Generalized gaugings and the field-antifield formalism

Jan De Rydt

KATHOLIEKE UNIVERSITEIT
LEUVEN

15th European Workshop on String Theory -
September 7, 2009 - Zürich

Based on work in collaboration with:
Frederik Coomans, Antoine Van Proeyen

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

- Non-zero gauge coupling constant: Gauged supergravity

  examples: $d = 4, \mathcal{N} = 8$ supergravity with SO(8) gauge group [de Wit, Nicolai], non-compact versions of SO(8),. . .
- Massive deformations

  example: Romans massive deformation of IIA. [Romans]
- ...

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

- Non-zero gauge coupling constant: Gauged supergravity

  examples: $d = 4, \mathcal{N} = 8$ supergravity with SO(8) gauge group [de Wit, Nicolai], non-compact versions of SO(8),. . .
- Massive deformations

  example: Romans massive deformation of IIA. [Romans]
- ...

Can the possible deformations be CLASSIFIED?

The embedding tensor formalism does exactly that. However, in general it has a complicated gauge structure with an open, soft and reducible gauge group.

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

- Non-zero gauge coupling constant: Gauged supergravity

  examples: $d = 4, \mathcal{N} = 8$ supergravity with SO(8) gauge group [de Wit, Nicolai], non-compact versions of SO(8),. . .
- Massive deformations

  example: Romans massive deformation of IIA. [Romans]
- ...

Can the possible deformations be CLASSIFIED?

The embedding tensor formalism does exactly that. However, in general it has a complicated gauge structure with an open, soft and reducible gauge group.

The field-antifield (Batalin-Vilkovisky) formalism gives us a framework to tackle these complicated gauge structures and provides a much simpler formulation.

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

Part 1: Gauged supergravity

Can the possible deformations be CLASSIFIED?

The embedding tensor formalism does exactly that. However, in general it has a complicated gauge structure with an open, soft and reducible gauge group.

The field-antifield (Batalin-Vilkovisky) formalism gives us a framework to tackle these complicated gauge structures and provides a much simpler formulation.

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

Part 1: Gauged supergravity

Can the possible deformations be CLASSIFIED?

Part 2: The embedding tensor formalism

The field-antifield (Batalin-Vilkovisky) formalism gives us a framework to tackle these complicated gauge structures and provides a much simpler formulation.

Introduction and outline

Pure (ungauged) supergravity theories:

No charged matter fields, no scalar potential, only global symmetries and Abelian gauge groups.

They can have different DEFORMATIONS:

Part 1: Gauged supergravity

Can the possible deformations be CLASSIFIED?

Part 2: The embedding tensor formalism

Part 3: Field-antifield formalism

Part 1. Gauging: the conventional approach

Ungauged Supergravity
(4D)

No charged matter fields, no potential

Part 1. Gauging: the conventional approach

Ungaged Supergravity

(4D)

No charged matter fields, no potential

- Rigid symmetry group
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\mathrm{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$

Part 1. Gauging: the conventional approach

Ungauged Supergravity
(4D)

No charged matter fields, no potential

- Rigid symmetry group
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\mathrm{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$
- Invariance group of action
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

Part 1. Gauging: the conventional approach

Ungauged Supergravity
(4D)

No charged matter fields, no potential

$\xrightarrow{\text{deformation}}$

Gauged Supergravity
(4D)

Masslike terms, potential

- Rigid symmetry group
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\text{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$
- Invariance group of action
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

Part 1. Gauging: the conventional approach

Ungauged Supergravity (4D) $\xrightarrow{\text{deformation}}$ **Gauged Supergravity** (4D)

No charged matter fields, no potential

Masslike terms, potential

- Rigid symmetry group
  - $Sp(2n_V+2,\mathbb{R})\times\mathrm{Iso}(\mathcal{M}_{\mathrm{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N}=8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56,\mathbb{R})$
- Invariance group of action ⟶ promote subgroup to local symmetry
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

  - minimal couplings w/ electric vectors

  example: $SO(8)$ [de Wit, Nicolai]

Part 1. Gauging: the conventional approach

Ungauged Supergravity (4D) $\xrightarrow{\text{deformation}}$ **Gauged Supergravity** (4D)

No charged matter fields, no potential

Masslike terms, potential

- Rigid symmetry group
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\text{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$
- Invariance group of action ⟶ promote subgroup to local symmetry
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

  - minimal couplings w/ electric vectors

  example: $SO(8)$ [de Wit, Nicolai]
- Possible gauge groups depend on the selected duality frame. Is there a systematic way to find all possibilities?

Part 1. Gauging: the conventional approach

Ungauged Supergravity (4D) $\xrightarrow{\text{deformation}}$ **Gauged Supergravity** (4D)

No charged matter fields, no potential

Masslike terms, potential

- Rigid symmetry group ⟶ breaks duality covariance
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\mathrm{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$

↑

- Invariance group of action ⟶ promote subgroup to local symmetry
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

  - minimal couplings w/ electric vectors

  example: $SO(8)$ [de Wit, Nicolai]

- Possible gauge groups depend on the selected duality frame. Is there a systematic way to find all possibilities?

Part 1. Gauging: the conventional approach

Ungauged Supergravity (4D) $\xrightarrow{\text{deformation}}$ **Gauged Supergravity** (4D)

No charged matter fields, no potential

Masslike terms, potential

- Rigid symmetry group $\longrightarrow$ breaks duality covariance
  - $Sp(2n_V + 2, \mathbb{R}) \times \mathrm{Iso}(\mathcal{M}_{\mathrm{scalar}})$
  - symplectic duality covariance

  example: pure $\mathcal{N} = 8$, [Cremmer, Julia]

  $E_{7(7)}$ embedded in $Sp(56, \mathbb{R})$

$\uparrow$

- Invariance group of action $\longrightarrow$ promote subgroup to local symmetry
  - subgroup of rigid symmetry group

  example: (non-unique) subgroup of $E_{7(7)}$

  - minimal couplings w/ electric vectors

  example: $SO(8)$ [de Wit, Nicolai]

- Possible gauge groups depend on the selected duality frame. Is there a systematic way to find all possibilities?
- Is there a way to maintain duality invariance?

Part 2. Generalized gaugings - the embedding tensor formalism

[de Wit, Samtleben, Trigiante]

Part 2. Generalized gaugings - the embedding tensor formalism

[de Wit, Samtleben, Trigiante]

Central object: embedding tensor ${\Theta_M}^{\alpha}$

- It determines the decomposition of gauge generators into generators of the rigid symmetry group (δ_α):

$$X_M = \sum_{\alpha} {\Theta_M}^{\alpha} \delta_\alpha$$

Part 2. Generalized gaugings - the embedding tensor formalism

[de Wit, Samtleben, Trigiante]

Central object: embedding tensor ${\color{red}\Theta_M{}^\alpha}$

- It determines the decomposition of gauge generators into generators of the rigid symmetry group (δ_α):

$$X_M = \sum_\alpha {\color{red}\Theta_M{}^\alpha} \delta_\alpha$$

- It characterizes the gauging completely (i.e. covariant derivatives, masslike terms, scalar potential), e.g.:

$$D_\mu \phi = \left(\partial_\mu - A_\mu{}^M {\color{red}\Theta_M{}^\alpha} \delta_\alpha\right) \phi \quad \text{with} \quad A_\mu{}^M = \begin{pmatrix} A_\mu{}^\Lambda \\ A_{\mu\Lambda} \end{pmatrix} \begin{matrix} \longleftarrow \text{electric} \\ \longleftarrow \text{magnetic} \end{matrix}$$

$\Rightarrow$ Both electric and magnetic vectors appear.

Part 2. Generalized gaugings - the embedding tensor formalism

[de Wit, Samtleben, Trigiante]

Central object: embedding tensor ${\Theta_M}^\alpha$

- It determines the decomposition of gauge generators into generators of the rigid symmetry group (δ_α):

$$X_M = \sum_\alpha {\Theta_M}^\alpha \delta_\alpha$$

- It characterizes the gauging completely (i.e. covariant derivatives, masslike terms, scalar potential), e.g.:

$$D_\mu \phi = \left(\partial_\mu - {A_\mu}^M {\Theta_M}^\alpha \delta_\alpha\right)\phi \quad \text{with} \quad {A_\mu}^M = \begin{pmatrix} {A_\mu}^\Lambda \\ A_{\mu\Lambda} \end{pmatrix} \begin{matrix} \longleftarrow \text{electric} \\ \longleftarrow \text{magnetic} \end{matrix}$$

$\Rightarrow$ Both electric and magnetic vectors appear.

- Conventional gaugings: only coupling to electric vectors: ${\Theta_M}^\alpha = \begin{pmatrix} {\Theta_\Lambda}^\alpha \\ 0 \end{pmatrix}$
- Generalized gaugings appear in
  - Flux compactifications with electric and magnetic fluxes,
  - Scherk-Schwarz reductions,
  - . . .

  The deformation parameters (such as flux parameters) appear as components of the embedding tensor.

Problems solved...

In the conventional gauging procedure, we encountered two main drawbacks:

1. Possible gauge groups depend on the selected duality frame. Is there a systematic way to find all possibilities?
2. Is there a way to maintain duality invariance?

Problems solved...

In the conventional gauging procedure, we encountered two main drawbacks:

1. Possible gauge groups depend on the selected duality frame. Is there a systematic way to find all possibilities?
2. Is there a way to maintain duality invariance?

The embedding tensor formalism provides a positive answer:

1. Admissible embedding tensors can be characterized and determine all possible gaugings.

   This requires solving the constraints:

   - Closure of the gauge algebra: $[X_M, X_N] = -X_{MN}{}^P X_P$
   - Locality: $\sum_\Lambda \Theta^{\Lambda[\alpha}\Theta_\Lambda{}^{\beta]} = 0$
   - Linear constraint: $X_{(MN}{}^Q\Omega_{P)Q} = 0$

     In chiral $\mathcal{N} = 1$ theories, anomaly cancellation can be achieved by changing this constraint to $X_{(MN}{}^Q\Omega_{P)Q} = \Theta_M{}^\alpha\Theta_N{}^\beta\Theta_P{}^\gamma d_{\alpha\beta\gamma}$ [Schmidt, Trigiante, Van Proeyen, Zagermann, DR]. The tensor $d_{\alpha\beta\gamma}$ characterizes the anomaly.
2. The formalism restores duality covariance (as long as we treat $\Theta_M{}^\alpha$ as a spurious object).

Extra complications...

The covariant treatment causes some difficulties:

Closure constraint: $[X_M, X_N] = -X_{MN}{}^P X_P$

- Consistency requires that $X_{(MN)}{}^P X_P = 0 \quad \Rightarrow \quad X_{MN}{}^P = X_{[MN]}{}^P + X_{(MN)}{}^P$
- The Jacobi identity is violated:

$$X_{[MN]}{}^P X_{[QP]}{}^R + \text{cyclic} = -\tfrac{1}{3}\left(X_{[MN]}{}^P X_{(QP)}{}^R + \text{cyclic}\right)$$

Extra complications...

The covariant treatment causes some difficulties:

$$\text{Closure constraint: } [X_M, X_N] = -X_{MN}{}^P X_P$$

- Consistency requires that $X_{(MN)}{}^P X_P = 0 \quad \Rightarrow \quad X_{MN}{}^P = X_{[MN]}{}^P + X_{(MN)}{}^P$
- The Jacobi identity is violated:

$$X_{[MN]}{}^P X_{[QP]}{}^R + \text{cyclic} = -\frac{1}{3}\left(X_{[MN]}{}^P X_{(QP)}{}^R + \text{cyclic}\right)$$

Solution:

- Extra gauge transformations:

$$\delta A_\mu{}^M = D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{\lceil NP \rfloor}$$

- Extra 2-forms, with gauge transformations:

$$\delta B_{\mu\nu}{}^{\lceil MN \rfloor} = \partial_\mu \Xi_\nu{}^{\lceil MN \rfloor} + \ldots$$

Extra complications...

The covariant treatment causes some difficulties:

$$\text{Closure constraint: } [X_M, X_N] = -X_{MN}{}^P X_P$$

- Consistency requires that $X_{(MN)}{}^P X_P = 0 \quad \Rightarrow \quad X_{MN}{}^P = X_{[MN]}{}^P + X_{(MN)}{}^P$
- The Jacobi identity is violated:

$$X_{[MN]}{}^P X_{[QP]}{}^R + \text{cyclic} = -\tfrac{1}{3}\left(X_{[MN]}{}^P X_{(QP)}{}^R + \text{cyclic}\right)$$

Solution:

- Extra gauge transformations:

$$\delta A_\mu{}^M = D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{\lceil NP \rfloor}$$

- Extra 2-forms, with gauge transformations:

$$\delta B_{\mu\nu}{}^{\lceil MN \rfloor} = \partial_\mu \Xi_\nu{}^{\lceil MN \rfloor} + \dots$$

Invariant action:

- Kinetic terms: $F^M \wedge * F^N$
- Topological terms: $B^{\lceil MN \rfloor} \wedge B^{\lceil PQ \rfloor}$
- Chern-Simons terms: $A^M \wedge A^N \wedge dA^P$

Tensor hierarchy

So far we have introduced the fields $A_\mu{}^M$ and $B_{\mu\nu}{}^{\lceil MN \rfloor}$ with gauge transformations

$$\begin{aligned} \delta A_\mu{}^M &= D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{NP} \\ \delta B_{\mu\nu}{}^{\lceil MN \rfloor} &= \partial_{[\mu} \Xi_{\nu]}{}^{\lceil MN \rfloor} + \ldots \end{aligned}$$

Tensor hierarchy

So far we have introduced the fields ${A_\mu}^M$ and ${B_{\mu\nu}}^{\lceil MN \rfloor}$ with gauge transformations

$$\begin{aligned} \delta {A_\mu}^M &= D_\mu \Lambda^M - {X_{(NP)}}^M {\Xi_\mu}^{NP} \\ \delta {B_{\mu\nu}}^{\lceil MN \rfloor} &= \partial_{[\mu} {\Xi_{\nu]}}^{\lceil MN \rfloor} + \ldots + {Y^{\lceil MN \rfloor}}_{\lceil P \lceil QR \rfloor \rfloor} {\Phi_{\mu\nu}}^{\lceil P \lceil QR \rfloor \rfloor} \end{aligned}$$

This list can be extended to higher order tensors ${C_{\mu\nu\rho}}^{\lceil M \lceil NP \rfloor \rfloor}, \ldots$ with gauge transformations:

$$\delta {C_{\mu\nu\rho}}^{\lceil M \lceil NP \rfloor \rfloor} = \partial_{[\mu} {\Phi_{\nu\rho]}}^{\lceil M \lceil NP \rfloor \rfloor} + \ldots , \text{ etc.}$$

Tensor hierarchy

So far we have introduced the fields $A_\mu{}^M$ and $B_{\mu\nu}{}^{\lceil MN \rfloor}$ with gauge transformations

$$\begin{aligned} \delta A_\mu{}^M &= D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{NP} \\ \delta B_{\mu\nu}{}^{\lceil MN \rfloor} &= \partial_{[\mu} \Xi_{\nu]}{}^{\lceil MN \rfloor} + \ldots + Y^{\lceil MN \rfloor}{}_{\lceil P \lceil QR \rfloor \rfloor} \Phi_{\mu\nu}{}^{\lceil P \lceil QR \rfloor \rfloor} \end{aligned}$$

This list can be extended to higher order tensors $C_{\mu\nu\rho}{}^{\lceil M \lceil NP \rfloor \rfloor}, \ldots$ with gauge transformations:

$$\delta C_{\mu\nu\rho}{}^{\lceil M \lceil NP \rfloor \rfloor} = \partial_{[\mu} \Phi_{\nu\rho]}{}^{\lceil M \lceil NP \rfloor \rfloor} + \ldots, \text{ etc.}$$

This is the **TENSOR HIERARCHY**.

Tensor hierarchy

So far we have introduced the fields ${A_\mu}^M$ and ${B_{\mu\nu}}^{\lceil MN \rfloor}$ with gauge transformations

$$\begin{aligned} \delta {A_\mu}^M &= D_\mu \Lambda^M - {X_{(NP)}}^M {\Xi_\mu}^{NP} \\ \delta {B_{\mu\nu}}^{\lceil MN \rfloor} &= \partial_{[\mu} {\Xi_{\nu]}}^{\lceil MN \rfloor} + \ldots + Y^{\lceil MN \rfloor}{}_{\lceil P \lceil QR \rfloor \rfloor} {\Phi_{\mu\nu}}^{\lceil P \lceil QR \rfloor \rfloor} \end{aligned}$$

This list can be extended to higher order tensors ${C_{\mu\nu\rho}}^{\lceil M \lceil NP \rfloor \rfloor}, \ldots$ with gauge transformations:

$$\delta {C_{\mu\nu\rho}}^{\lceil M \lceil NP \rfloor \rfloor} = \partial_{[\mu} {\Phi_{\nu\rho]}}^{\lceil M \lceil NP \rfloor \rfloor} + \ldots, \text{ etc.}$$

This is the **TENSOR HIERARCHY.**

- Different versions of the hierarchy exist. The field content depends on whether we implement the constraints or not.

e.g.
- without linear constraint: $\lceil MN \rfloor \sim (MN) \longrightarrow {B_{\mu\nu}}^{(MN)}$
- with linear constraint: $\lceil MN \rfloor \sim \alpha \longrightarrow {B_{\mu\nu}}^{\alpha}$

Tensor hierarchy

So far we have introduced the fields $A_\mu{}^M$ and $B_{\mu\nu}{}^{\lceil MN\rfloor}$ with gauge transformations

$$\begin{aligned}\delta A_\mu{}^M &= D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{NP}\\ \delta B_{\mu\nu}{}^{\lceil MN\rfloor} &= \partial_{[\mu}\Xi_{\nu]}{}^{\lceil MN\rfloor} + \ldots + Y^{\lceil MN\rfloor}{}_{\lceil P\lceil QR\rfloor\rfloor}\Phi_{\mu\nu}{}^{\lceil P\lceil QR\rfloor\rfloor}\end{aligned}$$

This list can be extended to higher order tensors $C_{\mu\nu\rho}{}^{\lceil M\lceil NP\rfloor\rfloor}, \ldots$ with gauge transformations:

$$\delta C_{\mu\nu\rho}{}^{\lceil M\lceil NP\rfloor\rfloor} = \partial_{[\mu}\Phi_{\nu\rho]}{}^{\lceil M\lceil NP\rfloor\rfloor} + \ldots, \text{ etc.}$$

This is the **TENSOR HIERARCHY**.

- Different versions of the hierarchy exist. The field content depends on whether we implement the constraints or not.

  e.g.
  - without linear constraint: $\lceil MN\rfloor \sim (MN) \longrightarrow B_{\mu\nu}{}^{(MN)}$
  - with linear constraint: $\lceil MN\rfloor \sim \alpha \longrightarrow B_{\mu\nu}{}^{\alpha}$
- 3-, 4-forms appear in the action as Lagrange multipliers for the constraints.

Tensor hierarchy

So far we have introduced the fields $A_\mu{}^M$ and $B_{\mu\nu}{}^{\lceil MN \rfloor}$ with gauge transformations

$$\begin{aligned} \delta A_\mu{}^M &= D_\mu \Lambda^M - X_{(NP)}{}^M \Xi_\mu{}^{NP} \\ \delta B_{\mu\nu}{}^{\lceil MN \rfloor} &= \partial_{[\mu} \Xi_{\nu]}{}^{\lceil MN \rfloor} + \ldots + Y^{\lceil MN \rfloor}{}_{\lceil P \lceil QR \rfloor \rfloor} \Phi_{\mu\nu}{}^{\lceil P \lceil QR \rfloor \rfloor} \end{aligned}$$

This list can be extended to higher order tensors $C_{\mu\nu\rho}{}^{\lceil M \lceil NP \rfloor \rfloor}, \ldots$ with gauge transformations:

$$\delta C_{\mu\nu\rho}{}^{\lceil M \lceil NP \rfloor \rfloor} = \partial_{[\mu} \Phi_{\nu\rho]}{}^{\lceil M \lceil NP \rfloor \rfloor} + \ldots, \text{ etc.}$$

This is the **TENSOR HIERARCHY.**

- Different versions of the hierarchy exist. The field content depends on whether we implement the constraints or not.

  e.g.
  - without linear constraint: $\lceil MN \rfloor \sim (MN) \longrightarrow B_{\mu\nu}{}^{(MN)}$
  - with linear constraint: $\lceil MN \rfloor \sim \alpha \longrightarrow B_{\mu\nu}{}^{\alpha}$
- 3-, 4-forms appear in the action as Lagrange multipliers for the constraints.
- Gauge transformations that leave the action invariant lead to an open (on-shell) algebra.

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}
[\delta(\Lambda_1),\delta(\Lambda_2)]A_\mu{}^M &= \delta(\Lambda_3)A_\mu{}^M + \delta(\Xi_3)A_\mu{}^M \\
[\delta(\Lambda_1),\delta(\Lambda_2)]B_{\mu\nu}{}^{\lceil MN \rfloor} &= \delta(\Lambda_3)B_{\mu\nu}{}^{\lceil MN \rfloor} + \delta(\Xi_3)B_{\mu\nu}{}^{\lceil MN \rfloor} + \left(E\,\frac{\partial S}{\partial B}\right)_{\mu\nu}^{\lceil MN \rfloor}
\end{aligned}$$

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}
[\delta(\Lambda_1), \delta(\Lambda_2)] A_\mu{}^M &= \delta(\Lambda_3) A_\mu{}^M + \delta(\Xi_3) A_\mu{}^M \\
[\delta(\Lambda_1), \delta(\Lambda_2)] B_{\mu\nu}{}^{\lceil MN \rfloor} &= \delta(\Lambda_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \delta(\Xi_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \left(E \frac{\partial S}{\partial B} \right)_{\mu\nu}^{\lceil MN \rfloor}
\end{aligned}$$

- terms proportional to the field equations ⇒ OPEN ALGEBRA

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}
[\delta(\Lambda_1), \delta(\Lambda_2)] A_\mu{}^M &= \delta(\Lambda_3) A_\mu{}^M + \delta(\Xi_3) A_\mu{}^M \\
[\delta(\Lambda_1), \delta(\Lambda_2)] B_{\mu\nu}{}^{\lceil MN \rfloor} &= \delta(\Lambda_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \delta(\Xi_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \left(E \frac{\partial S}{\partial B} \right)_{\mu\nu}^{\lceil MN \rfloor}
\end{aligned}$$

- terms proportional to the field equations ⇒ OPEN ALGEBRA
- Λ_3 and Ξ_3 are field dependent ⇒ SOFT ALGEBRA

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}
[\delta(\Lambda_1), \delta(\Lambda_2)] A_\mu{}^M &= \delta(\Lambda_3) A_\mu{}^M + \delta(\Xi_3) A_\mu{}^M \\
[\delta(\Lambda_1), \delta(\Lambda_2)] B_{\mu\nu}{}^{\lceil MN \rfloor} &= \delta(\Lambda_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \delta(\Xi_3) B_{\mu\nu}{}^{\lceil MN \rfloor} + \left(E \frac{\partial S}{\partial B} \right)_{\mu\nu}^{\lceil MN \rfloor}
\end{aligned}$$

- terms proportional to the field equations ⇒ OPEN ALGEBRA
- Λ_3 and Ξ_3 are field dependent ⇒ SOFT ALGEBRA

Gauge transformations: e.g.

$$\begin{aligned}
\delta(\Lambda) A_\mu{}^M &= \Lambda^N \left(\delta_N A_\mu{}^M \right) \\
\delta(\Xi) A_\mu{}^M &= \Xi_\nu{}^{\lceil NP \rfloor} \left(\delta^\nu_{\lceil NP \rfloor} A_\mu{}^M \right)
\end{aligned}$$

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}[\delta(\Lambda_1),\delta(\Lambda_2)]A_\mu{}^M &= \delta(\Lambda_3)A_\mu{}^M + \delta(\Xi_3)A_\mu{}^M \\ [\delta(\Lambda_1),\delta(\Lambda_2)]B_{\mu\nu}{}^{\lceil MN\rfloor} &= \delta(\Lambda_3)B_{\mu\nu}{}^{\lceil MN\rfloor} + \delta(\Xi_3)B_{\mu\nu}{}^{\lceil MN\rfloor} + \left(E\,\frac{\partial S}{\partial B}\right)^{\lceil MN\rfloor}_{\mu\nu}\end{aligned}$$

- terms proportional to the field equations ⇒ OPEN ALGEBRA
- Λ_3 and Ξ_3 are field dependent ⇒ SOFT ALGEBRA

Gauge transformations: e.g.

$$\begin{aligned}\delta(\Lambda)A_\mu{}^M &= \Lambda^N\left(\delta_N A_\mu{}^M\right) \\ \delta(\Xi)A_\mu{}^M &= \Xi_\nu{}^{\lceil NP\rfloor}\left(\delta^\nu_{\lceil NP\rfloor}A_\mu{}^M\right)\end{aligned}$$

Then there exist tensors $V, W \neq 0$ such that

$$\left(\delta_N A_\mu{}^M\right) V^N + \left(\delta^\nu_{\lceil NP\rfloor}A_\mu{}^M\right) W_\nu^{\lceil NP\rfloor} = 0$$

The gauge transformations are not independent ⇒ REDUCIBLE ALGEBRA

Gauge algebra: properties

Non-zero commutators:

$$\begin{aligned}
[\delta(\Lambda_1),\delta(\Lambda_2)]A_\mu{}^M &= \delta(\Lambda_3)A_\mu{}^M+\delta(\Xi_3)A_\mu{}^M \\
[\delta(\Lambda_1),\delta(\Lambda_2)]B_{\mu\nu}{}^{\lceil MN\rfloor} &= \delta(\Lambda_3)B_{\mu\nu}{}^{\lceil MN\rfloor}+\delta(\Xi_3)B_{\mu\nu}{}^{\lceil MN\rfloor}+\left(E\,\frac{\partial S}{\partial B}\right)_{\mu\nu}^{\lceil MN\rfloor}
\end{aligned}$$

- terms proportional to the field equations ⇒ OPEN ALGEBRA
- Λ_3 and Ξ_3 are field dependent ⇒ SOFT ALGEBRA

Gauge transformations: e.g.

$$\begin{aligned}
\delta(\Lambda)A_\mu{}^M &= \Lambda^N\left(\delta_N A_\mu{}^M\right) \\
\delta(\Xi)A_\mu{}^M &= \Xi_\nu{}^{\lceil NP\rfloor}\left(\delta^\nu_{\lceil NP\rfloor}A_\mu{}^M\right)
\end{aligned}$$

Then there exist tensors $V, W \neq 0$ such that

$$\left(\delta_N A_\mu{}^M\right)V^N+\left(\delta^\nu_{\lceil NP\rfloor}A_\mu{}^M\right)W_\nu^{\lceil NP\rfloor}=0$$

The gauge transformations are not independent ⇒ REDUCIBLE ALGEBRA

- We call V^N, $W_\mu^{\lceil NP\rfloor}$, . . . zero modes.
- V^N and $W_\mu^{\lceil NP\rfloor}$ also have zero modes ⇒ zero modes for zero modes, etc.

Part 3. the field-antifield formalism

The algebra has a complicated structure: open, soft and reducible.

Part 3. the field-antifield formalism

The algebra has a complicated structure: open, soft and reducible.

These are the properties for which the field-antifield (Batalin-Vilkovisky) formalism was developed [Batalin,Vilkovisky-1983].

Part 3. the field-antifield formalism

The algebra has a complicated structure: open, soft and reducible.

These are the properties for which the field-antifield (Batalin-Vilkovisky) formalism was developed [Batalin,Vilkovisky-1983].

Main features:

- The formalism was originally constructed as an extension of the BRST formalism for quantization.
- Unphysical fields (such as ghosts, ghosts for ghosts, etc.) are introduced to compensate for the effects of gauge invariance.
- It is convenient to have these features already at the classical level. (We will add extra terms to the classical action that depend on the unphyiscal fields.)
- The extended action, subject to one equation (the classical master equation), WILL INCORPORATE ALL THE PROPERTIES OF THE GAUGE THEORY.

Ingredients of the field-antifield formalism (1)

Fields Φ^A:

		parity
fields:	ϕ^i	+
every gauge parameter → ghost:	${\mathcal{C}_{(0)}}^{a_0}$	-
every zero mode → ghost for ghost:	${\mathcal{C}_{(1)}}^{a_1}$	+
…		

Ingredients of the field-antifield formalism (1)

Fields Φ^A:

		parity	ghost ♯
fields:	ϕ^i	+	0
every gauge parameter → ghost:	${\mathcal{C}_{(0)}}^{a_0}$	-	1
every zero mode → ghost for ghost:	${\mathcal{C}_{(1)}}^{a_1}$	+	2

. . .

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯
Fields Φ^A:	fields:	ϕ^i	+	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1
↓	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2
	. . .			
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2
	. . .			

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1	0
$\downarrow$	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	${\mathcal{C}_{(0)}}^{a_0}$	-	1	0
↓	every zero mode → ghost for ghost:	${\mathcal{C}_{(1)}}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		${A_\mu}^M$	${B_{\mu\nu}}^{[MN]}$ . . .

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1	0
↓	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		$A_\mu{}^M$	$B_{\mu\nu}{}^{[MN]}$ . . .
ghosts:	$\mathcal{C}_{(0)}{}^M$	$\mathcal{C}_{(0)\mu}{}^{[MN]}$	. . .

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1	0
$\downarrow$	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		$A_\mu{}^M$	$B_{\mu\nu}{}^{\lceil MN\rfloor}$ . . .
ghosts:	$\mathcal{C}_{(0)}{}^M$	$\mathcal{C}_{(0)\mu}{}^{\lceil MN\rfloor}$	. . .
ghosts for ghosts:	$\mathcal{C}_{(1)}{}^{\lceil MN\rfloor}$	$\mathcal{C}_{(1)\mu}{}^{\lceil M\lceil NP\rfloor\rfloor}$	. . .
⋮	⋮	⋮	

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1	0
$\downarrow$	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2	0
	...				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	...				

fields:		$A_\mu{}^M$	$B_{\mu\nu}{}^{[MN]}$ ...
ghosts:	$\mathcal{C}_{(0)}{}^M$	$\mathcal{C}_{(0)\mu}{}^{[MN]}$	...
ghosts for ghosts:	$\mathcal{C}_{(1)}{}^{[MN]}$	$\mathcal{C}_{(1)\mu}{}^{[M[NP]]}$	...
⋮	⋮	⋮	

+ AntiFields

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	${\mathcal{C}_{(0)}}^{a_0}$	-	1	0
↓	every zero mode → ghost for ghost:	${\mathcal{C}_{(1)}}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		${A_\mu}^M$	${B_{\mu\nu}}^{[MN]}$ . . .
ghosts:	${\mathcal{C}_{(0)}}^M$	${\mathcal{C}_{(0)\mu}}^{[MN]}$	. . .
ghosts for ghosts:	${\mathcal{C}_{(1)}}^{[MN]}$	${\mathcal{C}_{(1)\mu}}^{[M[NP]]}$	. . .
⋮	⋮	⋮	

+ AntiFields

We have a hierarchy, both horizontally (1-forms, 2-forms, ...) and vertically (ghosts, ghosts for ghosts, ...).

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	${\mathcal{C}_{(0)}}^{a_0}$	-	1	0
↓	every zero mode → ghost for ghost:	${\mathcal{C}_{(1)}}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		${A_\mu}^M$	${B_{\mu\nu}}^{\lceil MN \rfloor}$	. . .	Ending: D-forms
ghosts:	${\mathcal{C}_{(0)}}^M$	${\mathcal{C}_{(0)\mu}}^{\lceil MN \rfloor}$	. . .		
ghosts for ghosts:	${\mathcal{C}_{(1)}}^{\lceil MN \rfloor}$	${\mathcal{C}_{(1)\mu}}^{\lceil M \lceil NP \rfloor \rfloor}$	. . .		
⋮	⋮	⋮			

+ AntiFields

We have a hierarchy, both horizontally (1-forms, 2-forms, ...) and vertically (ghosts, ghosts for ghosts, ...).

Ingredients of the field-antifield formalism (1)

			parity	ghost ♯	antifield ♯
Fields Φ^A:	fields:	ϕ^i	+	0	0
	every gauge parameter → ghost:	$\mathcal{C}_{(0)}{}^{a_0}$	-	1	0
↓	every zero mode → ghost for ghost:	$\mathcal{C}_{(1)}{}^{a_1}$	+	2	0
	. . .				
AntiFields Φ^*_A:	for fields:	ϕ^*_i	-	-1	1
	for ghosts:	$\mathcal{C}^*_{(0)\,a_0}$	+	-2	2
	. . .				

fields:		$A_\mu{}^M$	$B_{\mu\nu}{}^{\lceil MN\rfloor}$	. . .
ghosts:	$\mathcal{C}_{(0)}{}^M$	$\mathcal{C}_{(0)\mu}{}^{\lceil MN\rfloor}$	. . .	
ghosts for ghosts:	$\mathcal{C}_{(1)}{}^{\lceil MN\rfloor}$	$\mathcal{C}_{(1)\mu}{}^{\lceil M\lceil NP\rfloor\rfloor}$	. . .	
⋮	⋮	⋮		

Ending: ??

Ending: D-forms

+ AntiFields

We have a hierarchy, both horizontally (1-forms, 2-forms, ...) and vertically (ghosts, ghosts for ghosts, ...).

Ingredients of the field-antifield formalism (2)

Extended action: $S[\Phi^A, \Phi^*_A] = S_0[\phi^i] + \dots$

- Expansion in order of antifields
- $\mathrm{gh}(S) = 0$, $\mathrm{parity}(S) = +$
- $S[\Phi^A, \Phi^*_A = 0] = S_0$
- (proper) solution of the classical master equation:

$$(S, S) = 2 \sum_A \frac{\partial S}{\partial \Phi^A} \frac{\partial S}{\partial \Phi^*_A} = 0$$

Ingredients of the field-antifield formalism (2)

Extended action: $S[\Phi^A, \Phi^*_A] = S_0[\phi^i] + \ldots$

- Expansion in order of antifields
- $\mathrm{gh}(S) = 0\,,\;\; \mathrm{parity}(S) = +$
- $S[\Phi^A, \Phi^*_A = 0] = S_0$
- (proper) solution of the classical master equation:

$$(S, S) = 2 \sum_A \frac{\partial S}{\partial \Phi^A} \frac{\partial S}{\partial \Phi^*_A} = 0$$

We propose the following form:

$$\begin{array}{rclll c}
 & & & & & \text{antifield } \sharp \\ \cline{6-6}
S[\Phi^A, \Phi^*_A] & = & S_0[\phi^i] & + & \phi^*_i R^i_{a_0} \mathcal{C}_{(0)}{}^{a_0} & 1 \\
 & & & + & \mathcal{C}^*_{(0)\,a_0} \left(Z_{(1)}{}^{a_0}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \frac{1}{2} T^{a_0}_{b_0 c_0} \mathcal{C}_{(0)}{}^{b_0} \mathcal{C}_{(0)}{}^{c_0} + \ldots \right) & 2 \\
 & & & + & \phi^*_i \phi^*_j \left(\frac{1}{2} V_{(1)}{}^{ij}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \frac{1}{4} E^{ji}_{a_0 b_0} \mathcal{C}_{(0)}{}^{a_0} \mathcal{C}_{(0)}{}^{b_0} \right) & 2 \\
 & & & + & \ldots & \vdots
\end{array}$$

Up to now, $R^i_{a_0}$, $Z_{(1)}{}^{a_0}_{a_1}$, $\ldots$ are arbitrary tensors.

Ingredients of the field-antifield formalism (2)

Extended action: $S[\Phi^A, \Phi^*_A] = S_0[\phi^i] + \dots$

- Expansion in order of antifields
- $\mathrm{gh}(S) = 0\,,\ \ \mathrm{parity}(S) = +$
- $S[\Phi^A, \Phi^*_A = 0] = S_0$
- (proper) solution of the classical master equation:

$$(S,S) = 2\sum_A \frac{\partial S}{\partial \Phi^A}\frac{\partial S}{\partial \Phi^*_A} = 0$$

We propose the following form:

$$\begin{array}{rcllc}
 & & & & \text{antifield } \sharp \\
S[\Phi^A, \Phi^*_A] & = & S_0[\phi^i] & + \ \phi^*_i R^i_{a_0} \mathcal{C}_{(0)}{}^{a_0} & 1 \\
 & & & + \ \mathcal{C}^*_{(0)\,a_0}\left(Z_{(1)}{}^{a_0}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \frac{1}{2} T^{a_0}_{b_0 c_0} \mathcal{C}_{(0)}{}^{b_0} \mathcal{C}_{(0)}{}^{c_0} + \dots \right) & 2 \\
 & & & + \ \phi^*_i \phi^*_j \left(\frac{1}{2} V_{(1)}{}^{ij}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \frac{1}{4} E^{ji}_{a_0 b_0} \mathcal{C}_{(0)}{}^{a_0} \mathcal{C}_{(0)}{}^{b_0} \right) & 2 \\
 & & & & \vdots \\
 & & & + \ \dots &
\end{array}$$

Up to now, $R^i_{a_0}$, $Z_{(1)}{}^{a_0}_{a_1}$, $\dots$ are arbitrary tensors.

Next: if we impose the master equation, S contains all the relevant information about the gauge structure.

Solving the master equation...

$$\begin{aligned} S[\Phi^A, \Phi^*_A] \quad = \quad S_0[\phi^i] \quad & + \quad \phi^*_i R^i_{a_0} \mathcal{C}_{(0)}{}^{a_0} \\ & + \quad \mathcal{C}^*_{(0)\,a_0} \left(Z_{(1)}{}^{a_0}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \tfrac{1}{2} T^{a_0}_{b_0 c_0} \mathcal{C}_{(0)}{}^{b_0} \mathcal{C}_{(0)}{}^{c_0} + \dots \right) \\ & + \quad \phi^*_i \phi^*_j \left(\tfrac{1}{2} V_{(1)}{}^{ij}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \tfrac{1}{4} E^{ji}_{a_0 b_0} \mathcal{C}_{(0)}{}^{a_0} \mathcal{C}_{(0)}{}^{b_0} \right) \\ & + \quad \dots \end{aligned}$$

Solving the master equation...

$$\begin{array}{rclcl} S[\Phi^A,\Phi^*_A] & = & S_0[\phi^i] & + & \phi^*_i R^i_{a_0}\mathcal{C}_{(0)}{}^{a_0} \\ & & & + & \mathcal{C}^*_{(0)\,a_0}\left(Z_{(1)}{}^{a_0}_{a_1}\mathcal{C}_{(1)}{}^{a_1} + \frac{1}{2}T^{a_0}_{b_0c_0}\mathcal{C}_{(0)}{}^{b_0}\mathcal{C}_{(0)}{}^{c_0} + \ldots\right) \\ & & & + & \phi^*_i\phi^*_j\left(\frac{1}{2}V_{(1)}{}^{ij}_{a_1}\mathcal{C}_{(1)}{}^{a_1} + \frac{1}{4}E^{ji}_{a_0b_0}\mathcal{C}_{(0)}{}^{a_0}\mathcal{C}_{(0)}{}^{b_0}\right) \\ & & & + & \ldots \end{array}$$

If we impose the master equation $(S,S)=0$, we have

- $R^i_{a_0}$: gauge transformations
- $Z_{(1)}{}^{a_0}_{a_1}$: zero modes
- $T^{a_0}_{b_0c_0}$: structure functions
- $E^{ji}_{a_0b_0}$: dependence of the algebra on the equations of motion
- ...

Solving the master equation...

$$\begin{aligned} S[\Phi^A, \Phi^*_A] \quad = \quad S_0[\phi^i] \quad & + \quad \phi^*_i R^i_{a_0} \mathcal{C}_{(0)}{}^{a_0} \\ & + \quad \mathcal{C}^*_{(0)\,a_0} \left(Z_{(1)}{}^{a_0}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \tfrac{1}{2} T^{a_0}_{b_0 c_0} \mathcal{C}_{(0)}{}^{b_0} \mathcal{C}_{(0)}{}^{c_0} + \ldots \right) \\ & + \quad \phi^*_i \phi^*_j \left(\tfrac{1}{2} V_{(1)}{}^{ij}_{a_1} \mathcal{C}_{(1)}{}^{a_1} + \tfrac{1}{4} E^{ji}_{a_0 b_0} \mathcal{C}_{(0)}{}^{a_0} \mathcal{C}_{(0)}{}^{b_0} \right) \\ & + \quad \ldots \end{aligned}$$

If we impose the master equation $(S, S) = 0$, we have

- $R^i_{a_0}$: gauge transformations
- $Z_{(1)}{}^{a_0}_{a_1}$: zero modes
- $T^{a_0}_{b_0 c_0}$: structure functions
- $E^{ji}_{a_0 b_0}$: dependence of the algebra on the equations of motion
- ...

Conclusion: all information about the gauge algebra is contained in the tensors that form the extended action S.

Summary and conclusions

- Conventional gaugings break duality covariance since only electric charges are turned on. This makes it hard to classify all possible gaugings.
- The embedding tensor formalism cures this problem by introducing magnetic vectors, such that duality covariance is restored. Solving the constraints on the embedding tensor leads to the classification of all possible gaugings.
- The formulation in terms of the embedding tensor leads to an open, soft and reducible algebra. These are the features for which the field-antifield formalism was designed.
- We found a hierarchy in fields, but also in ghosts, ghosts for ghosts, etc.
- The master equation $(S, S) = 0$ reproduces the entire gauge structure, such as structure functions, Jacobi identity, etc.