(Supersymmetric) Godel Space from Wrapped M2 Branes

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in collaboration with:

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Overview

- **Introduction**

- **Our system**
  - Reduction of 11d sugra to 3d effective action

- **Solutions:**
  - Anti-de Sitter space
  - Godel space
  - (Super)symmetries

- **Superglue**
  - Gluing Godel space to anti de Sitter space

- **Conclusions and outlook**
Introduction
Introduction/Motivation

- **Problem:**
  - $AdS_3 \times S^2 \times CY_3$ background + probe M2 wrapping $S^2$ Backreaction?

- **Original motivation:** Black Hole entropy/microstates
  - Constituent counting: D-branes/CFT states
  - Supergravity regime?
    - Fuzzball proposal
    - Gaiotto-Strominger-Yin (2005), GSY + Denef-Van den Bleeken (2007):
      - Type IIA on CY gives N=2 supergravity in 4d
      - D0-D4 black hole split as D0-D4 background and probe D0, counting agrees with entropy
1) **Gaiotto-Strominger-Yin:**

- D0-D4 BH: background core D0-D4 + probe D0s
- Background near-horizon: $AdS_2 \times S^2 \times CY_3$
- AdS superconformal quantum mechanics
- Large # (non-abelian) D0s: puff up to D2s (conjecture)
Introduction/Motivation

- **GSY+Denef-Van den Bleeken:**
  - D0-D4 BH: background + probe
    
    \[ D_6 \; \rightarrow \; D_6 \] purely fluxed
  - Two interesting regions in scaling limit
    - **FAR region:** quotient of $AdS_3 \times S^2$
      (not surprising, D0-D4 BH near horizon)
    - **NEAR region:** global $AdS_3 \times S^2$
      (surprising)

- **Entropy?**
  - Start in near region (take D0 charge to infinity)
  - Bring in D0 charge as D2/M2 branes (non-abelian degrees of freedom)
  - M2’s wrap near horizon \( S^2 \): spinning particles in \( AdS_3 \)
  - Counting ok in leading order à la GSY

- **ISSUES:**
  - Myers effect?
  - **Backreaction** of probe branes on \( S^2 \)
    Either solves black hole problems or is interesting new solution
Our System
Our System
From M2 probes to an ansatz for the backreaction

- Backreaction of M2 branes on $S^2$ ? 11 dimensional picture

D6-anti D6 near horizon:

$$ AdS_3 \times S^2 \times CY_3 $$
Our System
From M2 probes to an ansatz for the backreaction

- Backreaction of M2 branes on $S^2$?

11 dimensional picture

**M2 brane probes**
(Susy when rotating with constant angular velocity in global $AdS_3$, see later)

$$\text{AdS}_3 \times S^2 \times CY_3$$
Our System
From M2 probes to an ansatz for the backreaction

- Backreaction of M2 branes on $S^2$? 11 dimensional picture

**M2 brane probes**
(Susy when rotating with constant angular velocity in global $AdS_3$, see later)

$AdS_3 \times S^2 \times CY_3$

Backreacted system:
(ASSUME...)

$g_{11}$

$F^{(4)}$ Red and blue cycles (on $S^2$) $\Lambda < 0$

$M_3 \times S^2 \times CY_3$
Our System
Specifying our ansatz in 11 dimensions

- Backreaction of M2 branes on $S^2$?

\[ ds_{11}^2 = \frac{1}{\tau_2^{2/3}} (ds_3^2 + \frac{\ell^2}{4} d\Omega_2^2) + ds^2(CY_3) \]

\[ F^{(4)} = p^A D_A \wedge dvol_{S^2} + \frac{1}{\tau_2} \ast_3 d\tau_1 \wedge dvol_{S^2} \]

\[ \tau_2 = CY_3 \text{ volume} \]

\[ d\tau_1 = \frac{1}{\tau_2} \ast_5 dA^{(3)} = \frac{1}{\tau_2} \ast_3 dA \]
Our System
Specifying our ansatz in 11 dimensions

- Backreaction of M2 branes on $S^2$?

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\[ g_{11} \]

\[ F^{(4)} \]

\[ \begin{array}{c}
\text{Redux} \\
\text{Redux}
\end{array} \]

\[ \begin{array}{c}
\tau_2 = \text{CY}_3 \text{ volume} \\
d\tau_1 = \frac{1}{\tau_2^2} \star_5 dA^{(3)} = \frac{1}{\tau_2^2} \star_3 dA
\end{array} \]

\[ \text{3d with } \Lambda < 0 \]

+ two scalars $\tau_1, \tau_2$
Our System
From 11d to 5d to 3d

- Reduced to 5 dimensions over $CY_3$
  - Gives five dimensional N=1 supergravity with vector multiplets and one hypermultiplet:
    \[
    S_5 = \int d^5x \sqrt{-g} \left( R - \frac{1}{2} \partial_\mu \tau \partial_\mu \bar{\tau} \right) - \frac{1}{2} \int G_{AB} F^A \wedge * F^B + \frac{D_{ABC}}{6} \int A^A \wedge F^B \wedge F^C
    \]
    \[
    F^A = \rho^A d\text{vol}_{S^2}
    \]
    \[
    \sim *_5 dA^3
    \]
    \[
    \tau = \tau_1 + i\tau_2 \quad \text{complex scalar in universal hypermultiplet}
    \]
    \[
    \text{CY volume}
    \]

- Reduced to 3 dimensions over $S^2$
  - $\mu = 1$: no backreaction, AdS
  - $\mu = \frac{3}{2}$: backreaction $= ???$
  - Constant sphere radius? None of the other fields couple to this modulus!
  - General for codimension 2 branes
Our System
Equations of motion in the 3d system

- **Action and equations of motion in 3d:**
  \[ S_3 = \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} - (\mu - 1) \frac{\partial_\mu \tau \partial^\mu \tilde{\tau}}{\tau^2} \right) \]

  - Einstein eqns.  \[ R_{\alpha\beta} + \frac{2}{\ell^2} g_{\alpha\beta} = (\mu - 1) \frac{\partial_{(\alpha} \tau \partial_{\beta)} \tilde{\tau}}{\tau^2} \]
  - Scalar field eqn. \[ \partial_\alpha \left( \sqrt{-g} g^{\alpha\beta} \partial_{\beta} \tau \right) + i \sqrt{-g} g^{\alpha\beta} \frac{\partial_\alpha \tau \partial_{\beta} \tau}{\tau^2} = 0 \]
  - We search for stationary solutions

- **Inspiration: codimension 2 branes are special!**
  - Greene, Shapere, Vafa, Yau (1990) (Stringy cosmic strings)
  - Gibbons, Green, Perry (1995) (D7 branes)

- **3D part of metric:**
  \[ -dt^2 + dx^2 + dy^2 - dz d\bar{z} \]

  - Remains valid when spatial part of \( \sqrt{-g} g^{\alpha\beta} \) is “flat”
  - Flat space: \[ -dt^2 + e^{2\phi(z,\bar{z})} dz d\bar{z} \]
  - Our case: \[ -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z} \]
Let's put everything together:

- Wrapped branes on the sphere in $M_3 \times S^2 \times CY_3$
- Leads to three-dimensional action:
\[
S_3 = \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} - (\mu - 1) \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2} \right)
\]

- Ansatz:
\[
ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z, \bar{z})} dz d\bar{z} \quad \tau(z)\\
\chi = \chi(z, \bar{z}) dz + \chi_\bar{z}(z, \bar{z}) d\bar{z}
\]

- Einstein eqns.:
\[
d\chi = \frac{i e^{2\phi} dz \wedge d\bar{z}}{2}\\
\partial \bar{\partial} \phi - \frac{e^{2\phi}}{4} = - (\mu - 1) \frac{\partial_\tau \partial \bar{\tau}}{4\tau_2^2}
\]

- We will focus on eqn. for $\phi$: sourced Liouville equation
\[
\mu = \frac{3}{2} \quad \text{Our case, we will show this is timelike stretched } \text{AdS}_3\\
\mu = 1 \quad \text{AdS}_3
\]
Let's put everything together:

- Wrapped branes on the sphere in $M_3 \times S^2 \times CY_3$
- Leads to three-dimensional action:

$$S_3 = \int dx^3 \sqrt{-g} \left( R + \frac{2}{\ell^2} - (\mu - 1) \frac{\partial_\mu \tau \partial_\mu \bar{\tau}}{\tau_2^2} \right)$$

- Ansatz:

$$ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z}$$

$$\chi = \chi(z, \bar{z}) dz + \chi_\bar{z}(z, \bar{z}) d\bar{z}$$

- Einstein eqns.:

$$d\chi = \frac{i e^{2\phi}}{2} dz \wedge d\bar{z}$$

$$\partial \bar{\partial} \phi - \frac{e^{2\phi}}{4} = -(\mu - 1) \frac{\partial \tau \bar{\partial} \bar{\tau}}{4 \tau_2^2}$$

- We will focus on eqn. for $\phi$: sourced Liouville equation

Take $\mu$ arbitrary

Our case, we will show this is timelike stretched $AdS_3$

$\mu = \frac{3}{2}$ $AdS_3$

$\mu = 1$ $AdS_3$
Solutions
Solutions
Solving for our ansatz

• Solving for general value of $\mu$
  
  - Ansatz and Einstein equations:
    \[
    \begin{align*}
    ds_3^2 &= -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z} \\
    \tau(z) &
    \end{align*}
    \]
  
  - Specifics:
    
    • Spatial base has topology of a disk (UHP) (cf. $AdS$)
    • Imaginary part of $\tau$ positive: lives on UHP
    • Possible poles of $\tau$ only on the boundary of space
      meromorphic function from disk/UHP to UHP
Solutions
Solving for our ansatz

- Solving for general value of $\mu$

  - Ansatz and Einstein equations:

    \[
    \begin{cases} 
    ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} \, dz \, d\bar{z} \\
    \tau(z) 
    \end{cases}
    \]

    \[
    d\chi = \frac{i e^{2\phi} \, dz \wedge d\bar{z}}{2} 
    \]

    \[
    \partial \bar{\partial} \phi - \frac{e^{2\phi}}{4} = -(\mu - 1) \frac{\partial \tau \bar{\partial} \bar{\tau}}{4 \tau_2^2} 
    \]

  - Specifics:
    
    - Spatial base has topology of a disk (UHP) (cf. $AdS$)
    - Imaginary part of $\tau$ positive: lives on UHP
    - Possible poles of $\tau$ only on the boundary of space
    - Meromorphic function from disk/UHP to UHP

  - Build solutions:

    \[
    \chi = 2 \text{Im} \left( \partial \phi + (1 - \mu) \partial \ln \tau_2 \right) + df
    \]

    \[
    D e^{2\phi} = -(\mu - 1) \frac{\partial \tau \bar{\partial} \bar{\tau}}{2 \tau_2^2} = D \left( \mu \frac{\partial \tau \bar{\partial} \bar{\tau}}{\tau_2^2} \right)
    \]

    Notice earlier gauge freedom!

  - We can take:

    \[
    e^{2\phi} = \mu \frac{\partial \tau \bar{\partial} \bar{\tau}}{\tau_2^2}
    \]

    - Literature (sourced Liouville eqn.)
    - Uniqueness? Locally: 3D gravity
We express the Godel metric in coordinates where the spatial base is either the Poincare disk or UHP.

\[ ds^2 = -(dt + d\chi)^2 + e^{2\phi(z, \bar{z})} dz d\bar{z} \]

\[ e^{2\phi} = \mu \frac{\partial \tau \partial \bar{\tau}}{\tau^2} \]

- **Poincare Disk**
  
  \[ z = re^{i\varphi} \]
  
  \[ \tau(z) = i \frac{1 + z}{1 - z} \]
  
  \[ ds^2 = \frac{\mu \ell^2}{4} \left[ -\mu (d\tilde{t} + \frac{2r^2}{1 - r^2} d\varphi)^2 + 4 \frac{dr^2 + r^2 d\varphi^2}{(1 - r^2)^2} \right] \]
  
  - \( \mu < 1 \) Timelike Squashed AdS (TMG): unphysical!
  - \( \mu = 1 \) AdS
  - \( \mu > 1 \) Timelike Stretched AdS

- **Upper Half Plane (UHP)**
  
  \[ w = x + iy \]
  
  \[ \tau(w) = w \]
  
  \[ ds^2 = \frac{\mu \ell^2}{4} \left[ -\mu (dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2} \right] \]
Solutions: Godel Space
Properties of original and our Godel space

- **Original Godel space**
  - Metric in four dimensions:
    3d solution above ($\mu = 2$) plus 1 extra dimension
  - Solution to Einstein eqns. with a pressureless fluid source:
    \[ T_{\mu\nu} = \rho u_\mu u_\nu, \quad u^\mu = \frac{2}{i} \delta^\mu_0 \quad \rho \text{ constant energy density} \]
  - Closed Timelike Curves (CTCs)
  - Godel rotates around every point: \( \ast_3 (u \wedge du) \neq 0 \)

- **Our Godel space**
  - 3d solution ($\mu = 3/2$)
  - Non-trivial complex scalar $\tau(w)$ with EM tensor of the “Godel” form
  - CTCs
  - Pole of $\tau$ on the boundary ($w = i\infty$)
    Infinite U(1)- charge of gauge field:
    \[ d\tau_1 \sim \ast_3 dA \]
(Super)Symmetries
Comparison of supersymmetries: probe/backreaction

- **Background geometry**
  
  \[ AdS_3 \times S^2 \times CY_3 \]

  - Bosonic symmetry group: \( SL(2, R)_L \times SU(2)_L \times SL(2, R)_R \)
  - Supergroup (8 supersymmetries): \( SU(1, 1|2) \)

- **Probe (wrapped) M2 Branes**
  
  - Static w.r.t. \( l_0 \) generator of \( SL(2, R)_L \) (i.e. \( t \) in UHP coords, physically rotating M2!)
  - Minimal energy: \( L_0 = Z \), \( Z = \) mass of brane
  - \( \frac{1}{2} \) BPS state: (4 supersymmetries)

- **Is 3d Godel Space = Backreacted (wrapped) M2 branes?**
  
  - Bosonic symmetry group \( U(1)_L \times SU(2)_L \times SL(2, R)_R \)
  - Same (4 supersymmetries) of probe
  - Check: M2 branes in Godel background do not break any susy
Superglue
**Motivation**: why glue to Anti de Sitter space?

- Black hole motivation
- AdS/CFT
  - Embedding of Godel in AdS:
    - **SIMPLEST REALIZATION**: domain wall canceling the M2 charge
    - Outside wall: locally AdS (3d gravity)
- Analogy with enhancon etc.
  - Resolving CTCs of Godel-type spacetime

**Setting up the domain wall**

- Cancel energy-momentum sourcing Godel space
- Need M2 brane charge

Domain wall built up out of M2 branes wrapping internal smeared in AdS on a dimension 1 domain “wall”

**Action**

\[ S = S_3 + S_{probe} \]

- Which wall? M2 branes couple to CY volume \((\tau_2)\) try constant \(\tau_2\)
Superglue

Strategy

- **Action**
  \[ S = \int d^3x (L_3 + L_{probe}) \]
  with
  \[ dA = \frac{1}{\tau_2^2} \star_3 d\tau_1 \]
  \[ X^0(\sigma, \lambda) = \sigma \quad X^1(\sigma, \lambda) = \lambda \quad X^2(\sigma, \lambda) = Y \]

  Solution supersymmetric (remember static probe branes in UHP are susy!)

- **Domain Wall**

- **Ansatz:**

  \begin{align*}
  \tau &= x + iy \\
  \epsilon(y - Y) &\leq 0 \\
  dS_3^2 &= dS_{Godel}^2 \\
  \tau &= \tau_0 \text{ (cst)} \\
  \epsilon(y - Y) &\geq 0 \\
  dS_3^2 &= N^2 dy^2 + h_{ab} dx^a dx^b
  \end{align*}
Superglue

Strategy

- Junction conditions

<table>
<thead>
<tr>
<th>$\tau = x + iy$</th>
<th>$\varepsilon(y - Y) \leq 0$</th>
<th>$ds^2_3 = ds^2_{Godel}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = \tau_0$ (cst)</td>
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<td>$ds^2_3 = N^2 dy^2 + h_{ab} dx^a dx^b$</td>
</tr>
</tbody>
</table>

- For metric, complex scalar
  (= CY volume and gauge field)

- Need to match:
  - Continuity across domain wall
  - EOM scalar/metric across wall:
  - Einstein eqn. AdS part: no source

\[ \varepsilon = \text{the sign of the M2 brane tension!} \]
\[ \text{Godel in/out determined by brane tension neg/pos} \]

Metric:

\[ \varepsilon(y - Y) \leq 0 \quad ds^2 = \frac{\mu l^2}{4} \left[ -\mu (dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2} \right] \]

\[ \varepsilon(y - Y) \geq 0 \quad ds^2 = \frac{\mu l^2}{4} \left( -(dt + \frac{dx}{y})^2 + \left( f(y) dx^2 + f^{-1}(y) dy^2 \right) \right) \]

\[ f(y) = \mu + (1 - \mu) \frac{Y^2}{y^2} \]

Global AdS!
• Problems:
  
  – CTCs:
    – Remember poincare disk:
      Domain walls are circles tangent to the boundary in $z = 1$
    – Even worse:
      – Out(in)side AdS space can be brought to global coordinates
      – Identification of AdS-angle requires extra timelike identification in Godel
  
  – Black hole charges? Hoped for, but not realized!
Conclusions & Outlook
Conclusions & Discussion

• Summary?
  - Backreaction of M2 branes wrapped on $S^2$ in $AdS_3 \times S^2 \times CY_3$ background?
  - Gödel space + complex scalar in 3 dimensions:
    • Infinite M2 charge on one boundary point
    • Supersymmetric ($\frac{1}{2}$ BPS)
  - Connect to to asymptotically AdS spacetime: domain wall of M2 branes
    • PROBLEMS: CTCs, no BH equivalent

• Questions?
  - What corresponds to backreacted setup of Gaiotto-Denef-Strominger-Van den Bleeken-Yin? We would expect asymptotics = quotient of $AdS_3 \times S^2$
  - Other alternatives
    • For Gödel solutions? Complex scalar solution?
    • of making Domain Wall?
      - 11d picture, need codimension 1 object – other brane sources:
        • M2?
        • M5?
  - Use this technology to resolve problematic solutions (3d conical defect of AdS…)

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End
Extra Slides
As an appetizer, let's solve the case

\[ S_3 = \int d^5x \sqrt{-g} \left( R + \frac{2}{l^2} \right) \]

- We know this should be AdS
- Equations of motion become

\[ \partial \bar{\partial} \phi - \frac{e^{2\phi}}{4} = 0 \]
\[ d\chi = \frac{ie^{2\phi}}{2} dz \wedge d\bar{z} \]

\[ ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z, \bar{z})} dz d\bar{z} \]

\[ e^{2\phi} = \frac{4\partial g \bar{\partial} \bar{g}}{(1 - g \bar{g})^2} \]
\[ \chi = 2\text{Im}\partial \phi + df \]

in terms of an arbitrary holomorphic function \( g(z) \)

- We can show this is AdS with the coordinate transformation:

\[ g = \tanh(\rho)e^{i(\psi - \sigma)} \]
\[ \sigma = \frac{t + f}{2} \]

\[ ds^2 = l^2 \left( -\cosh^2 \rho \, d\sigma^2 + d\rho^2 + \sinh^2 \rho \, d\psi^2 \right) \]

For later use, two main coordinate systems:

- The Poincare Disk
- The Upper Half Plane (UHP)
Solutions: Godel Space
Godel Space in Disk and UHP coordinates

- We express the AdS metric in coordinates where the spatial base is either the Poincare disk or UHP.

\[ ds^2 = -(dt + d\chi)^2 + e^{2\phi(z, \bar{z})} dz \, d\bar{z} \]

\[ e^{2\phi} = \frac{4\partial g \bar{g}}{(1 - g \bar{g})^2} \]

\[ \chi = 2\text{Im} \partial \phi + df \]

- Poincare Disk

\[ z = r e^{i \varphi} \]

\[ r = 1 \]

\[ r = 1/\sqrt{\mu} \]

\[ \tau(z) = i \frac{1 + z}{1 - z} \]

\[ ds^2 = \frac{\ell^2}{4} \left[ -(d\tilde{t} + \frac{2r^2}{1 - r^2} d\varphi)^2 + \frac{4}{(1 - r^2)^2} (dr^2 + r^2 d\varphi^2) \right] \]

- Upper Half Plane (UHP)

\[ w = x + iy \]

\[ \tau(w) = w \]

\[ ds^2 = \frac{\ell^2}{4} \left[ -\mu (dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2} \right] \]

- Timelike Squashed AdS (TMG)
- AdS
- Timelike Stretched AdS