A =$ abelian fiber, $\quad \mathcal{E}_I = \frac{1}{2}(\Theta_I - \Theta'_I), \quad H = \frac{1}{2}(\Theta_I + \Theta'_I) - \frac{1}{6}A,$

    gives the desired intersections for $X_{1,1}$.

    $-\frac{1}{6}A$? Only effects self-intersection of $[H]$
    $\Rightarrow$ Basis for H from sugra harmonic form has small mismatch w. $H_2(\mathbb{Z})$.
- ✔ Wall's classification theorem for 3 folds:(Wall; Žubr)
  $(c_1, c_2, C_{IJK}) \Rightarrow$ unique CY up to homotopy type.

Conclusions

✔ Two complimentary constructions of the IIA duals of $\mathrm{T}^6/\mathbb{Z}_2$:

1. Monodromy/string-junction (analog of F-theory for T^4 fibers),
2. Relative Jacobian of a genus-2 fibered surface S (for $m, n = 1, 1$).

✔ We have constructed the Mordell-Weil lattice of rational sections, to obtain the required D_N lattice.

✘ In Case 1, D3 tadpole condition $\Leftrightarrow$ total monodromy $= 1$.
✘ All criteria for Wall's theorem (c_1, c_2, C_{IJK}) satisfied in Case 2.

✔ Stage set for studying related issues in this setting:
e.g., warped KK reduction, D-instantons, attractor bhs(Hsu et.al.)...

✔ Future studies: duality with other $\mathcal{N} = 2$ string vacua, w.(o.) fluxes, e.g. Heterotic-IIA; More generic CY orientifolds with flux; connecting to $\mathcal{N} = 1$ flux vacua.

Thank You!