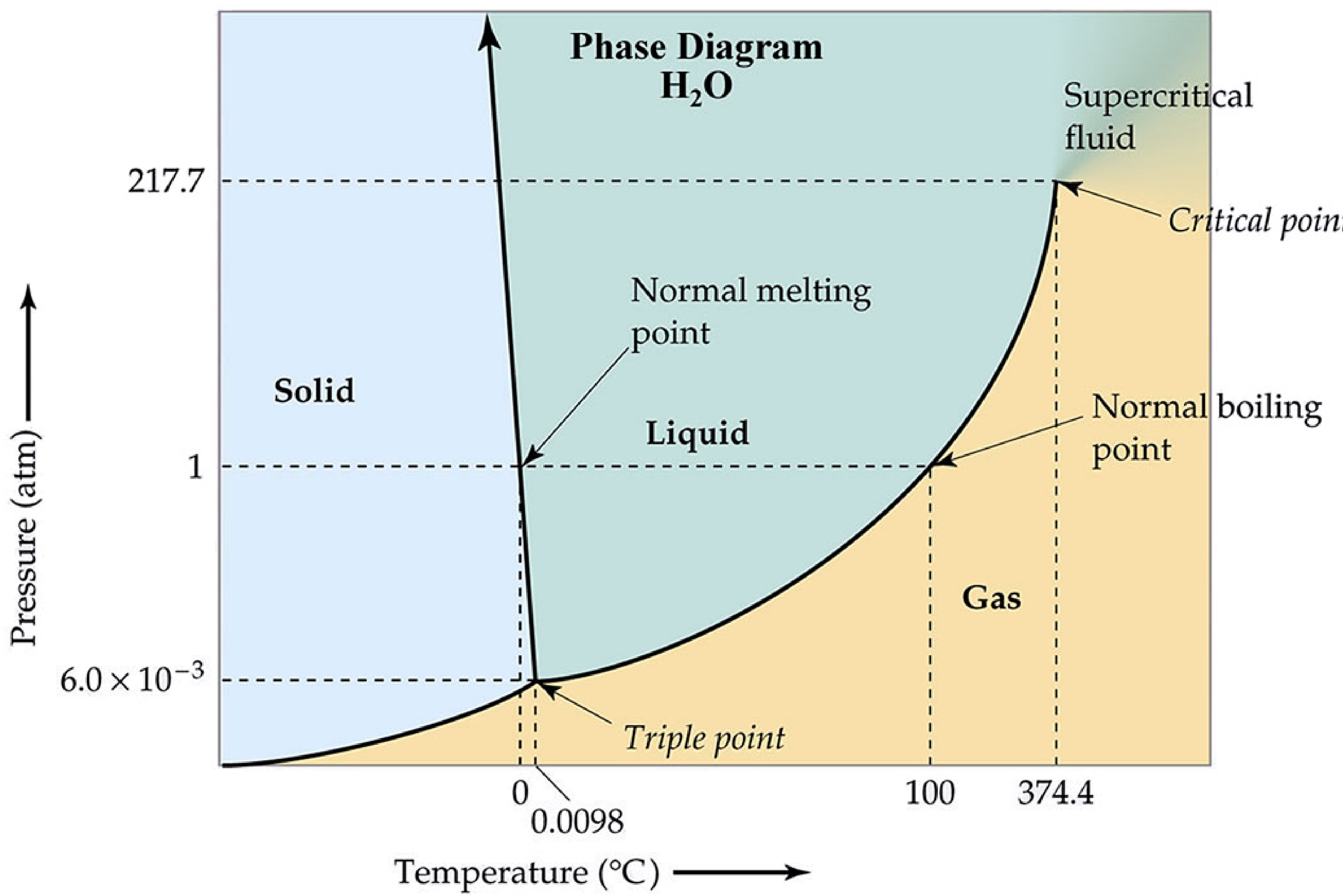
Phases of Gauge Theories and Surface Operators

Sergei Gukov

based on:

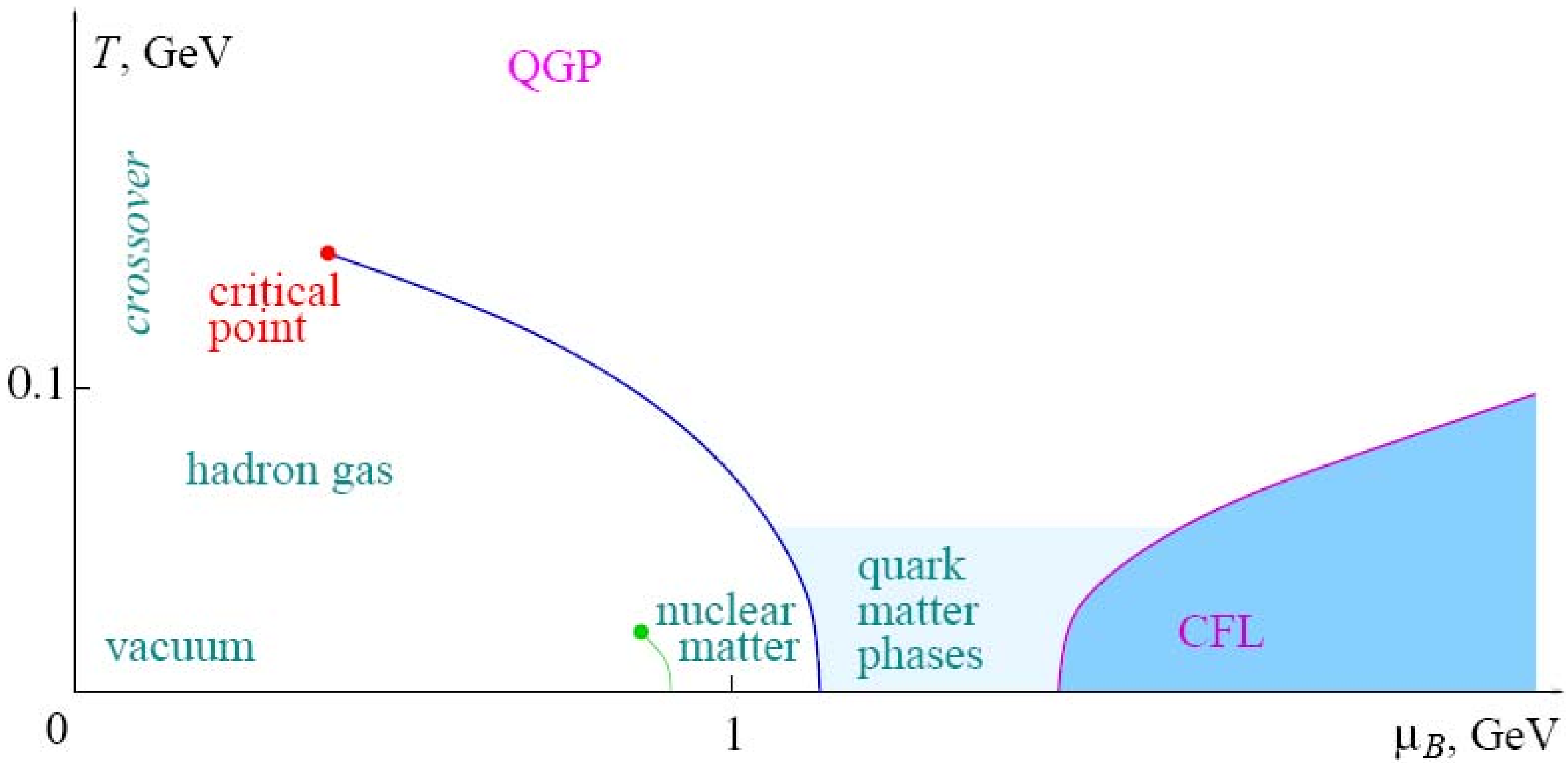
- S.G., E.Witten, "Rigid Surface Operators," arXiv:0804.1561
- S.G., E.Witten, "Gauge theory, ramification, and the geometric Langlands program," hep-th/0612073
- work in progress with N.Seiberg

Phase Diagram of Water



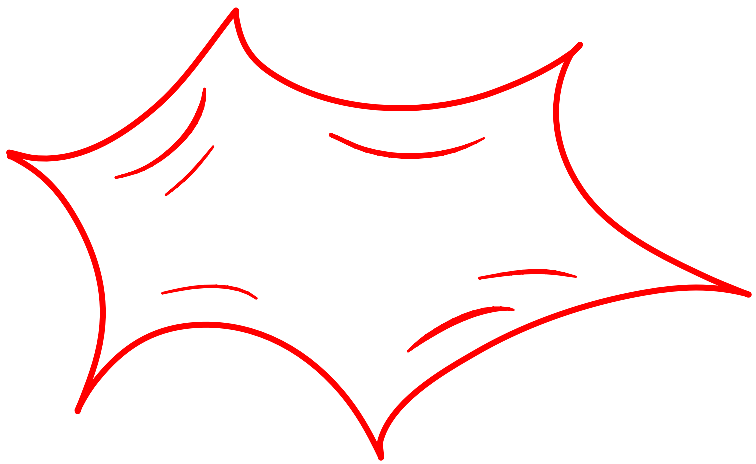
Phase Diagram of QCD

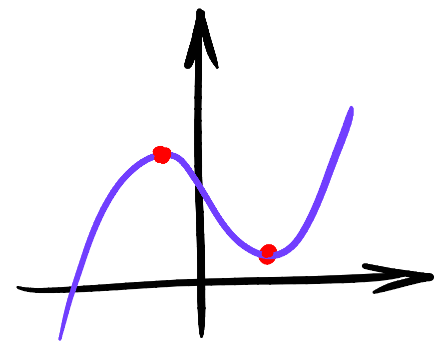
[M.Stephanov]



Phases of N=1 Gauge Theories

- Moduli space of **N=1** SYM with an adjoint matter Φ and a superpotential $W(\Phi)$





$$U(N) \rightarrow U(N_1) \times U(N_2)$$

- Phases not distinguished by traditional order parameters, like Wilson and 't Hooft operators

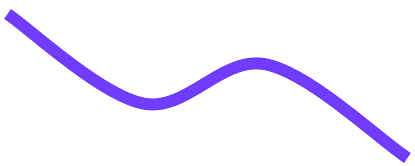
[F.Cachazo, N.Seiberg, E.Witten]

Operators in 4D Gauge Theory

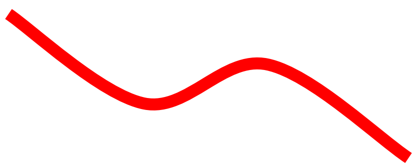
- Codimension 4: Local operators

  much studied in AdS/CFT

- Codimension 3: Line operators:



Wilson line



't Hooft line

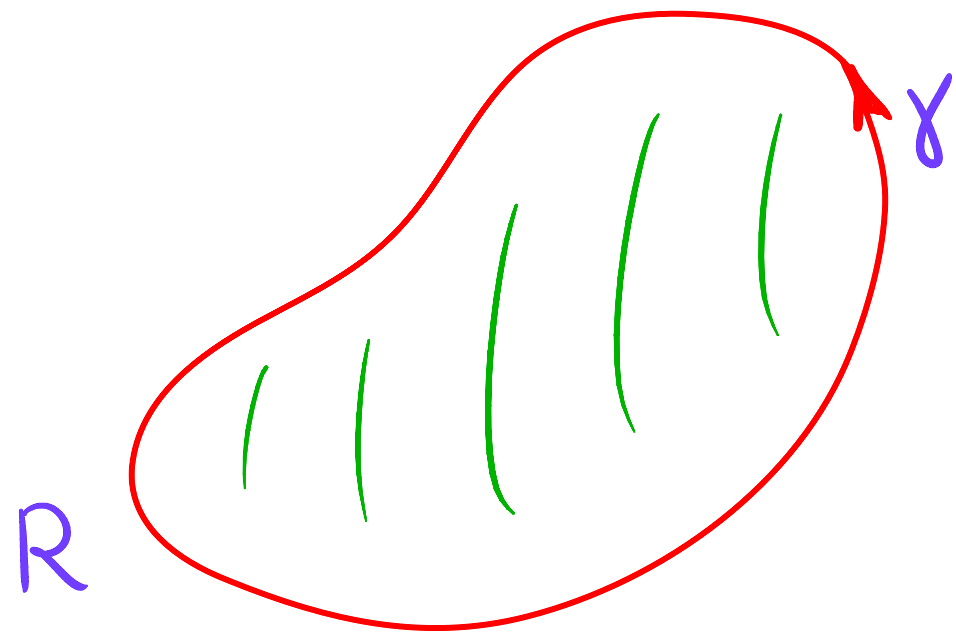
- Codimension 2: Surface operators

- Codimension 1: Boundaries

Wilson Operators

$$W_R(\gamma) = \mathrm{Tr}_R\, \mathrm{Hol}_\gamma(A) = \mathrm{Tr}_R\left(P\exp \oint_\gamma A\right)$$

representation of
the gauge group G



Wilson Operators

in abelian gauge theory:

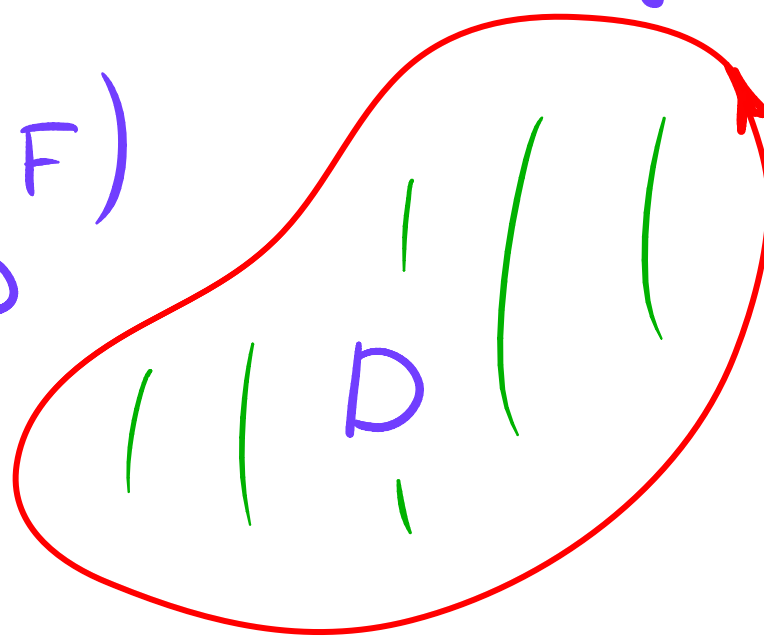
external charge

$$W_q(\gamma) = \exp\left(iq \oint_\gamma A\right)$$

$$= \exp\left(iq \int_D F\right)$$

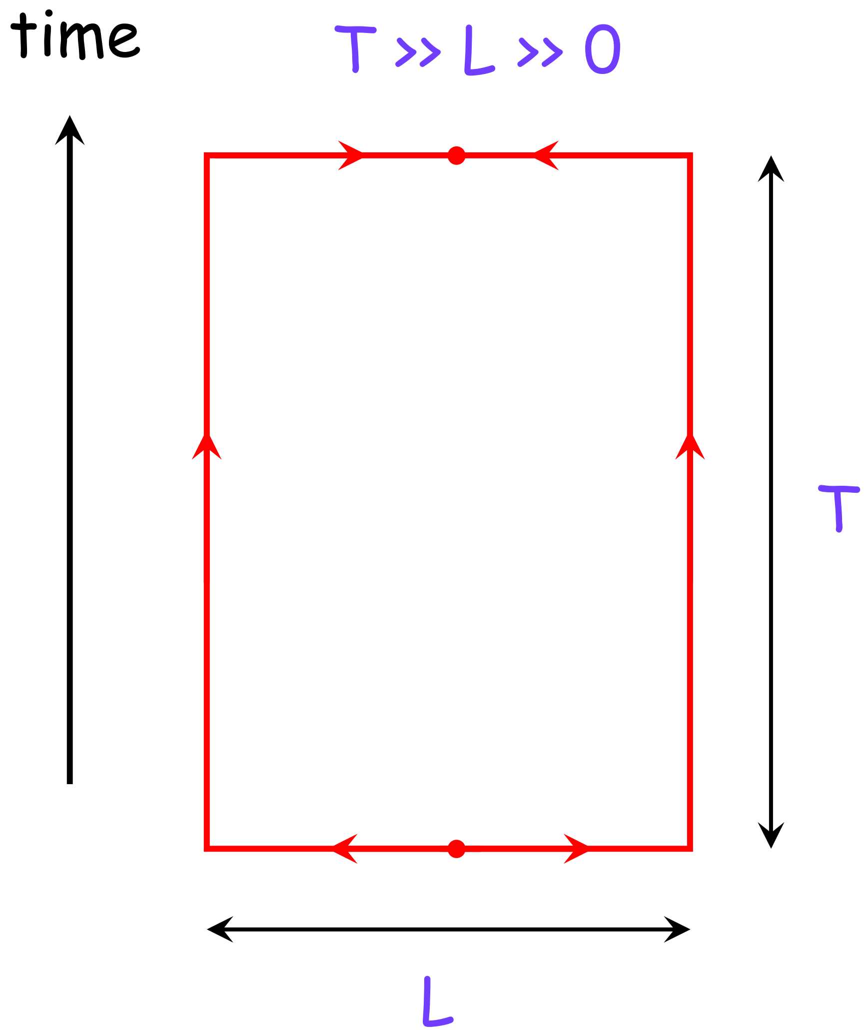
"surface operator"

$\gamma = \partial D$



Wilson Operators

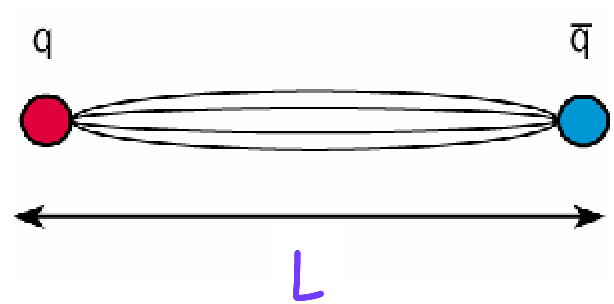
$T \gg L \gg 0$



$$\langle W_q(\gamma) \rangle = const \cdot e^{-T\,V(L)}$$

$V(L)$ = interaction energy of the charges at distance L

(quark-antiquark potential)



Phases

- Coulomb:

$$V(L) \sim \frac{1}{L} \rightsquigarrow \langle W(\gamma) \rangle \sim e^{-T/L}$$

- Higgs:

$$V(L) \sim \text{const} \rightsquigarrow \langle W(\gamma) \rangle \sim e^{-L_\gamma}$$

- confinement:

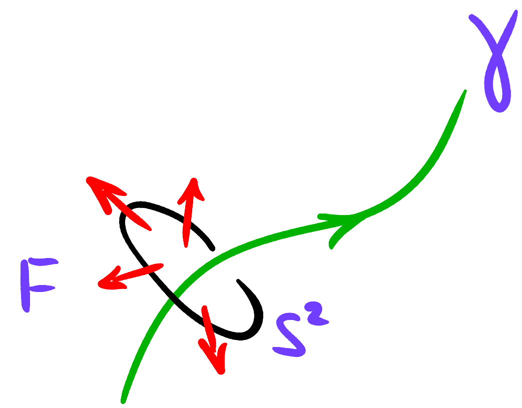
$$V(L) \sim L \rightsquigarrow \langle W(\gamma) \rangle \sim e^{-S_\gamma}$$

't Hooft Operators

- remove γ from M $\rightsquigarrow$ $M \setminus \gamma$ has S^2

$$T_q(\gamma): \qquad \int_{S^2} F = 2\pi q$$

- For general G: $\quad T_\rho(\gamma)$



$\rho: U(1) \to G$

cf. $\rho: SU(2) \to G$
for surface operators

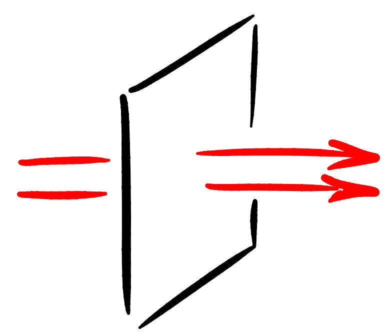
't Hooft Operators Detect Spontaneous Symmetry Breaking

- in Abelian Higgs model:

$$\mathcal{L}(A,\phi) = \frac{1}{2e^2} F_A^2 + \frac{1}{2e^2} |D_A|^2 + \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2$$

$v^2 > 0$: Higgs phase, mass gap

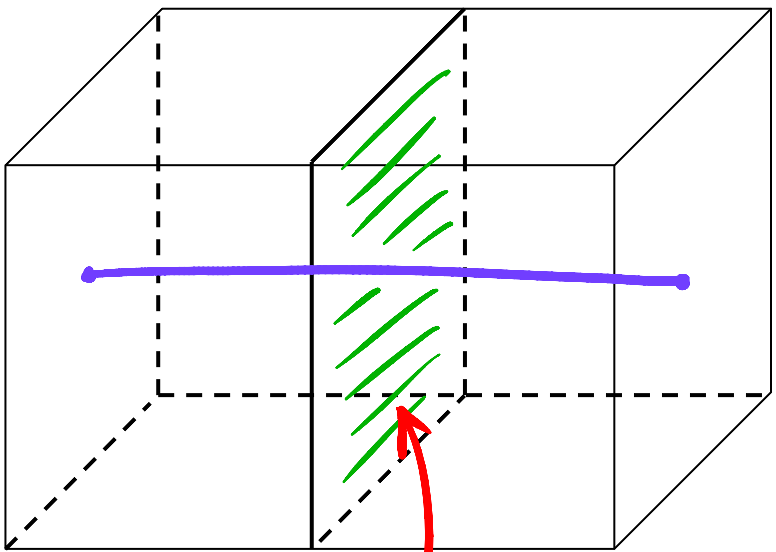
vortices



$$G = U(1) \longrightarrow \mathbb{1}$$

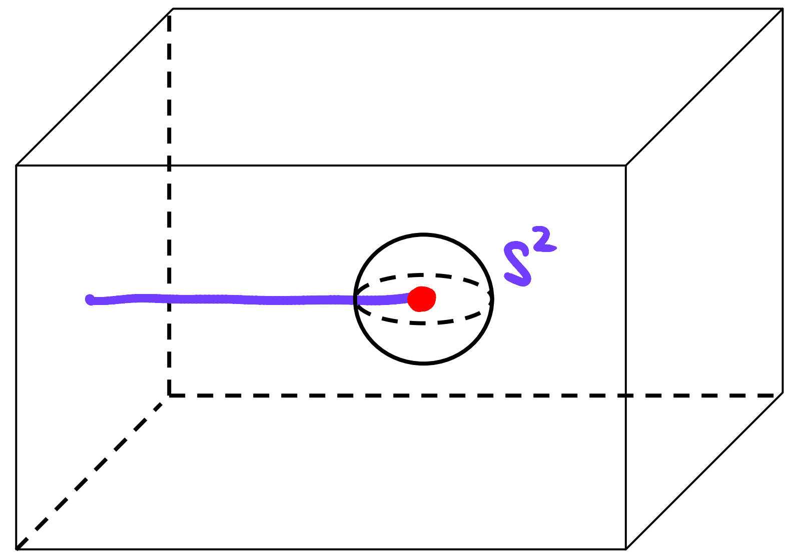
't Hooft Operators Detect Spontaneous Symmetry Breaking

vortex:



$$\int \frac{F}{2\pi} = 1$$

suppose it has a boundary:



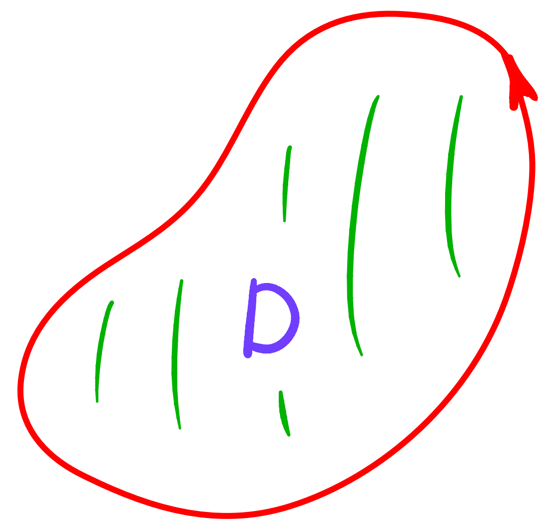
$$\int_{S^2} \frac{F}{2\pi} = 1$$

't Hooft Operators Detect Spontaneous Symmetry Breaking

$T_{q=1}(\gamma)$ = boundary of a vortex supported on a surface D, s.t. $\partial D = \gamma$

$$\langle T_{q=1}(\gamma) \rangle \sim e^{-c\cdot \mathrm{Area}(D)}$$

in Higgs phase



Electric-Magnetic Duality

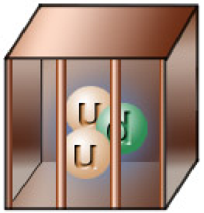
$$G \longleftrightarrow {}^L G$$

$$\tau \longleftrightarrow -\frac{1}{\tau}$$

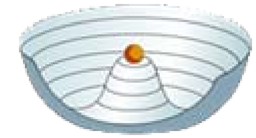
$$R \longleftrightarrow {}^L R$$

$$W_R(\gamma) \longleftrightarrow T_{{}^L R}(\gamma)$$

Electric-Magnetic Duality



Confinement (mass gap) ⟷ Higgs (mass gap)



$\langle W \rangle \sim e^{-\text{Area}}$ ⟷ $\langle T \rangle \sim e^{-\text{Area}}$

$\langle T \rangle \sim e^{-\text{Length}}$ ⟷ $\langle W \rangle \sim e^{-\text{Length}}$

Coulomb (no mass gap)

Surface Operators

- supported on a surface **D** in a space-time manifold **M**
- defined by introducing a singularity for the gauge field (for simplicity, take **G=U(1)**):

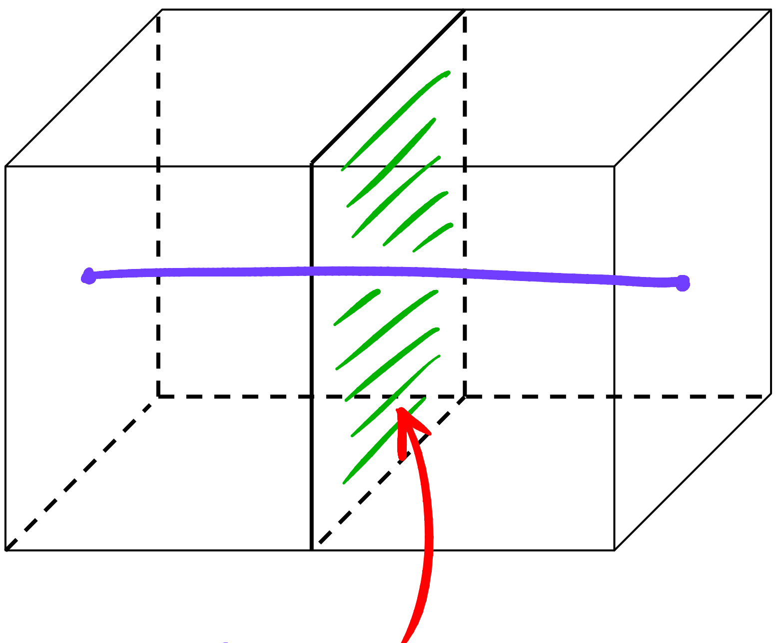
$$F = 2\pi\alpha\,\delta_D$$

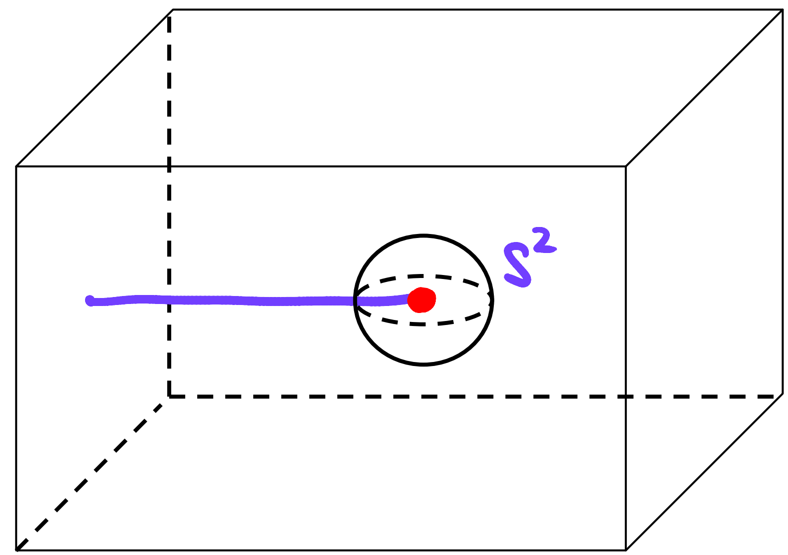
- and a phase factor in the path integral:

$$\exp\left(i\eta\int_D F\right)$$

Surface Operators

suppose D has a boundary:





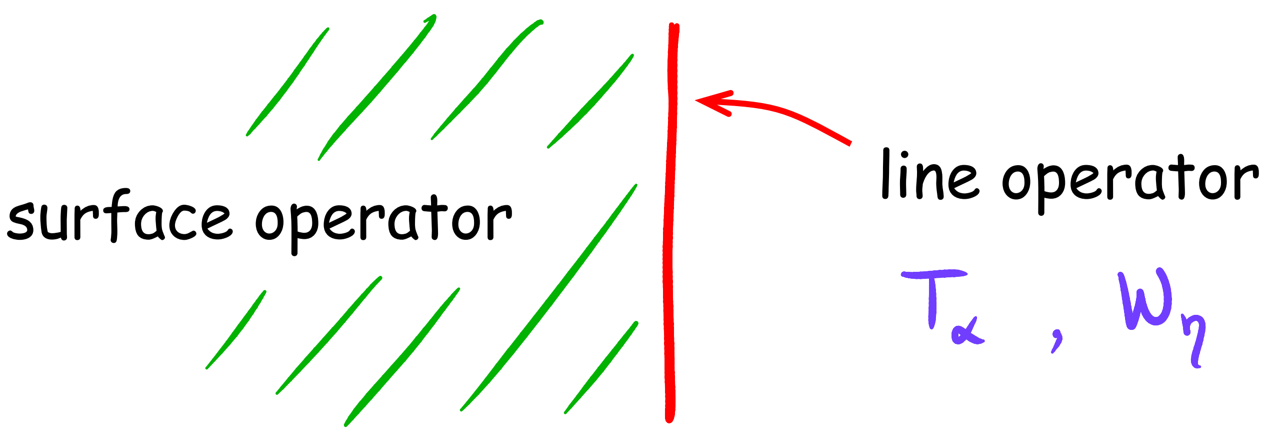
$$\int \frac{F}{2\pi} = \alpha$$

$$\int_{S^2} \frac{F}{2\pi} = \alpha$$

α = magnetic charge

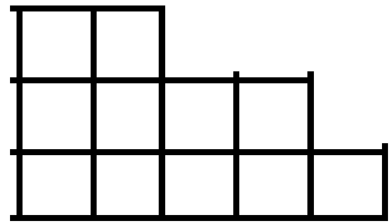
Surface Operators

⟹ surface operator with parameters (α,η) can be thought of as a Dirac string of a dyon with magnetic charge α and electric charge η



- One might expect that surface operators are labeled by representations of the gauge group G (or the dual group ${}^L G$), just like electric and magnetic charges.

- Indeed, for $G = U(N)$, there are different types of surface operators labeled by partitions of N:

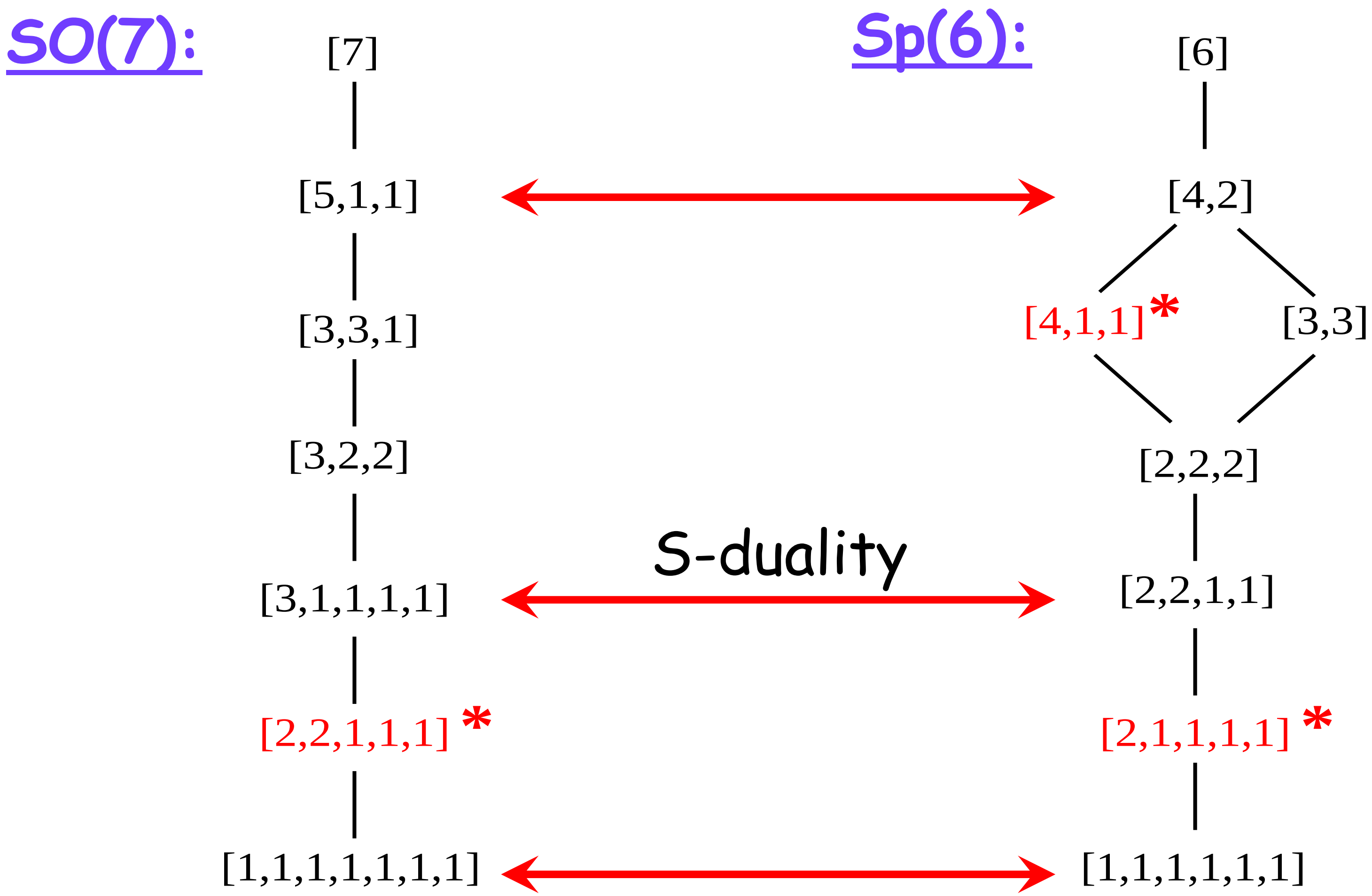


$$N = 3 + 3 + 2 + 2 + 1$$

- analog for general G:

$$\rho : SU(2) \to G$$

- in $SO(N)$ and $Sp(N)$ gauge theory, correspond to partitions of N with certain constraints

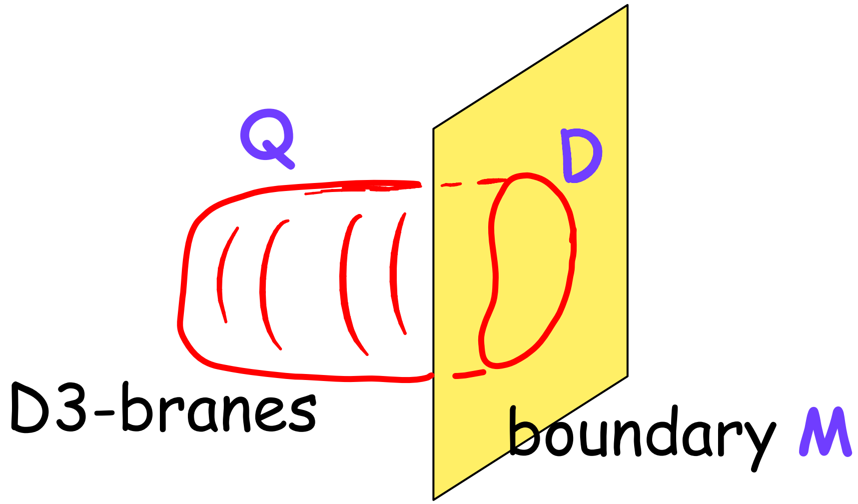


- Surface operators shown in red and labeled by * appear to spoil S-duality. In order to restore a nice match, one has to introduce a larger class of surface operators.

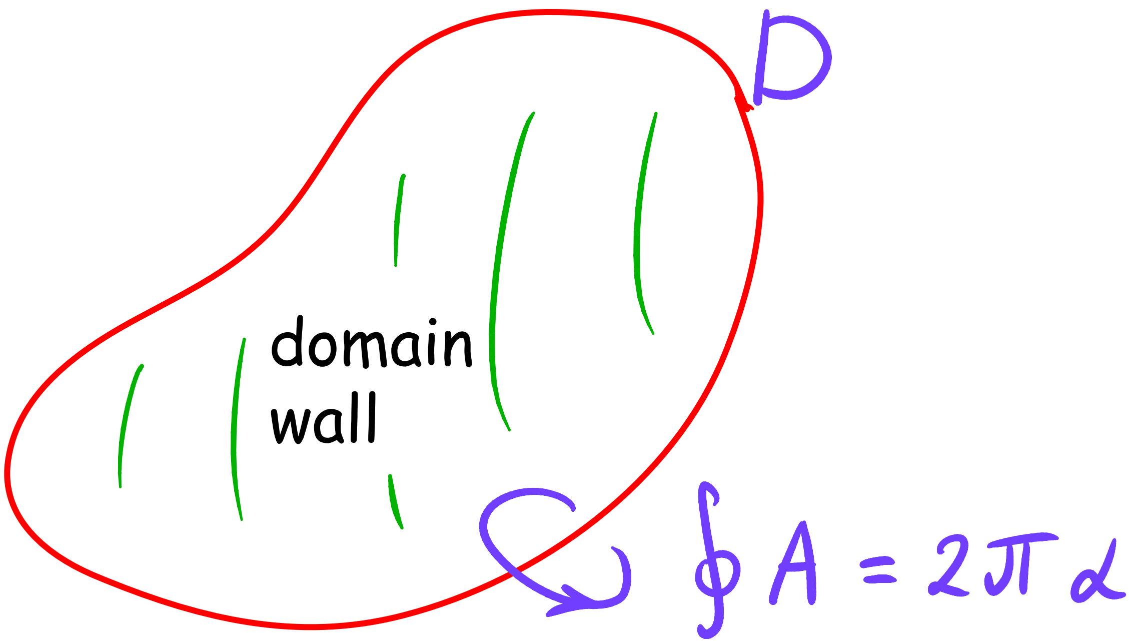
Holographic Dual

- In the limit of large N and large 't Hooft coupling, such surface operators can be described as D3-branes in $AdS_5 \times S^5$ with world-volume $Q \times S^1$ where $S^1 \subset S^5$ and $Q \subset AdS_5$ is a volume minimizing 3-manifold with boundary

$$\partial Q = D \subset M$$



- Surface operators exhibit a "volume law" when theory admits *domain walls*, which can end on a surface operator



- Examples of such theories include **N=1** Dijkgraaf-Vafa type theories.

Thermal Phase Transition

- To study thermal phase transition in $N=4$ SYM theory, we compactify the time direction on a circle of circumference $\beta = 2\pi/T$ and study the theory on a space-time manifold $M = S^1_\beta \times S^3$ with thermal (anti-periodic) boundary conditions on fermions.
- It is dual to IIB string theory on $X \times S^5$ whereC

$$X = \begin{cases} \text{thermal AdS} \quad B^4 \times S^1 & \text{(low temperature)} \\ \text{AdS black hole} \quad S^3 \times B^2 & \text{(high temperature)} \end{cases}$$

Low Temperature

- temporal surface operator ($D = \gamma \times S^1_\beta$):

$$\langle \mathcal{O}_{temporal} \rangle = 0$$

since S^1_β is not contractible in X, and so there is no minimal submanifold Q bounded by D

- spatial surface operator ($D \subset S^3$):

$$\langle \mathcal{O}_D \rangle = e^{-Area(D)}$$

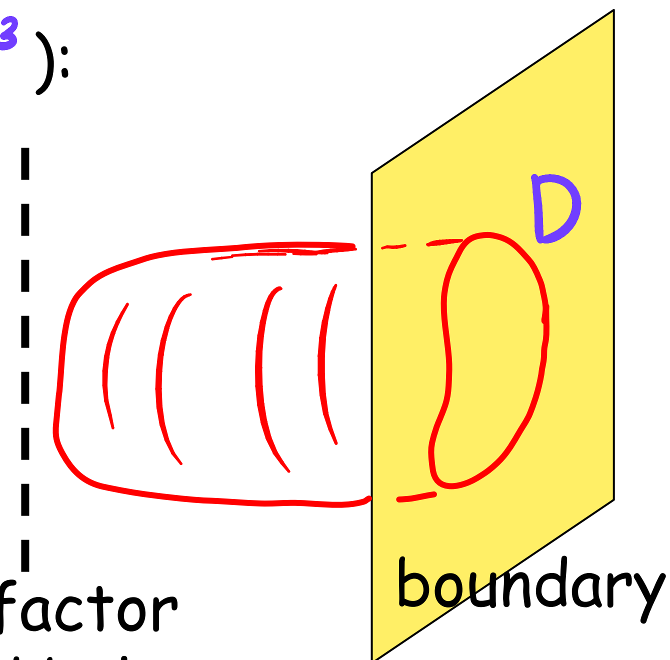
High Temperature

- temporal surface operator ($D = \gamma \times S^1_\beta$):

$$\langle \mathcal{O}_{\text{temporal}} \rangle \neq 0$$

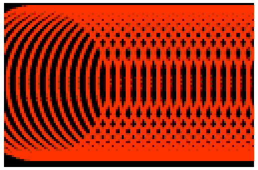
- spatial surface operator ($D \subset S^3$):

$$\langle \mathcal{O}_D \rangle = e^{-\text{Volume}(D)}$$



the warp factor
is bounded below

From Surfaces to Lines



- Note, in the high temperature limit (β -> 0) the theory reduces to a pure (non-supersymmetric) three-dimensional Yang-Mills theory on S^3. (Scalars acquire a mass from loops.)
- In this limit, a temporal surface operator turns into a a line operator (supported on γ) in the 3D theory.
- Therefore, surface operators in the four-dimensional gauge theory exhibit volume (resp. area) law whenever the corresponding line operators in the 3D theory exhibit area (resp. circumference) law.