CFT description of the self-dual orbifold

Balasubramanian, Parsons & SFR

Simon Ross

Centre for Particle Theory, Durham University

Monte Verita 29 July 2010

Outline

- Motivation: laboratory for studying AdS_2
- Review of self-dual orbifold spacetime
- Two boundaries: entangled state
- Boundary conditions
- Asymptotically SDO geometries?
- Conclusions

Motivation

- AdS_2 is everywhere:
  - Generic near-horizon geometry of extremal black hole—entropy function formalism.
  - AdS_2 in near-horizon of Reissner-Nordström AdS used to describe non-Fermi liquids.
  - Kerr/CFT.
- Some puzzles:
  - Nature of dual CFT? 0+1? 1+1 chiral theory?
  - Geometry has two boundaries, causally connected through bulk.
  - No asymptotically AdS_2 spacetimes?

Self-dual orbifold is a quotient of AdS_3, circle fibration over AdS_2. Simple context to address these issues.

Review of SDO

Coussaert
Henneaux

Balasubramanian
Naqvi
Simon

Balasubramanian
de Boer
Sheikh-Jabbari
Simon

Quotient of AdS_3 by $\xi = J_{02} + J_{13}$. $\|\xi\|^2 = 1$. $SL(2,\mathbb{R}) \times U(1)$ isometries. Define adapted coordinates:

$$U + X = \frac{1}{\sqrt{2}} e^{\phi}(e^{z}\cos t - e^{-z}\sin t) = e^{r+u},$$

$$U - X = \frac{1}{\sqrt{2}} e^{-\phi}(e^{-z}\cos t - e^{z}\sin t) = \frac{1}{2}(e^{-r-u} + 2ve^{r-u}),$$

$$V + Y = \frac{1}{\sqrt{2}} e^{\phi}(e^{-z}\cos t + e^{z}\sin t) = \frac{1}{2}(e^{-r+u} - 2ve^{r+u}),$$

$$V - Y = \frac{1}{\sqrt{2}} e^{-\phi}(e^{z}\cos t + e^{-z}\sin t) = e^{r-u}.$$

In these coordinates, $\xi = \partial_\phi = \partial_u$. AdS_3 metric is

$$ds^2 = -\cosh^2 2z dt^2 + dz^2 + (d\phi + \sinh 2z dt)^2$$

and

$$ds^2 = -e^{4r}dv^2 + dr^2 + (du + e^{2r}dv)^2.$$

(t, ϕ, z) global; (u, v, r) are like Poincare coordinates.

Review of SDO: Boundary geometry

Boundary is at $z \to \pm\infty$.
On boundary, (t, ϕ) are null coordinates; boundary metric $\tilde{ds}^2 = -2dtd\phi$.
Related to usual global coordinates on AdS_3 (τ, θ) by

$$\tan(\tau - \theta) = \mp \frac{1}{\sinh 2\phi}, \quad \tan(\tau + \theta) = \tan 2t.$$

$\phi \to \pm\infty$ corresponds to $\tau - \theta = 0, \pi$. These lines divide the boundary into two strips, corresponding to $z = \pm\infty$.
Boundary two null cylinders

In (u, v, r) coordinates, boundary at $r \to \infty$, $\tilde{ds}^2 = -2dudv$.
This covers the $z = \infty$ boundary for $t \in (-\pi/2, \pi/2)$.

Review of SDO: Limit of BTZ

Strominger

Balasubramanian
de Boer
Sheikh-Jabbari
Simon

From extremal BTZ black hole

$$ds^2 = -\frac{(r^2 - r_+^2)^2}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)^2} dr^2 + r^2 (d\phi - \frac{r_+^2}{r^2} dt)^2,$$

define coordinates $u = r_+(t - \phi)$, $v = \frac{\epsilon}{2r_+}(t + \phi)$, $r^2 - r_+^2 = \epsilon e^{2r'}$.
The limit $\epsilon \to 0$ gives SDO.

- Obtain CFT dual of SDO from CFT dual of BTZ;
  - BTZ: CFT on spacelike cylinder, state $T_R = 0$, $T_L = r_+/2\pi$
  - Near-horizon limit involves infinite boost: DLCQ
  - Chiral $1+1$ CFT on null cylinder, thermal state for surviving sector.
  - Unlike Kerr-CFT, different J are states in a single CFT.
- Can also see DLCQ in quotient picture: u is spacelike at finite r, becomes null as we remove UV cutoff.

Note these coordinates only see one boundary.

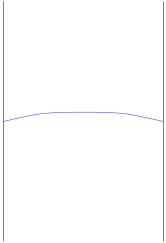
CFT description

Spacetime has two boundaries, which are causally connected. Is the dual a single CFT, or two copies on the two boundaries?

CFT description

Spacetime has two boundaries, which are causally connected. Is the dual a single CFT, or two copies on the two boundaries?

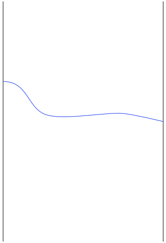
Our understanding of non-extremal BTZ suggests it should be an entangled state in two copies of the CFT.



- Bulk time translation gives two independent Hamiltonians.
- However, causal connection $\Rightarrow$ translating spacelike surface in bulk can't generate independent time-translations by arbitrary amounts.
- Spatial connection implies $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \neq 0$, obtained from entanglement. van Raamsdonk

CFT description

Spacetime has two boundaries, which are causally connected. Is the dual a single CFT, or two copies on the two boundaries?
Our understanding of non-extremal BTZ suggests it should be an entangled state in two copies of the CFT.

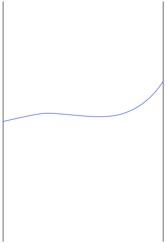


- Bulk time translation gives two independent Hamiltonians.
- However, causal connection $\Rightarrow$ translating spacelike surface in bulk can't generate independent time-translations by arbitrary amounts.
- Spatial connection implies $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \neq 0$, obtained from entanglement. van Raamsdonk

CFT description

Spacetime has two boundaries, which are causally connected. Is the dual a single CFT, or two copies on the two boundaries?

Our understanding of non-extremal BTZ suggests it should be an entangled state in two copies of the CFT.

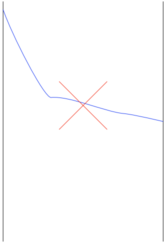


- Bulk time translation gives two independent Hamiltonians.
- However, causal connection $\Rightarrow$ translating spacelike surface in bulk can't generate independent time-translations by arbitrary amounts.
- Spatial connection implies $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \neq 0$, obtained from entanglement. van Raamsdonk

CFT description

Spacetime has two boundaries, which are causally connected. Is the dual a single CFT, or two copies on the two boundaries?

Our understanding of non-extremal BTZ suggests it should be an entangled state in two copies of the CFT.



- Bulk time translation gives two independent Hamiltonians.
- However, causal connection $\Rightarrow$ translating spacelike surface in bulk can't generate independent time-translations by arbitrary amounts.
- Spatial connection implies $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \neq 0$, obtained from entanglement. van Raamsdonk

AdS_3 in SDO coordinates

Consider usual $\mathrm{AdS}_3/\mathrm{CFT}_2$ duality in this coordinate system:

- Coordinate transformation is conformal transformation on boundary; maps $\mathbb{R} \times S^1$ boundary to two $\mathbb{R}^{1,1}$ regions.
- Indeed have independent dof in the two regions. Entanglement:
  - Toy model: free scalar field on $\mathbb{R} \times S^1$. Vacuum wrt Poincare time is an entangled state wrt u, v coords,

$$|0\rangle = e^{-i \int_0^\infty d\omega e^{-\pi\omega} b^{1\dagger}_{\omega,l} b^{2\dagger}_{\omega,l}} |0\rangle_1 \otimes |0\rangle_2,$$

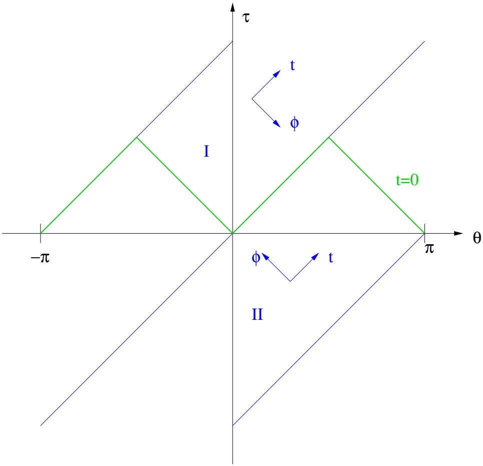
    where $b_l^{1,2\dagger}$ are creation operators for modes of frequency ω wrt the u coordinate. The corresponding modes $b_r^{1,2\dagger}$ wrt the v coordinate are not excited.
  - Boundary stress tensor: $T_{\mu\nu} = 0$ in vacuum in Poincare coordinates, so in (u, v) coordinates

$$T_{vv} = 0, \quad T_{uu} = \frac{c}{12}\left[\frac{\partial_u^3 x^+}{\partial_u x^+} - \frac{3}{2}\left(\frac{\partial_u^2 x^+}{\partial_u x^+}\right)^2\right] = -\frac{c}{6}.$$

    Can obtain same result from bulk by usual holographic calculation.

AdS_3 in SDO coordinates

- Causal connection in bulk is causal connection on boundary.
  - In particular, spacelike slices are portions of a single spacelike slice of $\mathbb{R} \times S^1$; this slice ceases to be acausal when the $z = \pm\infty$ slices are causally connected in bulk.



Effect of quotient on boundary

In a free theory, can obtain state on the quotient by method of images.
But quotient modifies causal relations:

- The quotient under $\phi \sim \phi + 2\pi r_+$ has fixed points in the boundary, at $\tau - \theta = 0, \pi$.
- Need to remove fixed points before quotienting: removes causal connection in boundary. Spacelike surfaces in $z = \pm\infty$ now never causally connected in boundary.
- Causal connection in bulk would predict

$$\langle [\mathcal{O}_1(0), \mathcal{O}_2(\Delta t)] \rangle = \Delta^{\phi}_{bulk}(\Delta t) \neq 0 \quad \text{for } \Delta t > \pi/2,$$

inconsistent with entanglement description in CFT.
Explicit interaction?

Entangled state from near-horizon limit

Can also obtain SDO as near-horizon, near-extremal limit from non-extremal BTZ:

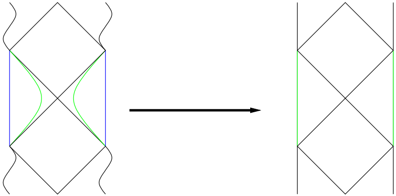
$$r^2 - r_+^2 = \epsilon r_+^2 \bar{r}^2, \quad r_+^2 - r_-^2 = \epsilon r_+^2,$$

$$\bar{t} = r_+ \epsilon t, \quad \bar{\phi} = r_+ \phi - r_- t.$$

Limit $\epsilon \to 0$ gives

$$ds^2 = -\bar{r}^2(\bar{r}^2 + 1) d\bar{t}^2 + \frac{d\bar{r}^2}{(\bar{r}^2 + 1)} + \left(d\bar{\phi} + \bar{r}^2 d\bar{t}\right)^2$$

Null coordinates on boundary are $\bar{u} = \frac{1}{2}\bar{t} - \bar{\phi}$, $\bar{v} = \bar{t}$. "Black hole coordinates"; covers the region $t \in (-\pi/4, \pi/4)$ on boundary.



- Non-extremal black hole has two boundaries: can see regions of both boundaries in SDO in this limit.
- Non-extremal BTZ is an entangled state of two CFTs, temperatures $T_L = \frac{r_+ + r_-}{4\pi}$, $T_R = \frac{r_+ - r_-}{4\pi}$.
  - DLCQ scaling keeps $T_{\bar{u}}$, $T_{\bar{v}}$ finite as $\epsilon \longrightarrow 0$.
- Near-horizon limit is DLCQ: no interactions between boundaries. Entangled state in CFT on these regions.
- Extremal black hole as limiting case.

Marolf
Yarom

CFT description from near-horizon limit

For both extreme and non-extreme BTZ, near-horizon limit covers regions of boundary which aren't in causal contact.

Propose CFT on these boundary regions has a saddle-point description by corresponding regions of SDO, but can't extend to global SDO spacetime

Why can't we extend the spacetime?

- In BTZ, region beyond Cauchy horizon isn't really physical:
  - Mass inflation instability
  - Bulk QFT stress tensor divergent on Cauchy horizon
- But this instability seems to shut off in near-horizon limit:
  - Must scale perturbation $\to 0$ in near-horizon limit to preserve asymptotically SDO structure in "black hole" region
  - QFT stress tensor regular on SDO
- Still, no UV source for CFT data on full SDO boundary.

Entanglement may play important role in this restriction.

Boundary conditions and symmetries

- Can impose standard Brown-Henneaux boundary conditions:

$$\delta g_{uu} \sim \delta g_{uv} \sim \delta g_{vv} \sim \mathcal{O}(1).$$

  Aymptotic isometries parametrized by two arbitrary functions, $\xi^+(u), \xi^-(v)$.

- Balasubramanian de Boer Sheikh-Jabbari Simon restrict boundary conditions to have $g_{vv} = \mathcal{O}(r^{-2})$: $\xi^- = A + Bv + Cv^2$. Asymptotic isometries Virasoro $\times SL(2, \mathbb{R})$. Chiral CFT.

- Contrasts with Kerr-CFT, which has a more relaxed boundary condition.

Boundary conditions and geometries

Boundary metric in all cases is a null cylinder; different coordinate systems distinguished by subleading components of the metric.

- With Brown-Henneaux boundary conditions, can "fill in" null cylinder in different ways. SDO in (t, ϕ, z) or "black hole" coordinates correspond to different states in a single CFT?
  - Black hole coordinates corr to non-chiral excited state.
  - No CFT dual to (t, ϕ, z) coordinates?
- Boundary condition with $g_{vv} = \mathcal{O}(r^{-2})$ picks a unique bulk: SDO in (u, v, r) coordinates.
- Note this also forbids "black hole" coordinates. Consistent with chiral CFT interpretation: no non-extremal states.

Asymptotically SDO geometries

For CFT, natural to consider other states. Geometrical description?
Focus on chiral states:

- More restrictive boundary conditions would only admit such states, $T_{vv} \equiv 0$.
- These states account for entropy of extremal BTZ.
- Dual geometries may preserve $SL(2, \mathbb{R})$; avoid problem with non-trivial asymptotically AdS_2 spacetimes? Maldacena Michelson Strominger
- But $SL(2, \mathbb{R})$ very restrictive: transitive symmetry on SDO spacetime.

For theory with Brown-Henneaux boundary conditions, would also be interesting to study non-chiral states.

Look for solutions by considering near-horizon limits.

Asymptotically SDO geometries

One successful example: dual of vacuum state $|0\rangle = |0\rangle_L \times |0\rangle_R$.

- Dual of this state on a spacelike cylinder is $M = 0$ BTZ black hole.
- Near-horizon limit gives a quotient of AdS_3 by $\xi = J_{01} - J_{13} + J_{02} - J_{23}$. $\|\xi\|^2 = 0$, but ξ is non-zero everywhere, so no fixed points.
- In Poincare coordinates,

$$ds^2 = -2r^2 dx^+ dx^- + \frac{dr^2}{r^2},$$

  $\xi = \partial_{x^-}$.
- Formal dual of ground state. Supersymmetric.
- Geometry has a single boundary, at $r \to \infty$.
- Geometry in Poincare coordinates is asymptotically SDO, even with restrictive boundary conditions.

Pure state, bulk geometry has a single boundary

Note solution has $SL(2,\mathbb{R}) \times U(1)$ isometries, but no AdS_2 factor.

Perturbative states

On asymptotically AdS_3 spacetimes, can generically study nearby states by considering linearized perturbations on bulk geometry.
Consider chiral perturbations of extremal BTZ: i.e., consider a perturbation with $\delta M = \delta J$ on a bh with $M = J$.

- For a scalar field $\phi = e^{i\omega t + im\phi} f_{\omega m}(r)$, want $\omega = m$. Implies $\omega_c = \omega - m = 0$.
- $\Box\phi - \mu^2\phi = e^{i\omega t + im\phi}\left[\frac{1}{r}\partial_r(rh(r)\partial_r f(r)) - \mu^2 f(r)\right]$, normalizable solution

$$f(r) = c(r^2 - r_+^2)^{-h_+/2}.$$

  This blows up on horizon!
- Can't analyse chiral states perturbatively
- Also true for vector fields.

Discussion

- Simple context to study CFT description of very-near horizon region.
- Dual description two CFTs on null cylinder, in an entangled state.
  - General idea: multiple boundaries have independent Hamiltonians, connections come from entangled states.
  - Can't describe causal connections; full spacetime not realised in CFT description.
- Is there any CFT description that can give these causal connections?
- Ground state in CFT on a null cylinder dual (at least formally) to lightlike quotient.
  - Analogous to Schrödinger spacetime
  - Accurate description of this state?
- Geometries dual to other chiral states?

  Not accessible perturbatively even on spacelike cylinder.
- Duality for non-chiral theory?