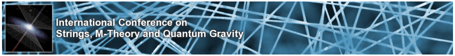
M2-branes and the novel Higgs mechanism

Sunil Mukhi
Tata Institute of Fundamental Research, Mumbai



Centro Stefano Franscini, Monte Verita, Ascona
28 July 2010

- Based on:

  "M2 to D2",
  SM and Costis Papageorgakis,
  arXiv:0803.3218 [hep-th], JHEP 0805:085 (2008).

  "M2-branes on M-folds",
  Jacques Distler, SM, Costis Papageorgakis and Mark van Raamsdonk,
  arXiv:0804.1256 [hep-th], JHEP 0805:038 (2008).

  "The Power of the Higgs Mechanism: Higher-Derivative BLG Theories",
  Bobby Ezhuthachan, SM, Costis Papageorgakis,
  arXiv:0903.0003 [hep-th], JHEP 0904:101 (2009).

  and work in progress.

Outline

Background: Multiple M2-branes

- M-theory is a theory of massless modes described by 11-d supergravity as well as stable M2-branes and M5-branes and ...

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Background: Multiple M2-branes

- M-theory is a theory of massless modes described by 11-d supergravity as well as stable M2-branes and M5-branes and ...
- Of specific interest to us in this talk will be the $(2+1)$-d world-volume field theory on M2-branes.
- For a single M2-brane, the theory is a simple free field theory (with higher-derivative corrections).

$$\text{M2} : 8\phi, 8\psi \qquad\qquad \text{in 2+1 d}$$

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- Many pieces of evidence have been accumulated to show that three-algebras are central to this question.
- Surprisingly three-algebras may also be relevant to multiple M5-branes (cf. the talk of Costis Papageorgakis in this conference).

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- This is a super-renormalisable theory that inherits its coupling from the string coupling g_s:

$$g_{YM} = \sqrt{\frac{g_s}{l_s}}$$

- In the M-theory limit, $g_{YM} \to \infty$ which is the infrared limit for the SYM theory.

- Thus we may define:

$$\mathcal{L}_{M2} = \lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{D2}$$

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- It must be an infrared fixed point and therefore a CFT. In particular it should have 8 scalars describing transverse motion of the branes and an SO(8) R-symmetry.
- For the Abelian case it is possible to derive the M2-brane theory via a duality transformation.

- The derivation is as follows:

$$\frac{1}{2g^2} dA \wedge {}^*dA \ \leftrightarrow \ B \wedge dA - \frac{g^2}{2} B \wedge {}^*B$$

where B is a 1-form field. Integrating out B gives the LHS.

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- Integrating out A instead, we find $dB = 0$ which implies $B = d\phi$ for some scalar ϕ. The action then becomes:

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- The full DBI approximation can be dualised similarly.

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- The class of theories subsequently found by [Aharony-Bergman-Jafferis-Maldacena] satisfy (iii) but not manifestly (i) and (ii).
- Because the ABJM theory is more complicated, in this talk I will restrict my attention to BLG theory. But the phenomena to be discussed here hold also in ABJM theory.

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- Then I will discuss its relation to some more general (and older) works on $(2+1)$-d topologically massive field theories [Deser-Jackiw-Templeton], [Deser-Jackiw].

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- This problem was solved [BLG] by the key insight that a non-dynamical Chern-Simons gauge field should be added.
- With this, they discovered a superconformal theory whose interactions are governed by a mathematical structure called a 3-algebra.

- This involves generators T^A and a "three-bracket" satisfying:

$$[T^A, T^B, T^C] = f^{ABC}{}_D T^D$$

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- The 3-algebra idea was inspired by old work of [Nambu, Filippov] and more recent work of [Basu-Harvey].

- The scalars X^I and fermions Ψ are three-algebra valued and the interactions are:

$$\sim \mathrm{Tr}\Big([X^A, X^B, X^C]^2\Big) \quad \text{and} \quad \sim \mathrm{Tr}\Big([\bar{\Psi}^A, X^B, \Psi^C]X^D\Big)$$

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- And there is a gauge field A_μ^{AB} with minimal couplings to the scalars and fermions, and a Chern-Simons interaction:

$$\frac{k}{2\pi}\,\varepsilon^{\mu\nu\lambda}\Big(f_{ABCD} A_\mu{}^{AB} \partial_\nu A_\lambda{}^{CD} + \tfrac{2}{3} f_{AEF}{}^{G} f_{BCDG}\, A_\mu{}^{AB} A_\nu{}^{CD} A_\lambda{}^{EF}\Big)$$

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where k is the quantised level.

- Surprisingly, the only consistent solution of the fundamental identity (with Euclidean 3-algebra metric) turns out to be:

$$f^{ABCD} = \epsilon^{ABCD}, \quad A, B, C, D = 1, \cdots, 4$$

- By taking suitable linear combinations [van Raamsdonk] of A_μ^{AB} one finds a pair of $SU(2)$ gauge fields $\boldsymbol{A}_\mu, \tilde{\boldsymbol{A}}_\mu$.

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$$\frac{k}{4\pi}\,\mathrm{tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A}+\frac{2}{3}\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}-\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}}-\frac{2}{3}\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right)$$

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- In this way the theory reduces to a conventional gauge theory, which conserves parity if we send $\boldsymbol{A}\leftrightarrow\tilde{\boldsymbol{A}}$.
- The integer parameter k is a puzzle.

The novel Higgs mechanism

- Take $k = 1$ to start with. In [SM-Papageorgakis] it was shown that on giving a vev v to one component of the scalars,

$$L_{BLG}\Big|_{vev\ \ v} = \frac{1}{v^2} L_{SYM}^{U(2)} + \mathcal{O}\left(\frac{1}{v^3}\right)$$

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- In comparison with the usual Higgs mechanism in $(2+1)$-d:

Usual:	$A_\mu^{YM,massless}$,	ϕ	$\rightarrow$	$A_\mu^{YM,massive}$
DOF:	1	1		2
Novel:	A_μ^{CS},	ϕ	$\rightarrow$	$A_\mu^{YM,massless}$
DOF:	0	1		1

- Here is a quick derivation of this novel Higgs mechanism:

$$\begin{aligned} L_{CS} &= \frac{1}{2}\,\mathrm{tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A}+\frac{2}{3}\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}-\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}}-\frac{2}{3}\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right) \\ &= 2\,\mathrm{tr}\left(\boldsymbol{B}\wedge \boldsymbol{F}(\boldsymbol{C})+\frac{1}{3}\boldsymbol{B}\wedge\boldsymbol{B}\wedge\boldsymbol{B}\right) \end{aligned}$$

where $\boldsymbol{B}=\frac{1}{2}(\boldsymbol{A}-\tilde{\boldsymbol{A}}), \boldsymbol{C}=\frac{1}{2}(\boldsymbol{A}+\tilde{\boldsymbol{A}})$, $\boldsymbol{F}(\boldsymbol{C})=d\boldsymbol{C}+\boldsymbol{C}\wedge\boldsymbol{C}$.

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$$\mathrm{tr}(D_\mu X)^2 = v^2\,\mathrm{tr}\,\boldsymbol{B}_\mu\boldsymbol{B}^\mu + \cdots$$

- Thus, $\boldsymbol{B}$ is massive – but not dynamical. Integrating it out:

$$-\frac{1}{v^2}\boldsymbol{F}(\boldsymbol{C})\wedge{}^*\boldsymbol{F}(\boldsymbol{C}) + \mathcal{O}\left(\frac{1}{v^3}\right)$$

  so $\boldsymbol{C}$ becomes a dynamical, massless Yang-Mills gauge field and $g_{YM} = v$.

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- It seems like the M2 brane theory has become a theory of two D2-branes with $g_{\mathsf{YM}} = v$.
- This appears related to the idea that M2-branes become D2-branes after compactifying a direction.

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- The resolution is that for any finite v, there are corrections to the Yang-Mills action that decouple only as $v \to \infty$.

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- This amounts to a proof that somewhere on its moduli space, the BLG theory describes a pair of M2-branes.
- Notice that nowhere do the postulated M-branes become weakly coupled D2-branes: when v is small then the corrections are important.

- The theory is not translation-invariant, so there must be something besides M2-branes in it. The "something" was proposed [Distler-SM-Papageorgakis-van Raamsdonk], [Lambert-Tong] to be a Z_{2k} orbifold. This would also explain the presence of the parameter k.

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- The standard orbifold was later studied by ABJM, leading to an impressive set of developments that won't be reviewed here.
- An important point is that although ABJM did not phrase it in that language, their theory too is a 3-algebra theory.

- Returning to BLG, once we introduce the Chern-Simons level k then the analysis of the novel Higgs mechanism is different [Distler-SM-Papageorgakis-van Raamsdonk]:

$$L_{BLG}\Big|_{vev\ \ v} = \frac{k}{v^2} L^{U(2)}_{SYM} + \mathcal{O}\left(\frac{k}{v^3}\right)$$

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- So this time M2-branes have genuinely reduced to D2-branes. But why?

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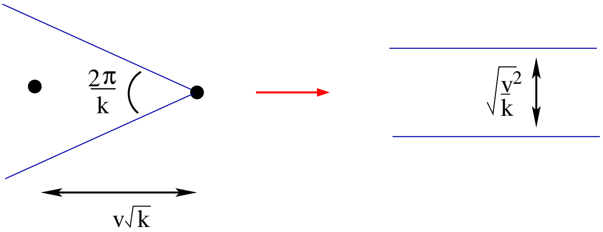

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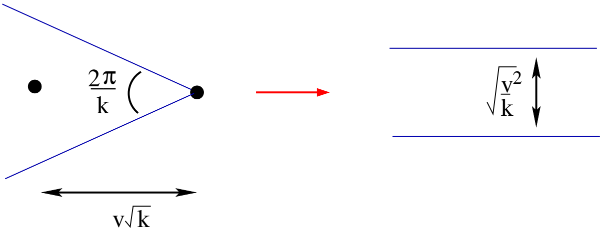

- With the joint scaling above, the branes see a cylinder of finite radius [Arkani-Hamed et al], i.e. an effectively compact transverse dimension. In this limit M2-branes should indeed reduce to D2-branes!

The power of the Higgs mechanism

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- To find the derivative corrections for the BLG theory, we simply wrote the most general 3-algebra expression at that order, Higgsed the theory and compared to D2-branes.
- Remarkably, all coefficients are determined uniquely by this procedure [Ezhuthachan-SM-Papageorgakis].
- This indicates that the 3-algebra structure extends to higher-derivative orders, much as the Yang-Mills commutator for D-branes extends to higher-derivatives.

$$
\begin{aligned}
S^{b}_{\ell_p^3} = (2\pi)^2 \ell_p^3 \int d^3x \,\mathrm{STr}\Big[&\tfrac{1}{4}\left(\tilde{D}^\mu X^I \tilde{D}_\mu X^J \tilde{D}^\nu X^J \tilde{D}_\nu X^I - \tfrac{1}{2}\tilde{D}^\mu X^I \tilde{D}_\mu X^I \tilde{D}^\nu X^J \tilde{D}_\nu X^J\right) \\
&- \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda}\left(X^{IJK}\tilde{D}_\mu X^I \tilde{D}_\nu X^J \tilde{D}_\lambda X^K\right) \\
&+ \tfrac{1}{4}\left(X^{IJK}X^{IJL}\tilde{D}^\mu X^K \tilde{D}_\mu X^L - \tfrac{1}{6}X^{IJK}X^{IJK}\tilde{D}^\mu X^L \tilde{D}_\mu X^L\right) \\
&+ \tfrac{1}{288}\left(X^{IJK}X^{IJK}X^{LMN}X^{LMN}\right)\Big] \,. \qquad (4.15)
\end{aligned}
$$

where now:

$$
X^{IJK} = [X^I, X^J, X^K] \,. \qquad (4.16)
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Unravelling the novel Higgs mechanism

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- As DJT explained, this theory has a single on-shell degree of freedom that is massive, spin $+1$. Because parity is violated, it is possible to have no spin -1 mode.

- Somewhat surprisingly, an alternative version of the same theory is Chern-Simons with an explicit mass term:

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- Classically, equivalence is shown as follows.

(i) ${}^*dA = mA \implies d\,{}^*dA = m\,dA,$ so $\mathcal{L}_2 \implies \mathcal{L}_1$

(ii) $d\,{}^*dA = m\,dA \implies d\,({}^*dA - m\,A) = 0 \implies {}^*dA - m\,A = d\lambda$

and a field re-definition $A \to A - \frac{1}{m} d\lambda$ gives:

$${}^*dA = mA$$

so $\mathcal{L}_1 \implies \mathcal{L}_2$.

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$$\begin{aligned}\mathcal{L}_1 &= \frac{1}{2}dA\wedge {}^*dA + \frac{m}{2}A\wedge dA \\ \mathcal{L}_2 &= \frac{1}{2}A\wedge dA - \frac{m}{2}A\wedge {}^*A\end{aligned}$$

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- The above discussion extends to the non-Abelian case.
- The [DJ] duality is however not (yet) the phenomenon that occurs on M2-branes.

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- Consider the following action, modelled on $\mathcal{L}_2$:

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- Also, denote the gauge fields A_I as (B_i, C_a) where $i = 1, 2, \cdots P$ and $a = 1, 2, \cdots Q$.

- Then the action is:

$$\mathcal{L} = \frac{1}{2}k_{ij}B_i \wedge dB_j + k_{ia}B_i \wedge dC_a + \frac{1}{2}k_{ab}C_a \wedge dC_b - \frac{1}{2}m_i B_i \wedge {}^* B_i$$

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- The equation of motion for the B_i is now algebraic and we get:

$$B_i = \frac{k_{ia}}{m_i} dC_a$$

from which:

$$\mathcal{L} = \frac{1}{2} \frac{k_{ia} k_{ib}}{m_i} dC_a \wedge {}^* dC_b$$

which is a massless theory!

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- In a basis where k_{IJ} rather than m_{IJ} is diagonal we have:

$$k_{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad m_{IJ} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

and we see the famous "difference Chern-Simons action".

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- It arises naturally in 3-algebra theories due to the twisted Chern-Simons term and the bi-fundamental nature of the scalars, but is much more generic.
- A more general form of the Higgs mechanism that allows $k_{ab} \neq 0$ was used by [Gaiotto-Tomasiello] to study M2-branes in the presence of a Romans mass.

Outline

Conclusions

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(ii) This mechanism arises in 3-algebra theories and uses a special class of dualities in $(2+1)$-d associated to the presence of a topological mass term.

(iii) It would be interesting if more general examples arise in M-theory.

(iv) The novel Higgs mechanism itself is more general and doesn't require 3-algebras or supersymmetry, just several Chern-Simons gauge fields and suitable mass terms. It may have applications elsewhere in physics.