# Exotic instantons and duality


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# Foreword

Mostly based on

- M. Billo, L. Ferro, M. Frau, L. Gallot, A. Lerda and I. Pesando, “Exotic instanton counting and heterotic/type I’ duality,” JHEP **0907** (2009) 092, arXiv:0905.4586 [hep-th].
- M. Billo, M. Frau, L. Gallot, A. Lerda and I. Pesando, “Classical solutions for exotic instantons?,”, JHEP **03** (2009) 056, arXiv:0901.1666 [hep-th].

It builds over a vast literature

- I apologize for missing references...

# Plan of the talk

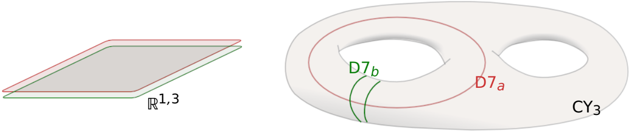
1. Introduction and motivations
2. “Exotic” instantons in type I’
3. Interpretation as 8d instanton solutions
4. The effective action
5. Conclusions and perspectives

# Introduction and motivations

# Non-perturbative sectors

in brane-worlds

- (Susy) gauge and matter sectors on the uncompactified part of (partially wrapped) D-branes
  - gauge couplings involve $1/g_s$ × different volumes → string scale not tied to 4d Planck scale
  - chiral matter, families from multiple intersections,...



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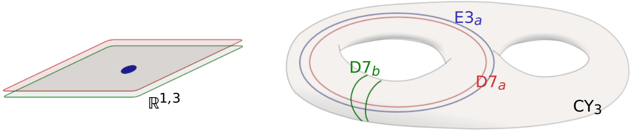
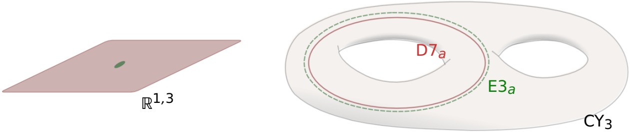



- Non-perturbative sectors from partially wrapped E(uclidean)-branes
  - Pointlike in the $\mathbb{R}^{1,3}$ space-time: "instanton configurations"
  - Tractable in String Theory, with techniques in rapid development

# Ordinary instantons

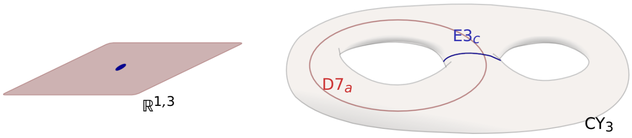
W.r.t. the gauge theory on a given D-brane stack,




- E-branes identical to D-branes in the internal directions: gauge instantons
  - ADHM from strings attached to the instantonic branes
    Witten, 1995; Douglas, 1995-1996; ...
  - non-trivial instanton profile of the gauge field
    Billo et al, 2001
  - Rules and techniques to embed the instanton calculus in string theory have been constructed
    Polchinksi, 1994; Green-Gutperle, 2000, ...; Turin/Rome/Münich/UPenn/Madrid,...

# Exotic instantons

W.r.t. the gauge theory on a given D-brane stack,




- E-branes different from D-branes in internal directions do not represent gauge instantons; they are called exotic or stringy instantons
  - May explain important terms in the effective action: neutrino Majorana masses, moduli stabilizing terms, . . .
    Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; ... ;
  - Exponentially suppressed but not just $\exp(-1/g^2)$, can involve volumes of different internal cycles
- Need to understand their status in the gauge theory and to construct precise rules for the "exotic" instanton calculus

# Our strategy

- Select a simple example: D(-1)/D7 in type I’ theory, sharing many features of stringy instantons
- Investigate the field-theory interpretation of D(-1)’s in this 8d gauge theory
  Billo et al, 2009a;
- Compute the non-perturbative effective action on the D7’s extending the rules of stringy instanton calculus to this “exotic” case.
- Check against the results in the dual Heterotic $SO(8)^4$ theory. Impressive quantitative check of this string duality.
  Billo et al, 2009b
- Apply the technology to tractable example leading to 4d models
  Work in progress, Turin + Tor Vergata

# “Exotic” instantons in type I’

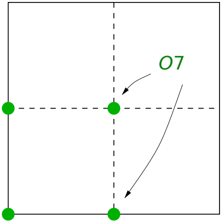
# A D(-1)/D7 system in type I'

- Type I' is type IIB on a two-torus $T_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\,\mathcal{I}_2$$

  where $\omega$ = w.s. parity, $F_L$ = left-moving fermion #, $I_2$ = inversion on $T_2$

- $\Omega$ has four fixed-points on $T_2$ where four O7-planes are placed



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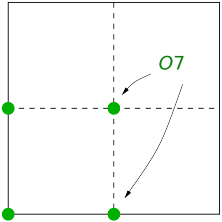

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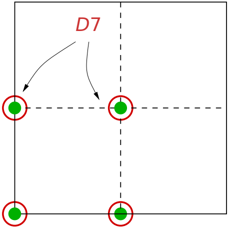

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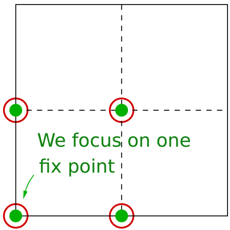

# The gauge theory on the D7's

- From the D7/D7 strings we get $\mathcal{N}=1$ vector multiplet in $d=8$ in the adjoint of SO(8):

$$\left\{A_\mu, \Lambda^\alpha, \phi_m\right\}\,, \quad \mu=1,\ldots 8\,, \quad m=8,9$$

- Can be assembled into a "chiral" superfield

$$\Phi(x,\theta)=\phi(x)+\sqrt{2}\,\theta\Lambda(x)+\frac{1}{2}\,\theta\gamma^{\mu\nu}\theta\,F_{\mu\nu}(x)+\ldots$$

  where $\phi=(\phi_9+i\phi_{10})/\sqrt{2}$.
  - Formally very similar to $\mathcal{N}=2$ in $d=4$

# Effective action on the D7

(tree level)

- Effective action in $F_{\mu\nu}$ and its derivatives: NABI ▸ Back

$$S = S_{(2)} + S_{(4)} + S_{(5)} + \cdots$$
$$= \frac{1}{8\pi g_s}\int d^8x \Big[ \frac{\mathrm{Tr}\left(F^2\right)}{(2\pi)^4 \alpha'^2} - \frac{t_8 \mathrm{Tr}\left(F^4\right)}{3\,(2\pi)^2} + \alpha' \mathcal{L}_{(5)}\left(F, DF\right) + \cdots \Big]$$

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- The quadratic Yang-Mills term $S_{(2)}$ has a dimensionful coupling $g_{\mathrm{YM}}^2 \equiv 4\pi g_s (2\pi\sqrt{\alpha'})^4$

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\end{aligned}
$$

- Contributions of higher order in $\alpha'$, whose rôle will be discussed later

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- The quartic term has a dimensionless coupling:

$$S_{(4)} = -\frac{1}{96\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right)$$

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\end{aligned}
$$

- Adding the WZ term, we can write

$$
S_{(4)} = -\frac{1}{4!\,4\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right) - 2\pi i\, C_0\, c_{(4)}
$$

where $c_{(4)}$ is the fourth Chern number

$$
c_{(4)} = \frac{1}{4!(2\pi)^4} \int \mathrm{Tr}\left(F \wedge F \wedge F \wedge F\right)
$$

# Effective action on the D7

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- Effective action in $F_{\mu\nu}$ and its derivatives: NABI Back

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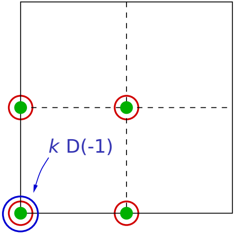
- Adding the fermionic terms, can be written using the superfield $\Phi(x,\theta)$ as

$$S_{(4)} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\ \mathrm{Tr}\left[\frac{i\pi}{12}\,\tau\,\Phi^4\right] + \text{c.c.}$$

where $\tau = C_0 + \frac{i}{g_s}$ is the axion-dilaton.

# Adding D-instantons

- Add $k$ D-instantons.
- D7/D(-1) form a 1/2 BPS system with 8 ND directions




  - D(-1) classical action

$$\mathcal{S}_{cl} = k\left(\frac{2\pi}{g_s} - 2\pi i C_0\right) \equiv -2\pi i k\tau ,$$

  - Coincides with the quartic action on the D7 for gauge fields $F$ with $c_{(4)} = k$ and

$$\int d^8x \, Tr\left(t_8 F^4\right) = -\frac{1}{2}\int d^8x \, Tr\left(\epsilon_8 F^4\right) = -\frac{4!}{2}(2\pi)^4 c_{(4)}$$

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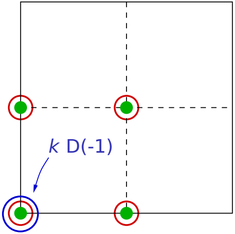
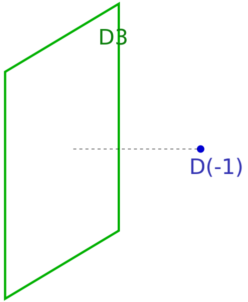



$$\mathcal{S}_{cl} = k\left(\frac{2\pi}{g_s} - 2\pi i C_0\right) \equiv -2\pi i k \tau \,,$$

  - Analogous to relation with self-dual YM config.s in D3/D(-1)
  - Suggests relation to some 8d instanton of the quartic action

# The size of the instanton solution

("gauge" instantons)




- For ordinary instantons, e.g. D3/D(-1), there are moduli $w_{\dot{\alpha}}$ related to the size $\rho$ of the instanton profile
- They com from the NS sector of mixed D3/D(-1) strings

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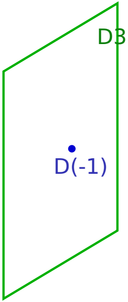


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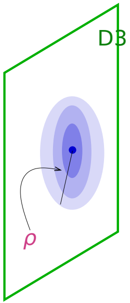


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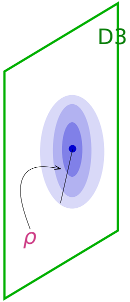


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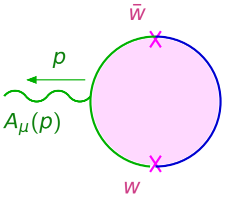
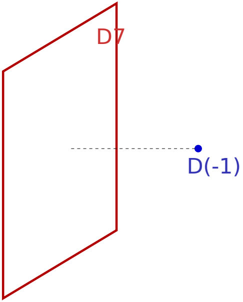
- The classical instanton profile arises from mixed disks Billo et al, 2001

$$A^i_\mu = 2\rho^2 \bar\eta^i_{\mu\nu} \frac{x^\nu}{|x|^4} + \ldots$$

(SU(2), sing. gauge, large-$|x|$, $2\rho^2 = tr\, \bar w_{\dot\alpha} w^{\dot\alpha}$ )



# No size for "exotic" instantons

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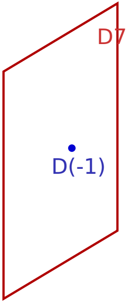


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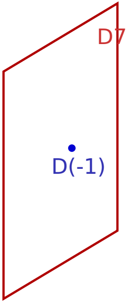


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- Can one still associate the D(-1) to the zero-size limit of some classical gauge configuration on the D7’s?

# Interpretation as 8d instanton solutions

# Expected features

- A D(-1) inside the D7's should correspond to the zero-size limit of some "instantonic" configuration of the SO(8) gauge field such that
  - has 4-th Chern number $c_{(4)} = 1$
  - the quartic action reduces to the D(-1) action, which requires
$$Tr(t_8 F^4) = -\frac{1}{2} Tr(\epsilon_8 F^4)$$
  - preserves SO(8) "Lorentz" invariance
  - corresponds to a 1/2 BPS config. in susy case

# The SO(8) instanton

- All our requirements met by the SO(8) instanton

[Grossmann e al, 1985]

$$\left[A_\mu(x)\right]^{\alpha\beta} = \frac{(\gamma_{\mu\nu})^{\alpha\beta}\, x^\nu}{r^2 + \rho^2}$$

with $\rho$ = instanton size and $r^2 = x_\mu x^\mu$, while $\alpha\beta \in$ adjoint of the SO(8) gauge group.

  - is "self-dual" in the sense that $F \wedge F = (F \wedge F)^*$
  - satisfies $t_8 F^4 = -1/2 \epsilon_8 F^4$ from Clifford Algebra
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  - has $c_{(4)} = 1$ and $S_{(4)} = -2\pi i\tau$

- However, it is not a solution of Y.M. e.o.m. in $d = 8$, for $\rho \neq 0$:

$$D^\mu F_{\mu\nu}(x) = \frac{4(d-4)\rho^2}{(r^2+\rho^2)^3}\,\gamma_{\mu\nu}x^\nu\ .$$

# Consistency conditions

- Eff. action on the D7 is the NABI action ▸ Recall
- To keep the quartic action and the instanton effects the field-theory limit must be

$$\alpha' \to 0\,, \qquad g_s \text{ fixed}$$

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- This limit is dangerous on the YM action $S_{YM}$ since $g^2_{YM} \propto g_s \alpha'^2$. On the SO(8) instanton, however, we have ($R$ regulates the volume):

$$S_{YM} \to \frac{\rho^4}{\alpha'^2\, g_s} \log\left(\rho/R\right) \,,$$

which vanishes in the zero-size limit $\rho \to 0$ if $\rho^2/\alpha'^2 \to 0$ (done before removing $R$)

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- To keep the quartic action and the instanton effects the field-theory limit must be

$$\alpha' \to 0\,, \qquad g_s \text{ fixed}$$

- Consider the higher order $\alpha'$ corrections to the NABI action. On the SO(8) instanton, by dimensional reasons, must be

$$\rho^{d-8}\sum_{n=1}^{\infty} a_n \left(\frac{\alpha'}{\rho^2}\right)^n\,,$$

- The coefficients $a_n$ should vanish for consistency!

# $O(F^5)$ terms in the NABI

- The first coefficient $a_1$ arises from the integral of $\mathcal{L}^{(5)}(F, DF)$, i.e.the term of order $\alpha'^3$ w.r.t to the YM action.
- We would like to check that it vanishes. Crucial point: which is the form of $\mathcal{L}^{(5)}(F, DF)$?

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- Various proposals in the literature

  Refolli et al, Koerber-Sevrin, Grasso, Barreiro-Medina, ...

  - obtained by different methods
  - differing by terms which vanish "on-shell", i.e. upon use of the YM e.o.m.

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  - obtained by different methods
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- One proposal is singled out by admitting a susy extension Collinucci et al, 2002

# Check at $O(F^5)$ in the NABI

- The bosonic part of the supersimmetrizable $O(\alpha'^3)$ lagrangian is

$$
\begin{aligned}
\mathcal{L}^{(5)} &= \frac{\zeta(3)}{2}\,\mathrm{Tr}\Big\{4\left[F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\left[\left[F_{\mu_1\mu_3},F_{\mu_2\mu_5}\right],F_{\mu_4\mu_5}\right] \\
&+ 2\left[F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\left[\left[F_{\mu_1\mu_2},F_{\mu_3\mu_5}\right],F_{\mu_4\mu_5}\right] \\
&+ 2\left[F_{\mu_1\mu_2},D_{\mu_5}F_{\mu_1\mu_4}\right]\left[D_{\mu_5}F_{\mu_2\mu_3},F_{\mu_3\mu_4}\right] \\
&- 2\left[F_{\mu_1\mu_2},D_{\mu_4}F_{\mu_3\mu_5}\right]\left[D_{\mu_4}F_{\mu_2\mu_5},F_{\mu_1\mu_3}\right] \\
&+ \left[F_{\mu_1\mu_2},D_{\mu_5}F_{\mu_3\mu_4}\right]\left[D_{\mu_5}F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\Big\}
\end{aligned}
$$

# Check at $O(F^5)$ in the NABI

- Plugging the instanton profile into $\frac{\alpha'}{g_s}\int d^8x\, \mathcal{L}^{(5)}$ we get [Using the CADABRA program by Kasper Peeters]

$$\frac{\alpha'\,\zeta(3)}{g_s}\,2^{d/2+9}\,\frac{\pi^{d/2}\,\Gamma(9-d/2)}{9!\,\rho^{10-d}}$$
$$\times\,(d-1)(d-2)(d-4)\left(-d\left(9-\frac{d}{2}\right)+\left(d+2\right)\frac{d}{2}\right)$$

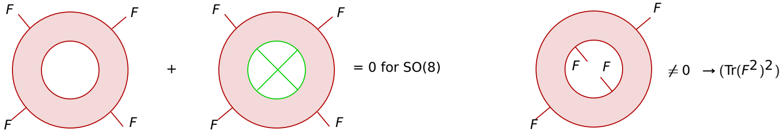
namely a result proportional to

$$d(d-1)(d-2)(d-4)(d-8)$$

- The quintic action vanishes on the SO(8) instanton! The check is successful

# The effective action

- At 1-loop we get contributions from annuli and Möbius diagrams. At the quartic level,



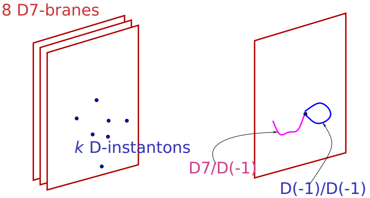

$$S_{(4)}^{1-\text{loop}} = \frac{1}{256\pi^4}\int d^8x \log\left(\text{Im}\tau\,\text{Im}U\,|\eta(U)|^4\right) t_8\left(\text{Tr}F^2\right)^2\Big]$$

$$= \frac{1}{(2\pi)^4}\int d^8x\, d^8\theta\, \left[\frac{1}{32}\log\left(\text{Im}\tau\,\text{Im}U\,|\eta(U)|^4\right)\left(\text{Tr}\Phi^2\right)^2\right] + \text{c.c.}$$

($U$ is the complex structure of the 2-torus $T_2$)

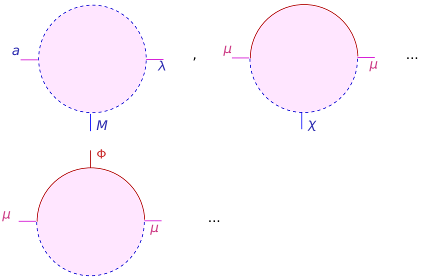
# Effective action from D-instantons

## Instanton moduli




- Open strings with at least one end on a D(–1) carry no momentum: they are moduli rather than dynamical fields.
- Need to determine their spectrum

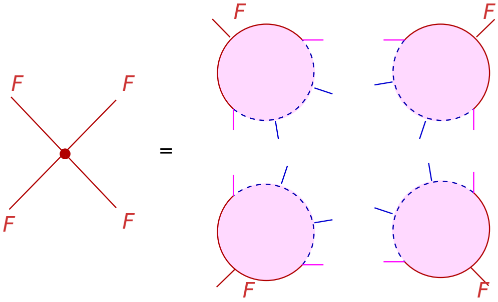
- Moduli interactions via disk diagrams encoded in $\mathcal{S}_{\text{inst}}$
- D7/D7 gauge fields interact with moduli through mixed disks



# Effective action from D-instantons



- Effective interactions between gauge fields can be mediated by D-instanton moduli through mixed disks

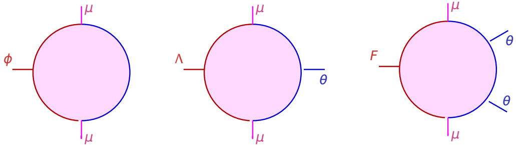



- In the r.h.s, integrate over the moduli with a weight $\exp(\mathcal{S}_{\text{inst}})$

# Effective action from D-instantons

## Field-dependent moduli action

- The effective interactions for the gauge multiplet $\Phi$ can be summarized by shifting the moduli action with $\Phi$-dependent terms arising from mixed disks




- In fact, we write the instanton action as

$$\mathcal{S}_{\text{inst}} = -2\pi i \tau k + \mathcal{S}(\mathcal{M}_{(k)}, \Phi)$$

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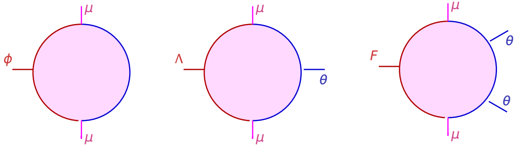


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  - Classical value

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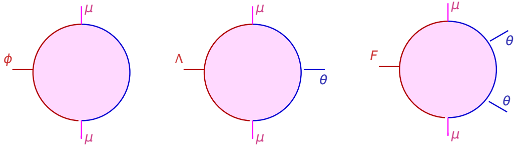


- In fact, we write the instanton action as

$$\mathcal{S}_{\text{inst}} = -2\pi i \tau k + \mathcal{S}(\mathcal{M}_{(k)}, \Phi)$$

  - Disk interactions

# Effective action from D-instantons



- Non-perturbative contributions to the effective action of the gauge degrees of freedom $\Phi$ arise integrating over the instanton moduli $\mathcal{M}_{(k)}$ and summing over all instanton numbers $k$

$$\mathcal{S}_{\text{n.p.}}(\Phi) = \sum_k e^{2\pi i \tau k} \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\mathcal{M}_{(k)},\Phi)}$$

# Effective action from D-instantons

Moduli integral

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- This procedure is by now well-established for instantonic brane systems corresponding to gauge instantons

Polchinksi, 1994; Green-Gutperle, 2000, ...; Turin/Rome/Münich/UPenn/Madrid,...

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- This is a very complicated matrix integral ...

# The moduli spectrum

Spectrum:

| Sector | | Name | Meaning | Chan-Paton | Dimension |
|---|---|---|---|---|---|
| $-1/-1$ | NS | $a_\mu$ | centers | symm $SO(k)$ | (length) |
| | | $\chi, \bar{\chi}$ | | adj $SO(k)$ | $(\text{length})^{-1}$ |
| | | $D_m$ | Lagr. mult. | adj $SO(k)$ | $(\text{length})^{-2}$ |
| | R | $M^\alpha$ | partners | symm $SO(k)$ | $(\text{length})^{\frac{1}{2}}$ |
| | | $\lambda_{\dot\alpha}$ | Lagr. mult. | adj $SO(k)$ | $(\text{length})^{-\frac{3}{2}}$ |
| $-1/7$ | R | $\mu$ | | $\mathbf{8} \times \mathbf{k}$ | (length) |
| | NS | $w$ | (auxiliary) | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^0$ |

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- The SO($k$) rep. is determined by the orientifold projection

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- Abelian part of $a_\mu$, $M_\alpha$ ~ Goldstone modes of the (super)translations on the D7 broken by D(-1)'s. Identified with coordinates $x_\mu$,$\theta_\alpha$

# The moduli spectrum

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| | NS | $w$ | (auxiliary) | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^0$ |

- For "mixed" strings, no bosonic moduli from the NS sector: characteristic of "exotic" instantons

# The moduli action

- The action reads:

$$\mathcal{S}(\mathcal{M}_{(k)},\Phi)=\mathrm{tr}\Big\{i\lambda_{\dot\alpha}\gamma_\mu^{\dot\alpha\beta}[a^\mu,M_\beta]+\frac{1}{2g_0^2}\lambda_{\dot\alpha}[\chi,\lambda^{\dot\alpha}]+M^\alpha[\bar\chi,M_\alpha]$$
$$+\frac{1}{2g_0^2}D_mD^m-\frac{1}{2}D_m(\tau^m)_{\mu\nu}\left[a^\mu,a^\nu\right]$$
$$+\left[a_\mu,\bar\chi\right][a^\mu,\chi]+\frac{1}{2g_0^2}[\bar\chi,\chi]^2\Big\}$$
$$+\mathrm{tr}\left\{\mu^T\mu\chi\right\}+\mathrm{tr}\left\{\mu^T\Phi(x,\theta)\mu\right\}+\mathrm{tr}\left\{w^T w\right\}$$

- The "supercoordinate" moduli $x,\theta$ only appear through $\Phi(x,\theta)$. The remaining "centred" moduli are denoted as $\widehat{\mathcal{M}}_{(k)}$

# All instanton numbers ...

... lead to quartic terms

- Effective action (using $q = e^{2\pi i\tau}$):

$$\mathcal{S}_{\text{n.p.}}(\Phi) = \int d^8x\, d^8\theta \sum_k q^k \int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\Phi(x,\theta))}$$

- In our "conformal" set-up, with with SO(8) gauge group on the D7, counting the dimensions of the moduli we get

$$\left[d\widehat{\mathcal{M}}_{(k)}\right] = (\text{length})^{-4}$$

- Thus

$$\int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\Phi(x,\theta))} = \text{quartic invariant in } \Phi(x,\theta)$$

- Integration over $d^8\theta$ leads to terms of the form "$t_8\, F^4$ "

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- Integration over $d^8\theta$ leads to terms of the form “$t_8 F^4$ ”
- The “non-conformal” case of $N \neq 8$ D7’s has been considered in Fucito et al, 2009

# One-instanton case

- For $k = 1$ things are particularly simple
  - The spectrum of moduli is reduced to $\{x, \theta, \mu\}$
  - The moduli action is simply $\mathcal{S}_{\text{inst}} = -2\pi i\tau + \mu^T \Phi(x, \theta)\mu$
- The instanton-induced interactions are thus

$$\int d^8x\, d^8\theta\, q \int d\mu\, e^{-\mu^T \Phi(x,\theta)\mu} \sim \int d^8x\, d^8\theta\, q\, \mathrm{Pf}\big(\Phi(x, \theta)\big)$$

- A new structure, associated to the SO(8) invariant " $t_8\mathrm{Pf}(F)$ ", appears in the effective action at the one-instanton level after the $d^8\theta$ integration

# Multi-instantons

- For $k > 1$ things are more complicated, but we can exploit the SUSY properties of the moduli action, which lead to:
  - an equivariant cohomological BRST structure
  - a localization of the moduli integrals (after suitable closed string deformations)
- Similar techniques have been successfully used to
  - compute the YM integrals in $d = 10, 6, 4$ and the D-instanton partition function
    Moore+Nekrasov+Shatashvili, 1998
  - compute multi-instanton effects in $\mathcal{N} = 2$ SYM in $d = 4$ and compare with the Seiberg-Witten solution
    Nekrasov, 2002; + ...
  - derive the multi-instanton calculus using D3/D(–1) brane systems
    Fucito et al, 2004; Billò et al, 2006; ...

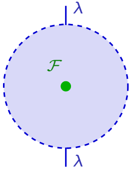
# Deformations from RR background

- Suitable deformations that help to fully localize the integral arise from RR field-strengths 3-form with one index on $T_2$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu z}\ , \quad \bar{\mathcal{F}}_{\mu\nu} \equiv F_{\mu\nu z}$$

- The $\mathcal{F}_{\mu\nu}$ is taken in an SO(7)$\subset$ SO(8) (Lorentz) with spinorial embedding

- Disk diagrams with RR insertions modify the moduli action




$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi) \to \mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F})$$

(here we introduced the v.e.v. $\varphi = \langle \Phi \rangle$)

- Single out one of the supercharges $Q_{\dot\alpha}$, say $Q = Q_8$. After relabeling some of the moduli:

$$M_\alpha \to M_\mu \equiv (M_m, -M_8)\ , \quad \lambda_{\dot\alpha} \to (\lambda_m, \eta) \equiv (\lambda_m, \lambda_8)$$

one has

$$Qa^\mu = M^\mu\ , \quad Q\lambda_m = -D_m\ , \quad Q\bar\chi = -i\sqrt{2}\eta\ , \quad Q\chi = 0\ , \quad Q\mu = w$$

- Moreover, on any modulus,

$$Q^2 \bullet = T_{SO(k)}(\chi)\bullet + T_{SO(8)}(\varphi)\bullet + T_{SO(7)}(\mathcal{F})\bullet$$

where
  - $T_{SO(k)}(\chi) =$ inf.mal SO($k$) rotation parametrized by $\chi$
  - $T_{SO(8)}(\varphi) =$ inf.mal SO(8) rotation parametrized by $\varphi$
  - $T_{SO(7)}(\mathcal{F}) =$ inf.mal SO(7) rotation parametrized by $\mathcal{F}$

# Symmetries of the moduli

- The action of the BRS charge $Q$ is thus determined by the symmetry properties of the moduli

| | SO($k$) | SO(7) | SO(8) |
|---|---|---|---|
| $a^\mu$ | symm | $\mathbf{8}_s$ | $\mathbf{1}$ |
| $M^\mu$ | symm | $\mathbf{8}_s$ | $\mathbf{1}$ |
| $D_m$ | adj | $\mathbf{7}$ | $\mathbf{1}$ |
| $\lambda_m$ | adj | $\mathbf{7}$ | $\mathbf{1}$ |
| $\bar{\chi}$ | adj | $\mathbf{1}$ | $\mathbf{1}$ |
| $\eta$ | adj | $\mathbf{1}$ | $\mathbf{1}$ |
| $\chi$ | adj | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mu$ | **k** | $\mathbf{1}$ | $\mathbf{8}_v$ |

- The (deformed) action is BRST-exact:

$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F}) = Q\Xi$$

- $\bar{\mathcal{F}}$ only appears in the "gauge fermion" Ξ: the final result does not depend on it

- The (deformed) BRST structure allows to suitably rescale the integration variables and show that the semiclassical approximation is exact

Moore+Nekrasov+Shatashvili, 1998; ...; Nekrasov, 2002; Flume+Poghossian, 2002; Bruzzo et al, 2003; ...

# Scaling to localization

- Many integrations reduce to quadratic forms:

$$Z_k(\varphi, \mathcal{F}) \equiv \int d\mathcal{M}_{(k)}\; e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F})} \;=\; \ldots \;=\; \ldots$$

$$= \int \{da\, dM\, dD\, d\lambda\, d\mu\, d\chi\}\; e^{-\mathrm{tr}\{\frac{g}{2}D^2 - \frac{g}{2}\lambda\widetilde{Q}^2\lambda + \frac{t}{4}a\widetilde{Q}^2 a + \frac{t}{4}M^2 + {}^{t}\mu\widetilde{Q}^2\mu\}}$$

$$\sim \int \{d\chi\}\; \frac{\mathrm{Pf}_\lambda\left(\widetilde{Q}^2\right)\, \mathrm{Pf}_\mu\left(\widetilde{Q}^2\right)}{\det_a\left(\widetilde{Q}^2\right)^{1/2}}$$

- The $\chi$ integrals can be done as contour integrals and the final result for $Z_k(\varphi, \mathcal{F})$ comes from a sum over residues

Moore+Nekrasov+Shatashvili, 1998

# The recipe

- From the explicit expression of $Z_k(\varphi, \mathcal{F})$, we can obtain the non-perturbative effective action. However:
- At instanton number $k$, there are disconnected contributions from smaller instantons $k_i$ (with $\sum_i k_i = k$). To isolate the connected components we have to take the log:

$$\mathcal{Z} = \sum_k Z_k(\varphi, \mathcal{F})\, q^k \;\rightarrow\; \log \mathcal{Z}$$

- In obtaining $Z_k(\varphi, \mathcal{F})$ we integrated also over $x$ and $\theta$ producing a factor of $\mathcal{E}^{-1} \sim \det(\mathcal{F})^{-1/2}$. To remove this contribution we have to multiply by $\mathcal{E}$

$$\log \mathcal{Z} \;\rightarrow\; \mathcal{E} \log \mathcal{Z}$$

before turning off the RR deformation.

- All in all, we obtain the non-perturbative part of the D7-brane effective action:

$$S_{(\mathrm{n.p.})} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\; F_{(\mathrm{n.p.})}\big(\Phi(x,\theta)\big)$$

with the "prepotential" $F_{(\mathrm{n.p.})}(\Phi)$ given by

$$F_{(\mathrm{n.p.})}(\Phi) = \mathcal{E} \log \mathcal{Z}\Big|_{\varphi\to\Phi,\mathcal{F}\to 0}$$

and with

$$\mathcal{Z} = \sum_k Z_k(\varphi,\mathcal{F})\, q^k \qquad , \qquad \mathcal{E} \sim \det(\mathcal{F})^{1/2}$$

# Explicit results

- Expanding in instanton numbers, $F^{(\mathrm{n.p.})} = \sum_k q^k F_k$, we have

$$F_1 = \mathcal{E} Z_1 \,,$$

$$F_2 = \mathcal{E} Z_2 - \frac{F_1^2}{2\mathcal{E}} \,,$$

$$F_3 = \mathcal{E} Z_3 - \frac{F_2 F_1}{\mathcal{E}} - \frac{F_1^3}{6\mathcal{E}^2}$$

$$F_4 = \mathcal{E} Z_4 - \frac{F_3 F_1}{\mathcal{E}} - \frac{F_2^2}{2\mathcal{E}} - \frac{F_2 F_1^2}{2\mathcal{E}^2} - \frac{F_1^4}{24\mathcal{E}^3} \,,$$

$$F_5 = \mathcal{E} Z_5 - \frac{F_4 F_1}{\mathcal{E}} - \frac{F_3 F_2}{\mathcal{E}} - \frac{F_3 F_1^2}{2\mathcal{E}^2} - \frac{F_2^2 F_1}{2\mathcal{E}^2} - \frac{F_2 F_1^3}{6\mathcal{E}^3} - \frac{F_1^5}{120\mathcal{E}^4} \,,$$

$$\ldots\ldots$$

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$$F_1 = 8\mathrm{P}f(\Phi) \,,$$

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$$F_5 = \mathcal{E}Z_5 - \frac{F_4F_1}{\mathcal{E}} - \frac{F_3F_2}{\mathcal{E}} - \frac{F_3F_1^2}{2\mathcal{E}^2} - \frac{F_2^2F_1}{2\mathcal{E}^2} - \frac{F_2F_1^3}{6\mathcal{E}^3} - \frac{F_1^5}{120\mathcal{E}^4}\ ,$$

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$$F_3 = \frac{32}{3}\mathrm{P}f(\Phi)\,,$$

$$F_4 = \frac{1}{4}\mathrm{Tr}\Phi^4 - \frac{1}{4}\left(\mathrm{Tr}\Phi^2\right)^2\,,$$

$$F_5 = \frac{48}{5}\mathrm{P}f(\Phi)\,,$$

......

# Explicit results

- The D-instanton induced effective "prepotential" is

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\left(q+\frac{4}{3}q^3+\frac{6}{5}q^5+\dots\right)+\mathrm{Tr}\Phi^4\left(\frac{1}{2}q^2+\frac{1}{4}q^4+\dots\right)$$
$$+\left(\mathrm{Tr}\Phi^2\right)^2\left(\frac{1}{4}q^2+\frac{1}{4}q^4+\dots\right)$$

- It is natural to generalize these results and write

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\sum_{k=1} d_{2k-1}\,q^{2k-1}+\frac{1}{2}\mathrm{Tr}\Phi^4\sum_{k=1}\left(d_k\,q^{2k}-d_k\,q^{4k}\right)$$
$$+\frac{1}{8}\left(\mathrm{Tr}\Phi^2\right)^2\sum_{k=1}\left(d_k\,q^{4k}-2d_k\,q^{2k}\right)$$

with $d_k = \sum_{\ell|k}\frac{1}{\ell}$ sum over the inverse divisors of $k$

# Complete result

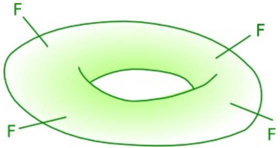
- Taking into account the contributions at tree-level for $\mathrm{Tr}F^4$ and at 1-loop for $\left(\mathrm{Tr}F^2\right)^2$, the full expression for the quartic terms in the effective action of the D7-branes reads

$$2\, t_8\, \mathrm{Pf}(F) \log \left| \frac{\eta(\tau + 1/2)}{\eta(\tau)} \right|^4 + \frac{t_8\, \mathrm{Tr}F^4}{4} \log \left| \frac{\eta(4\tau)}{\eta(2\tau)} \right|^4 + \frac{t_8\, (\mathrm{Tr}F^2)^2}{16} \log \left( \mathrm{Im}\,\tau\, \mathrm{Im}\, U \, \frac{|\eta(2\tau)|^8\, |\eta(U)|^4}{|\eta(4\tau)|^4} \right)$$

with $q = e^{2\pi i \tau}$

# Heterotic / Type I′ duality

- In the $SO(8)^4$ Heterotic String on $T_2$ the BPS-saturated quartic terms in $F$ arise at 1-loop




$$\frac{t_8\,\mathrm{Tr}F^4}{4}\,\log\left|\frac{\eta(4T)}{\eta(2T)}\right|^4 + \frac{t_8\,(\mathrm{Tr}F^2)^2}{16}\,\log\left(\mathrm{Im}\,T\,\mathrm{Im}\,U\,\frac{|\eta(2T)|^8\,|\eta(U)|^4}{|\eta(4T)|^4}\right)$$

Lerche+Stieberger, 1998; Gutperle, 1999; Kiritsis et al, 2000; ...

$$+2\,t_8\,\mathrm{Pf}(F)\,\log\left|\frac{\eta(T+1/2)}{\eta(T)}\right|^4$$

Gava et al, 1999

- Agrees with our Type I’ result under the duality map

$T$ : Kähler structure of the 2-torus $T_2 \longleftrightarrow \tau$ : axion-dilaton

world-sheet instantons $\longleftrightarrow$ D-instantons

# Conclusions and perspectives

# Remarks

- We have explicitly computed the effective couplings induced by stringy instantons in a simple 8d example, the D7/D(–1) system in Type I′, extending the philosophy used for "ordinary" instantons
- If we do not switch off the RR background $\mathcal{F}$ in the final expressions we get also non-perturbative gravitational corrections to $\mathrm{Tr}R^4$ and $\mathrm{Tr}R^2\mathrm{Tr}F^2$

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- If we do not switch off the RR background $\mathcal{F}$ in the final expressions we get also non-perturbative gravitational corrections to $\mathrm{Tr}R^4$ and $\mathrm{Tr}R^2\mathrm{Tr}F^2$
- The result checks out perfectly against the dual Heterotic SO(8) theory:
  - Assuming the duality, confirms our procedure to deal with the stringy instantons
  - Assuming the correctness of our computation, yields very non-trivial check of this fundamental string duality

# 4d exotic instanton calculus

- We're now investigating simple models where
  - the gauge theory lives in four dimensions
  - there are "exotic" instantons, with no "size" moduli from mixed strings having more than 4 ND directions, ...
  - all instanton numbers can contribute to the effective action at order $F^2$ ("conformal" situation)
  - there is the chance of checking the result against a dual heterotic theory

# A specific model

- In particular, we are considering Type I' theory on

$$\mathbb{R}^{1,3} \times T_4/\mathbb{Z}_2 \times T_2$$

  with 8 D7-branes and 8 D3-branes (T-dual on $T_2$ of PS-GP model)
    - D(-1)'s represent exotic instantons w.r.t. to the gauge theory on the D7's
- Preliminary analysis indicates that the calculus of the induced effective action is feasible with methods analogues to those presented here