International Conference on Strings, M-Theory and Quantum Gravity
Centro Stefano Franscini, Monte Verita, Ascona, 27 July 2010

Aspects of the BMS/CFT correspondence

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Overview

Classical gravitational aspects of AdS3/CFT2 correspondence

4d flat case, null infinity: asymptotic symmetries

3d flat case, null infinity: BMS3/CFT1 correspondence

4d flat case, null infinity: solution space

work done in collaboration with C. Troessaert

Asymptotic symmetries

Fefferman-Graham ansatz

$$g_{\mu\nu} = \begin{pmatrix} \frac{l^2}{r^2} & 0 \\ 0 & g_{AB} \end{pmatrix} \qquad \Lambda = -\frac{1}{l^2}$$

$$r \qquad t, \phi$$

$$g_{AB} = r^2 \bar{\gamma}_{AB}(x^C) + O(1)$$

2d metric

$\bar{\gamma}_{AB}$ conformal to flat metric on the cylinder $\bar{\gamma}_{AB} = e^{2\varphi}\eta_{AB}$

$$\eta_{AB}dx^A dx^B = -d\tau^2 + d\phi^2, \quad \tau = \frac{t}{l}, \quad \varphi = \varphi(x^A)$$

asymptotic symmetries $\mathcal{L}_\xi g_{rr} = 0 = \mathcal{L}_\xi g_{rA}, \quad \mathcal{L}_\xi g_{AB} = O(1),$

general solution determined by conformal Killing vector Y^A of η_{AB}

$$\begin{cases} \xi^r = -\frac{1}{2}\psi r, \\ \xi^A = Y^A + I^A, \quad I^A = -\frac{l^2}{2}\partial_B\psi \int_r^\infty \frac{dr'}{r'} g^{AB} = -\frac{l^2}{4r^2}\bar{\gamma}^{AB}\partial_B\psi + O(r^{-4}). \end{cases}$$

$$\psi = \bar{D}_A Y^A$$

AdS3/CFT2

Asymptotic symmetries

metric dependence $\xi^\mu = \xi^\mu(x,g) \qquad \delta^g_{\xi_1} g_{\mu\nu} = \mathcal{L}_{\xi_1} g_{\mu\nu}$

modified bracket $[\xi_1,\xi_2]^\mu_M = [\xi_1,\xi_2]^\mu - \delta^g_{\xi_1}\xi^\mu_2 + \delta^g_{\xi_2}\xi^\mu_1$

linear representation of conformal algebra

$$[\xi_1,\xi_2]^r_M = -\frac{1}{2}\widehat{\psi} r, \quad [\xi_1,\xi_2]^A_M = \widehat{Y}^A + \widehat{I}^A, \quad \widehat{Y}^A = [Y_1,Y_2]^A, \qquad \widehat{\psi} = \bar{D}_A \widehat{Y}^A$$

light-cone coordinates

$$x^\pm = \tau \pm \phi, \quad 2\partial_\pm = \frac{\partial}{\partial \tau} \pm \frac{\partial}{\partial \phi}, \quad \bar{\gamma}_{AB} dx^A dx^B = -e^{2\varphi} dx^+ dx^-$$

$$Y^\pm(x^\pm)\partial_\pm = \sum_{n\in\mathbb{Z}} c^n_\pm l^\pm_n, \quad l^\pm_n = i e^{inx^\pm}\partial_\pm$$

$$[l^\pm_m, l^\pm_n] = (m-n) l^\pm_m, \quad [l^\pm_m, l^\mp_n] = 0$$

include Weyl rescalings of boundary metric $\mathcal{L}_\xi g_{rr} = 0 = \mathcal{L}_\xi g_{rA}, \quad \mathcal{L}_\xi g_{AB} = 2\omega g_{AB} + O(1)$

direct sum with abelian algebra of Weyl rescalings

$$(\widehat{Y},\widehat{\omega}) = [(Y_1,\omega_1),(Y_2,\omega_2)]$$

$$\widehat{Y}^A = Y^B_1 \partial_B Y^A_2 - Y^B_2 \partial_B Y^A_1, \quad \widehat{\omega} = 0$$

AdS3/CFT2 Solution space

existence of general solution

integration “constants” $\Xi_{++} = \Xi_{++}(x^+), \quad \Xi_{--} = \Xi_{--}(x^-)$

when $\varphi = 0$

$$g_{AB}dx^A dx^B = -(r^2 + \frac{l^4}{r^2}\Xi_{++}\Xi_{--})dx^+dx^- + l^2\Xi_{++}(dx^+)^2 + l^2\Xi_{--}(dx^-)^2,$$

BTZ black hole $\Xi_{\pm\pm} = 2G(M \pm \frac{J}{l})$ AdS3 space $M = -\frac{1}{8G}, J = 0$

general solution $\varphi \neq 0$

$$g_{AB}dx^A dx^B = \Big(-e^{2\varphi}r^2 + 2\widehat{\gamma}_{+-} - r^{-2}e^{-2\varphi}(\widehat{\gamma}_{+-}^2 + \widehat{\gamma}_{++}\widehat{\gamma}_{--})\Big)dx^+dx^- + \\ +\widehat{\gamma}_{++}(1 - r^{-2}e^{-2\varphi}\widehat{\gamma}_{+-})(dx^+)^2 + \widehat{\gamma}_{--}(1 - r^{-2}e^{-2\varphi}\widehat{\gamma}_{+-})(dx^-)^2,$$

$$\widehat{\gamma}_{\pm\pm} = l^2\big[\Xi_{\pm\pm}(x^\pm) + \partial_\pm^2\varphi - (\partial_\pm\varphi)^2\big] \qquad \widehat{\gamma}_{+-} = l^2\partial_+\partial_-\varphi$$

AdS3/CFT2 Conformal properties

asymptotic symmetries transform solutions into solutions

$$g_{AB} = g_{AB}(x, \Xi, \varphi)$$

$$g_{AB}(x, -\delta\Xi, -\delta\varphi) = \mathcal{L}_\xi g_{AB}$$

conformal transformation properties

$$-\delta_{Y^+,Y^-,\omega}\Xi_{\pm\pm} = Y^\pm\partial_\pm\Xi_{\pm\pm} + 2\partial_\pm Y^\pm\Xi_{\pm\pm} - \frac{1}{2}\partial_\pm^3 Y^\pm$$

$$-\delta_{Y^+,Y^-,\omega}\varphi = \omega$$

asymptotically AdS3 gravity: dual to conformal boundary theory

AdS3/CFT2 Charge algebra

Hamiltonian approach Q_ξ surface charge generators, Dirac algebra

centrally extended charge representation of conformal algebra

Brown & Henneaux 1986

$$\mathcal{Q}_\xi[g-\bar{g},\bar{g}] = \frac{1}{8\pi G}\int_0^{2\pi} d\phi\,(Y^+\Xi_{++} + Y^-\Xi_{--})$$

$$\mathcal{Q}_{\xi_1}[\mathcal{L}_{\xi_2}g,\bar{g}] \approx \mathcal{Q}_{[\xi_1,\xi_2]_M}[g-\bar{g},\bar{g}] + K_{\xi_1,\xi_2},$$

$$K_{\xi_1,\xi_2} = \mathcal{Q}_{\xi_1}[\mathcal{L}_{\xi_2}\bar{g},\bar{g}] = \frac{1}{8\pi G}\int_0^{2\pi} d\phi\,(\partial_\phi Y_1^\tau \partial_\phi^2 Y_2^\phi - \partial_\phi Y_2^\tau \partial_\phi^2 Y_1^\phi)$$

modes

$$i\{\mathcal{L}_m^\pm, \mathcal{L}_n^\pm\} = (m-n)\mathcal{L}_{m+n}^\pm + \frac{c}{12}m(m^2-1)\delta_{n+m},$$

$$\{\mathcal{L}_m^\pm, \mathcal{L}_n^\mp\} = 0,$$

where $c = \frac{3l}{2G}$ is the central charge for the anti-de Sitter case.

Strominger 1998: combine with Cardy formula to provide a microscopic derivation of the Bekenstein-Hawking entropy of BTZ black hole

Asymptotically flat spacetimes

BMS ansatz

$$g_{\mu\nu} = \begin{pmatrix} e^{2\beta}\frac{V}{r} + g_{CD}U^C U^D & -e^{2\beta} & -g_{BC}U^C \\ -e^{2\beta} & 0 & 0 \\ -g_{AC}U^C & 0 & g_{AB} \end{pmatrix}$$

$$u \qquad\qquad r \qquad\qquad x^A = \begin{cases} \theta, \phi \\ \zeta, \bar{\zeta} \end{cases}$$

Minkowski $\quad u = t - r$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$g_{AB}dx^A dx^B = r^2 \bar{\gamma}_{AB} dx^A dx^B + O(r)$$

Sachs: unit sphere $\quad \bar{\gamma}_{AB} = e^{2\varphi}{}_0\gamma_{AB} \quad {}_0\gamma_{AB}dx^A dx^B = d\theta^2 + \sin^2\theta d\phi^2$

Riemann sphere $\quad \zeta = e^{i\phi}\cot\frac{\theta}{2}, \quad \bar{\gamma}_{AB}dx^A dx^B = e^{2\tilde{\varphi}} d\zeta d\bar{\zeta}$

$$d\theta^2 + \sin^2\theta d\phi^2 = P^{-2} d\zeta d\bar{\zeta}, \quad P(\zeta,\bar{\zeta}) = \frac{1}{2}(1+\zeta\bar{\zeta}), \qquad \tilde{\varphi} = \varphi - \ln P$$

determinant condition $\quad \det g_{AB} = \frac{r^4}{4} e^{4\tilde{\varphi}}$

fall-off conditions $\quad \beta = O(r^{-2}), \quad U^A = O(r^{-2}), \quad V/r = -\frac{1}{2}\bar{R} + O(r^{-1})$

Asymptotic symmetries

asymptotic symmetries $\mathcal{L}_\xi g_{rr} = 0, \quad \mathcal{L}_\xi g_{rA} = 0, \quad \mathcal{L}_\xi g_{AB} g^{AB} = 0,$

$$\mathcal{L}_\xi g_{ur} = O(r^{-2}), \quad \mathcal{L}_\xi g_{uA} = O(1), \quad \mathcal{L}_\xi g_{AB} = O(r), \quad \mathcal{L}_\xi g_{uu} = O(r^{-1})$$

general solution

$$\begin{cases} \xi^u = f, & \dot{f} = f\dot{\varphi} + \frac{1}{2}\psi \iff f = e^{\varphi}\big[T + \frac{1}{2}\int_0^u du' e^{-\varphi}\psi\big], \\ \xi^A = Y^A + I^A, \quad I^A = -f_{,B}\int_r^\infty dr'(e^{2\beta}g^{AB}), & \\ \xi^r = -\frac{1}{2}r(\bar{D}_A\xi^A - f_{,B}U^B + 2f\partial_u\varphi), & \psi = \bar{D}_A Y^A \end{cases}$$

$Y^A = Y^A(x^B)$ conformal Killing vectors of the sphere

$T = T(x^B)$ generators for supertranslations

spacetime vectors with modified bracket form linear representation of $\mathfrak{bms}_4$ algebra

$$[(Y_1, T_1), (Y_2, T_2)] = (\widehat{Y}, \widehat{T})$$

$$\begin{aligned} \widehat{Y}^A &= Y_1^B \partial_B Y_2^A - Y_1^B \partial_B Y_2^A, \\ \widehat{T} &= Y_1^A \partial_A T_2 - Y_2^A \partial_A T_1 + \frac{1}{2}(T_1 \partial_A Y_2^A - T_2 \partial_A Y_1^A) \end{aligned}$$

Sachs 1962

standard GR choice: restrict to globally well-defined transformations

$$SL(2,\mathbb{C})/\mathbb{Z}_2 \simeq SO(3,1)$$

Y^A generators of Lorentz algebra

New proposal

CFT choice : allow for meromorphic functions on the Riemann sphere

solution to conformal Killing equation $$Y^{\zeta} = Y^{\zeta}(\zeta), \quad Y^{\bar{\zeta}} = Y^{\bar{\zeta}}(\bar{\zeta})$$

generators

$$l_n = -\zeta^{n+1}\frac{\partial}{\partial \zeta}, \quad \bar{l}_n = -\bar{\zeta}^{n+1}\frac{\partial}{\partial \bar{\zeta}}, \quad n \in \mathbb{Z}$$

$$T_{m,n} = \zeta^m \bar{\zeta}^n, \quad m, n \in \mathbb{Z}$$

commutation relations

$$[l_m, l_n] = (m-n)l_{m+n}, \quad [\bar{l}_m, \bar{l}_n] = (m-n)\bar{l}_{m+n}, \quad [l_m, \bar{l}_n] = 0,$$
$$[l_l, T_{m,n}] = (\frac{l+1}{2} - m)T_{m+l,n}, \quad [\bar{l}_l, T_{m,n}] = (\frac{l+1}{2} - n)T_{m,n+l}.$$

Poincaré subalgebra $$l_{-1},\, l_0,\, l_1, \quad \bar{l}_{-1},\, \bar{l}_0,\, \bar{l}_1, \quad T_{0,0},\, T_{1,0},\, T_{0,1},\, T_{1,1},$$

Asymptotic symmetries

ansatz for asymptotically flat metrics

$$g_{\mu\nu} = \begin{pmatrix} e^{2\beta} V r^{-1} + r^2 e^{2\varphi} U^2 & -e^{2\beta} & -r^2 e^{2\varphi} U \\ -e^{2\beta} & 0 & 0 \\ -r^2 e^{2\varphi} U & 0 & r^2 e^{2\varphi} \end{pmatrix}$$

Minkowski spacetime

$$ds^2 = -du^2 - 2du dr + r^2 d\phi^2 \qquad u = t - r$$

fall-off conditions

$$\beta = O(r^{-1}), \quad U = O(r^{-2}) \quad V = -2r^2 \partial_u \varphi + O(r)$$

asymptotic symmetries

$$\mathcal{L}_\xi g_{rr} = 0 = \mathcal{L}_\xi g_{r\phi}, \quad \mathcal{L}_\xi g_{\phi\phi} = 0,$$

$$\mathcal{L}_\xi g_{ur} = O(r^{-1}), \quad \mathcal{L}_\xi g_{u\phi} = O(1), \quad \mathcal{L}_\xi g_{uu} = O(1)$$

$$\begin{cases} \xi^u = f, \\ \xi^\phi = Y + I, \quad I = -e^{-2\varphi} \partial_\phi f \int_r^\infty dr' \, r'^{-2} e^{2\beta} = -\frac{1}{r} e^{-2\varphi} \partial_\phi f + O(r^{-2}), \\ \xi^r = -r \big[\partial_\phi \xi^\phi - \partial_\phi f U + \xi^\phi \partial_\phi \varphi + f \partial_u \varphi \big], \end{cases}$$

$$\partial_u f = f \partial_u \varphi + Y \partial_\phi \varphi + \partial_\phi Y \iff f = e^\varphi \big[T + \int_0^u du' e^{-\varphi} (\partial_\phi Y + Y \partial_\phi \varphi) \big]$$

solution involves 2 arbitrary functions on the circle

$$Y = Y(\phi), \quad T = T(\phi)$$

spacetime vector form faithful representation of $\mathfrak{bms}_3$ algebra

$$[(Y_1, T_1), (Y_2, T_2)] = (\widehat{Y}, \widehat{T})$$

$$\widehat{Y} = Y_1 \partial_\phi Y_2 - (1 \leftrightarrow 2), \quad \widehat{T} = Y_1 \partial_\phi T_2 + T_1 \partial_\phi Y_2 - (1 \leftrightarrow 2)$$

BMS3/CFT1 Solution space and conformal properties

general solution parametrized by $\Theta = \Theta(\phi), \quad \Xi = \Xi(\phi)$

$$ds^2 = s_{uu} du^2 - 2dudr + 2s_{u\phi} dud\phi + r^2 e^{2\varphi} d\phi^2,$$
$$s_{uu} = e^{-2\varphi}\big[\Theta - (\partial_\phi\varphi)^2 + 2\partial_\phi^2\varphi\big] - 2r\partial_u\varphi,$$
$$s_{u\phi} = e^{-\varphi}\Big[\Xi + \int_0^u du' e^{-\varphi}\big[\tfrac{1}{2}\partial_\phi\Theta - \partial_\phi\varphi[\Theta - (\partial_\phi\varphi)^2 + 3\partial_\phi^2\varphi] + \partial_\phi^3\varphi\big]\Big].$$

$\mathfrak{bms}_3$ transformation properties

$$-\delta_{Y,T}\,\Theta = Y\partial_\phi\Theta + 2\,\partial_\phi Y\,\Theta - 2\partial_\phi^3 Y,$$
$$-\delta_{Y,T}\,\Xi = Y\partial_\phi\Xi + 2\,\partial_\phi Y\,\Xi + \frac{1}{2}T\partial_\phi\Theta + \partial_\phi T\,\Theta - \partial_\phi^3 T,$$

covariant charges

$$\mathcal{Q}_\xi[g-\bar{g},\bar{g}] \approx \frac{1}{16\pi G}\int_0^{2\pi} d\phi\,(\Theta T + 2\Xi Y)$$

$$K_{\xi_1,\xi_2} = \frac{1}{8\pi G}\int_0^{2\pi} d\phi\,\Big[\partial_\phi Y_1(T_2 + \partial_\phi^2 T_2) - \partial_\phi Y_2(T_1 + \partial_\phi^2 T_1)\Big]$$

Charge algebra

modes

$Y(\theta)$

$$J_n = \xi(T=0, Y=\exp(in\theta))$$

$$P_n \equiv \xi(T=\exp(in\theta), Y=0)$$

$$\mathrm{i}[J_m, J_n] = (m-n)J_{m+n},$$
$$\mathrm{i}[P_m, P_n] = 0,$$
$$\mathrm{i}[J_m, P_n] = (m-n)P_{m+n}.$$

1 copy of Wit algebra acting on $\mathfrak{i}_1$

$$\cup$$

$$\mathfrak{iso}(2,1)$$

charge algebra:

$$\mathrm{i}\{\mathcal{J}_m, \mathcal{J}_n\} = (m-n)\mathcal{J}_{m+n},$$
$$\mathrm{i}\{\mathcal{P}_m, \mathcal{P}_n\} = 0,$$
$$\mathrm{i}\{\mathcal{J}_m, \mathcal{P}_n\} = (m-n)\mathcal{P}_{m+n} + \frac{1}{4G}m(m^2-1)\delta_{n+m}.$$

relation to AdS_3 similar to contraction between $\mathfrak{so}(2,2) \to \mathfrak{iso}(2,1)$

$$\mathrm{i}[J_m, J_n] = (m-n)J_{m+n},$$
$$\mathrm{i}[P_m, P_n] = \frac{1}{l^2}(m-n)J_{m+n},$$
$$\mathrm{i}[J_m, P_n] = (m-n)P_{m+n},$$

$$L^{\pm}_m = \frac{1}{2}(lP_{\pm m} \pm J_{\pm m}) \qquad l \to \infty$$

collaboration with G. Compère

solution space

ansatz $$g_{AB}=r^2\bar\gamma_{AB}+rC_{AB}+D_{AB}+\frac{1}{4}\bar\gamma_{AB}C^C_DC^D_C+o(r^{-\epsilon})$$

determinant condition $C^A_A=0=D^A_A$

Sachs: power series and $D_{AB}=0$ guarantees absence of log terms

equations of motion imply $\beta=\beta(g_{AB})$

$$U^A=-\frac{1}{2}r^{-2}\bar D_BC^{BA}-\frac{2}{3}r^{-3}\Big[(\ln r+\frac{1}{3})\bar D_BD^{BA}-\frac{1}{2}C^A_B\bar D_CC^{CB}+N^A\Big]+o(r^{-3-\varepsilon}),$$

angular momentum aspect $N^A(u,x^A)$ u dependence fixed

log terms also absent when $D_{\zeta\zeta}=d(\zeta),\quad D_{\bar\zeta\bar\zeta}=\bar d(\bar\zeta),\quad D_{\zeta\bar\zeta}=0.$

Solution space

$$\frac{V}{r} = -\frac{1}{2}\bar{R} + r^{-1}2M + o(r^{-1-\epsilon})$$

mass aspect $M(u, x^A)$ u dependence fixed

news tensor $\partial_u C_{AB}(u, x^A)$ only arbitray function of u

general solution: 4 arbitrary functions of 3 variables & 3 arbitrary functions of 2 variables

$$g_{AB}(u_0, r, x^A) \qquad \partial_u C_{AB}(u, x^A) \qquad M(u_0, x^A) \qquad N^A(u_0, x^A)$$

for simplicity $\widetilde{\varphi} = 0$ $\bar{\gamma}_{AB}dx^A dx^B = d\zeta d\bar{\zeta}$

$$C_{\zeta\zeta} = c, \quad C_{\bar{\zeta}\bar{\zeta}} = \bar{c}, \quad C_{\zeta\bar{\zeta}} = 0$$

redefinitions $\tilde{M} = M - \bar{\partial}^2 c - \partial^2 \bar{c}$ $\tilde{N}^\zeta = -\frac{1}{12}[2N^\zeta + 7\bar{c}\bar{\partial}c + 3c\bar{\partial}\bar{c}]$

evolution equations $\partial_u \tilde{M} = -\dot{c}\dot{\bar{c}}$ $3\partial_u \tilde{N}^\zeta = -\bar{\partial}\tilde{M} - 2\bar{\partial}^3 c - (\bar{\partial}\bar{c} + 3\bar{c}\bar{\partial})\dot{c}$

Conformal properties

bms4 transformations

$$-\delta c = f\dot{c} + Y^A\partial_A c + (\frac{3}{2}\partial Y - \frac{1}{2}\bar{\partial}\bar{Y})c - 2\partial^2 f$$

$$-\delta d = Y^A\partial_A d + 2\partial Y d \qquad\qquad f = T + \frac{1}{2}u\psi$$

$$-\delta\dot{c} = f\ddot{c} + Y^A\partial_A\dot{c} + 2\partial Y\dot{c} - \partial^3 Y$$

$$-\delta\tilde{M} = -f\dot{c}\dot{\bar{c}} + Y^A\partial_A\tilde{M} + \frac{3}{2}\psi\tilde{M} + \bar{c}\partial^3 Y + c\bar{\partial}^3\bar{Y} + 4\partial^2\bar{\partial}^2\tilde{T}$$

$$-\delta\tilde{N}^\zeta = Y^A\partial_A\tilde{N}^\zeta + (\partial Y + 2\bar{\partial}\bar{Y})\tilde{N}^\zeta + \frac{1}{3}\partial(\psi\bar{d})$$
$$-\bar{\partial}f(\tilde{M} + 2\bar{\partial}^2 c + \bar{c}\dot{c}) - \frac{f}{3}[\bar{\partial}\tilde{M} + 2\bar{\partial}^3 c + (\bar{\partial}\bar{c} + 3\bar{c}\bar{\partial})\dot{c}]$$

Interpretation and consequences: work in progress

Conclusions and perspectives

4d gravity is dual to some conformal field theory

to be done:

- classifiy (non)-central extensions; study representation theory of bms4
- surface charge algebra
- non extremal Kerr/CFT correspondence ?

angular momentum problem in GR:

Lorentz = bms4(old)/supertranslations versus bms4(new)/supertranslations = Virasoro

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quantum understanding of BH entropy: what microstates are responsible?

physical toy model for BH: electromagnetic Coulomb solution

quantize EM field in the presence of external source Q

action

$$S^Q = \int d^4x \, [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu]$$

$$j^\mu = \delta^\mu_0 Q \delta^3(\vec{x})$$

modification of constraint associated with Gauss law $\phi_2^Q \equiv -\pi^i{}_{,i} + j^0 = 0$

Quantum Coulomb solution   BFV-BRST quantization

standard BFV-BRST operator quantization in Hilbert space with indefinite norm

modified BRST charge

$$\Omega^Q = \int d^3k \; c^*(\vec{k})[a(\vec{k}) - q(\vec{k})] + [a^*(\vec{k}) - q(\vec{k})]c(\vec{k})$$

$$q(\vec{k}) = \frac{Q}{(2\pi)^{3/2}\sqrt{2}k^{3/2}}$$

null oscillators

$$a(\vec{k}) = a_3(\vec{k}) + a_0(\vec{k})$$
$$b(\vec{k}) = \frac{1}{2}(a_3(\vec{k}) - a_0(\vec{k}))$$

$$[\hat{a}(\vec{k}), \hat{b}^\dagger(\vec{k}')] = \delta^3(\vec{k} - \vec{k}')$$

standard vacuum no longer BRST invariant !

shifted oscillators

$$a^Q(\vec{k}) = a(\vec{k}) - q(\vec{k}), \quad a^{*Q} = a^*(\vec{k}) - q(\vec{k})$$

new vacuum in the presence of source

$$\hat{a}^Q(\vec{k})|0\rangle^Q = 0$$

Quantum Coulomb solution   Coherent state of unphysical photons

solution in terms of old vacuum

$$|0\rangle^Q = \prod_{\vec{k}} \exp q(\vec{k})\hat{b}^\dagger(\vec{k})|0\rangle = \exp \int d^3k\; q(\vec{k})\hat{b}^\dagger(\vec{k})\; |0\rangle$$

null oscillator

$$^Q\langle 0|0\rangle^Q = \langle 0|0\rangle = 1$$

expectation values of electric and magnetic fields

$$^Q\langle 0|\hat{\pi}^i(x)|0\rangle^Q = \frac{Qx^i}{4\pi r^3}$$

$$^Q\langle 0|\vec{\nabla} \times \hat{\vec{A}}|0\rangle^Q = 0$$

quantum mechanical understanding of Coulomb solution only possible in Gupta-Bleuler/ BRST quantization

BH problem: one needs to “count” coherent states of unphysical degrees of freedom