Some national history. . .

The Rütlischwur is a legendary oath of the Old Swiss Confederacy

- between three cantons (Uri, Schwyz and Unterwalden)
- beginning of Switzerland

Some local physics' history...

But there is also the "Uetli Schwur" in physics:

[...] It was not long after the publication of Bohr's papers
that Stern and von Laue went for a walk up the Uetliberg,
a small mountain just outside Zürich.
On the top they sat down and talked about physics,
in particular about the new atom model.

There and then they made the "Uetli Schwur":
If that crazy model of Bohr turned out to be right,
then they would leave physics.

It did and they didn't.

[A. Pais]

p-branes on the waves

Ben Craps,[a] Frederik De Roo[a,b,1], Oleg Evnin[a] and Federico Galli[a]

[a] *Vrije Universiteit Brussel and*
The International Solvay Institutes
[b] *Universiteit Gent*
[1] *Aspirant FWO*

fderoo@tena4.vub.ac.be

September 8, 2009

Where does the universe come from?

Quantum gravity expected to resolve initial spacelike singularity

String theory still has problems in presence of

- singularities
- time-dependences

$\Rightarrow$ investigate singular and time-dependent backgrounds in string theory

Age of universe: ca. $14\,\mathrm{Gyr}$

$1\,\mathrm{yr} \rightarrow 7 \cdot 10^{-9}\,\%$

p-branes on the waves: outline

Singular and time-dependent backgrounds in string theory

- why plane waves?
- Matrix big bang
- p-branes embedded in plane waves

A family of 10-dimensional supergravity solutions [1]

D0-branes embedded in plane waves

[1] B. Craps, F.D.R., O. Evnin, F. Galli, arXiv: 0905.1843 [hep-th]

+ work in progress

Why plane waves?

Plane waves: first approximation to spacetime singularities

- obtained by Penrose limit
- capture tidal forces of singularities

[Blau e.a.]

Exact string theory solutions

- no α' corrections

[Horowitz, Steif; Amati, Klimčík]

Exactly solvable σ-models [Papadopoulos, Russo, Tseytlin]

Time-dependent waves possible

- add dilaton for background consistency (e.g.)

Matrix big bang

Flat Minkowski space + light-like linear dilaton

- $ds^2 = -2dX^+dX^- + \sum_{i=1}^{8} \left(dX^i\right)^2$
- $\phi = -QX^+$

DLCQ

- compactify X^- and focus on sector with $p^+ = 2\pi N/R$
- Lorentz boost
- T and S duality

$$\Downarrow$$

N D1-branes wrapped around x^1 in IIB

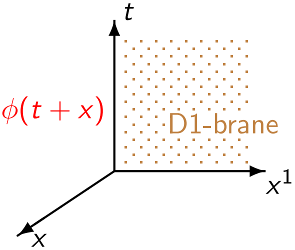
- $ds^2 = -2dudv + u\sum_{i=1}^{8} \left(dx^i\right)^2$
- $\phi = \log u$

[Craps, Sethi, Verlinde]

p-branes embedded in plane waves

Matrix big bang leads to D1 branes in a dilaton-gravity plane wave

- branes wrapped along x^1
- $ds^2 = -dt^2 + dx^2 + (t+x)\sum_{i=1}^{8}\left(dx^i\right)^2, \quad \phi = \log(t+x)$



Not supersymmetric, but static solutions exist (DBI analysis)

D1 along dilaton preserves susy $\Rightarrow$ easier supergravity solution?

p-branes on the waves: outline

Singular and time-dependent backgrounds in string theory

A family of 10-dimensional supergravity solutions

- restricted ansatz for extremal branes
- solution strategy in four steps
- solution in Brinkmann coordinates

D0-branes embedded in plane waves

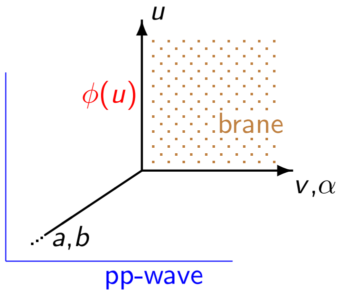
A family of ten-dimensional supergravity solutions

extended extremal supersymmetric Ramond-Ramond p-branes

embedded into dilaton-gravity plane waves

- time-dependent (lightcone time $u = t + x$)
- arbitrary profile $\phi(u)$
- isotropy in transverse coordinates x_a, $x_b \ldots$

brane world-volume parallel with propagation direction of the wave



Equations of motion in Einstein frame

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\sum_p \frac{1}{(p+2)!} e^{(3-p)\phi/2}\left[\mathcal{F}^2_{\mu\nu} - \frac{p+1}{8} g_{\mu\nu}\mathcal{F}^2\right]$$

$$\Box\phi = \frac{1}{4}\sum_p \frac{3-p}{(p+2)!} e^{(3-p)\phi/2}\mathcal{F}^2$$

$$\partial_\mu\left(\sqrt{-g}\, e^{(3-p)\phi/2}\mathcal{F}^{\mu\cdots}\right) = 0$$

$$\partial_{[\mu}\mathcal{F}_{\nu\ldots]} = 0$$

$$\mathcal{F}:\ (p+2)\text{–form},\ \ \mathcal{F}^2_{\mu\nu} = \mathcal{F}_{\mu\ldots}\mathcal{F}_{\nu}^{\ \cdots},\ \ \mathcal{F}^2 = \mathcal{F}_{\ldots}\mathcal{F}^{\cdots}$$

For extremal branes a restricted ansatz suffices

$$
\begin{aligned}
ds^2 &= A(u,r)\left(-2du dv + K(u,r)du^2 + dy_\alpha^2\right) + B(u,r)dx_a^2 \\
\phi &= \phi(u,r), & r = \sqrt{x_a^2} \\
\mathcal{F}_{uv\alpha_1\ldots\alpha_{p-1}a} &= \tfrac{x^a}{r}F(u,r)A^{(p+1)/2}B^{(p-7)/2}\epsilon_{\alpha_1\ldots\alpha_{p-1}}, & p<3,\ \text{Electric} \\
\mathcal{F}_{a_1\ldots a_{8-p}} &= \tfrac{x^a}{r}F(u,r)\epsilon_{a_1\ldots a_{8-p}a}, & p>3,\ \text{Magnetic}
\end{aligned}
$$

$K(u,r)$ captures the wave profile plus some corrections

What's new?

Relaxed assumptions for non-extremal branes

$$
\begin{aligned}
ds^2 = &A(u,r)\left(-2du dv + K(u,r)du^2 + L(u,r)dy_\alpha^2\right) \\
&+ g_{ua}(u,r)du\, dx_a + B(u,r)dx_a^2
\end{aligned}
$$

Restricted ansatz simplifies structure of Einstein's equations

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\sum_p \frac{1}{(p+2)!} e^{(3-p)\phi/2}\left[\mathcal{F}^2_{\mu\nu} - \frac{p+1}{8} g_{\mu\nu}\mathcal{F}^2\right]$$

Nonzero components

- uu, ua
- $uv = \alpha\alpha$
- $ab = \delta_{ab} + x_a x_b$

Electric ansatz satisfies Bianchi identity
Magnetic ansatz satisfies form equation of motion

Dilaton equation

Solution strategy in five steps

Step 1: Equations without time derivatives can be solved as for time-independent branes

- Dilaton equation;
- δ_{ab}, $x_a \cdot x_b$ and uv components of Einstein equations
- Form equation: integrate $\Rightarrow$ brane charge

Step 2: Promote all integration constants to functions of time

Step 3: String frame and coordinate choice

Step 4: Time-dependence is captured by uu and ua components of Einstein equations

Step 5: Plane wave asymptotics and coordinate choice

Step 1: Equations without time-derivatives can be solved as for time-independent branes

Take particular integrals for extremal branes

Dilaton equation $\left(r^{8-p}A^{(p+1)/2}B^{(7-p)/2}\left(\phi' - \frac{2(p-3)}{7-p}\frac{A'}{A}\right)\right)' = 0$

δ_{ab} equation

uv equation — Liouville equation

$x_a \cdot x_b$ equation — Energy conservation [Lü, Pope, Xu]
one constraint on integration constants

$\Rightarrow$ "$\phi(r)$", "$A(r)$", "$B(r)$"

Step 2: Promote all integration constants to functions of time

Integration constants

- from $\phi(u,r)$, $A(u,r)$, $B(u,r)$
- from $F(u,r)$: time-dependent brane charge
- one constraint on integrations constants

$\Rightarrow$ three time-dependent functions $h(u)$, $f(u)$, $\mu(u)$

Step 3: String frame and coordinate choice

Switch to string frame: $ds_S^2 = ds_E^2 e^{\phi/2}$

$$ds_S^2 = A_s(u,r)\left(-2dudv + K(u,r)du^2 + dy_\alpha^2\right) + B_s(u,r)dx_a^2$$

Coordinate choice for u: $g_{uv}dudv \to -2dudv$ when $r \to \infty$

$$A_s(u,r) = \left(1 + h(u)\frac{R^{7-p}}{r^{7-p}}\right)^{-1/2}$$

$$B_s(u,r) = \mu(u)\ \left(1 + h(u)\frac{R^{7-p}}{r^{7-p}}\right)^{1/2}$$

$$\phi(u,r) = f(u) + \frac{3-p}{4}\log\left(1 + h(u)\frac{R^{7-p}}{r^{7-p}}\right)$$

- has 8 supersymmetries
- constant R is related to brane charge

Remaining coordinate freedom ($\tilde{v}(u,v,r)$ and $\tilde{x}(u,x)$)

Step 4: *ua* and *uu* equations constrain time-dependence and determine wave profile $K(u, r)$

$$ds_S^2 = A_s(u,r)\left(-2dudv + K(u,r)du^2 + dy_\alpha^2\right) + B_s(u,r)dx_a^2$$

Further restrictions from remaining two equations

ua equation $\Rightarrow$ relation between $h(u)$, $\mu(u)$ and $f(u)$

uu equation $\Rightarrow$ $K(u,r) = \kappa_1(u) r^2 + \kappa_2(u) r^{p-5}$

$$h = e^f \sqrt{\mu}^{p-7}$$

$$\kappa_1(u) = \frac{1}{4}\mu\left[\frac{8}{9-p}\ddot{f} - 2\frac{\ddot{\mu}}{\mu} + \frac{\dot{\mu}^2}{\mu^2}\right]$$

$$\kappa_2(u) = \frac{1}{p-5} e^f \frac{R^{7-p}}{\sqrt{\mu}^{5-p}}\left[\ddot{f} - \dot{f}\frac{\dot{\mu}}{\mu} - \frac{\ddot{\mu}}{\mu} + \frac{9-p}{4}\frac{\dot{\mu}^2}{\mu^2}\right]$$

Step 5: Plane wave asymptotics and coordinate choice

Wave profile $K(u,r) = \kappa_1(u)\, r^2 + \kappa_2(u)\, r^{p-5}$

For $r \to \infty$ $\quad ds_S^2 = -2dudv + \kappa_1(u) r^2 du^2 + dy_\alpha^2 + \mu(u) dx_a^2$

$\phi = f(u)$

Brinkmann coordinates

$$ds^2 = -2dudv + \frac{2}{9-p}\ddot{f}(u) r^2 du^2 + dy_\alpha^2 + dx_a^2$$

$$\phi = f(u)$$

Rosen coordinates

$$ds^2 = -2dudv + dy_\alpha^2 + \mu(u) dx_a^2$$

$$\phi = f(u)$$

Coordinate transformation between Brinkmann and Rosen can be extended to our metrics for all r

Use remaining coordinate freedom to set $\mu(u) = 1$

Solution in Brinkmann coordinates

$$ds_S^2 = \frac{1}{\sqrt{\mathcal{H}(u,r)}} \frac{\ddot{f}(u)}{5-p} r^2 \left(2 + \frac{1-p}{9-p} - \mathcal{H}(u,r) \right) du^2$$

$$+ \frac{1}{\sqrt{\mathcal{H}(u,r)}} \left(-2du dv + dy_\alpha^2 \right) + \sqrt{\mathcal{H}(u,r)} dx_a^2$$

$$\phi = f(u) + \frac{3-p}{4} \log \mathcal{H}(u,r)$$

$$\mathcal{F}_{uv\alpha_1 \ldots \alpha_{p-1} a} = \frac{x^a}{r} e^{-f(u)} \, \frac{\partial}{\partial r} \mathcal{H}^{-1}(u,r) \, \epsilon_{\alpha_1 \ldots \alpha_{p-1}}, \quad p < 3$$

$$\mathcal{F}_{a_1 \ldots a_{8-p}} = \frac{x^a}{r} e^{-f(u)} \, \frac{\partial}{\partial r} \mathcal{H}(u,r) \, \epsilon_{a_1 \ldots a_{8-p} a}, \quad p > 3$$

$$\mathcal{H}(u,r) = 1 + e^{f(u)} \frac{R^{7-p}}{r^{7-p}}$$

p-branes on the waves: outline

Singular and time-dependent backgrounds in string theory

A family of 10-dimensional supergravity solutions

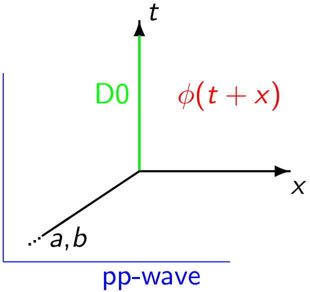
D0-branes embedded in plane waves

D0-branes embedded in plane waves

No aligment possible

Solution suggested by DBI analysis

Perturbation analysis



Summary

A family of ten-dimensional supergravity solutions

- p-branes embedded into dilaton-gravity plane waves
- brane world-volume parallel
  with propagation direction of the wave
- time-dependent, supersymmetric solutions
  and wave profile may be singular

Currently studying D0-branes
embedded into dilaton-gravity plane waves