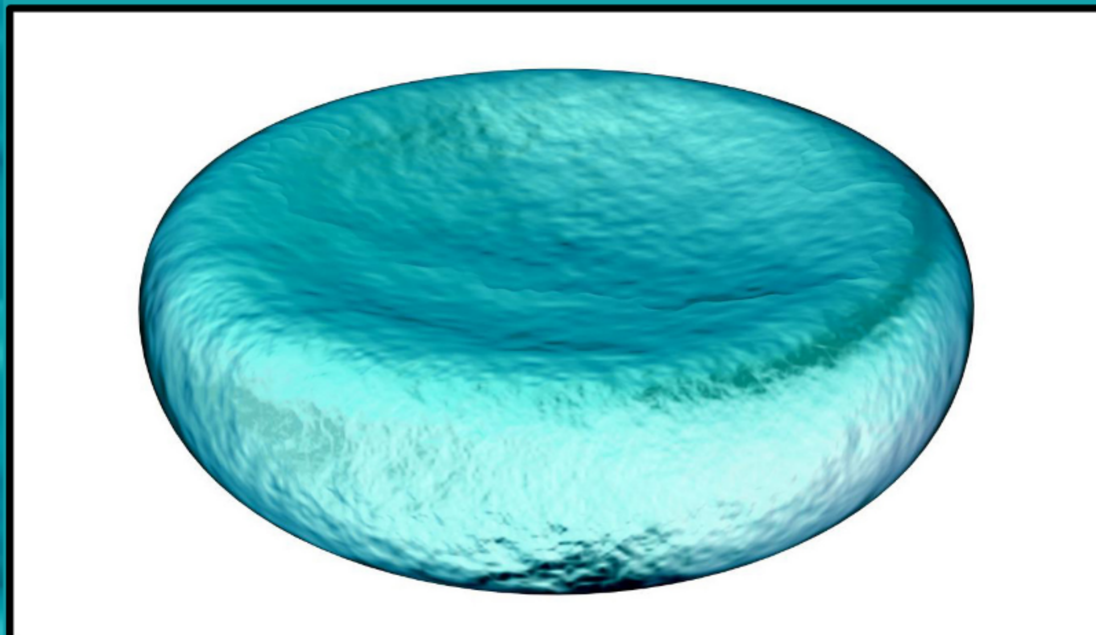
Black Holes and Biophysical (Mem)-branes



Jay Armas | Albert Einstein Center For Fundamental Physics

based on:

J.Armas, arXiv:1304.7773
J. Armas, arXiv:1312.0597
JA & T. Harmark, arXiv:1402.6330 and arXiv:1404.7813

Motivations (I):

. Perturbative construction of stationary black holes

. Probes of AdS/CFT at finite temperature

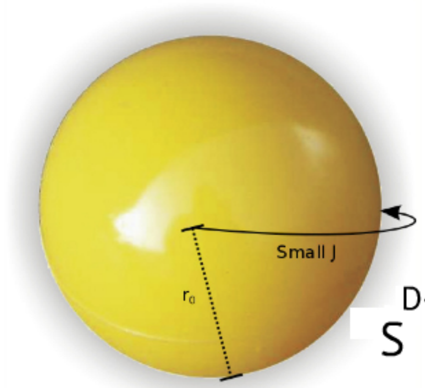
. More general theories of hydrodynamics (confined fluids)

. Fluid membranes | Cellular membranes

. Generic perturbations of black branes (viscous + elastic)

. Effective string actions, cosmic strings, entanglement entropy

Analytic Solutions :

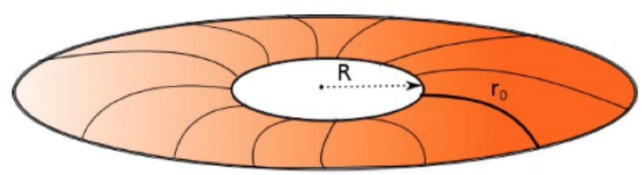


S^{D-2}

Kerr (1963)

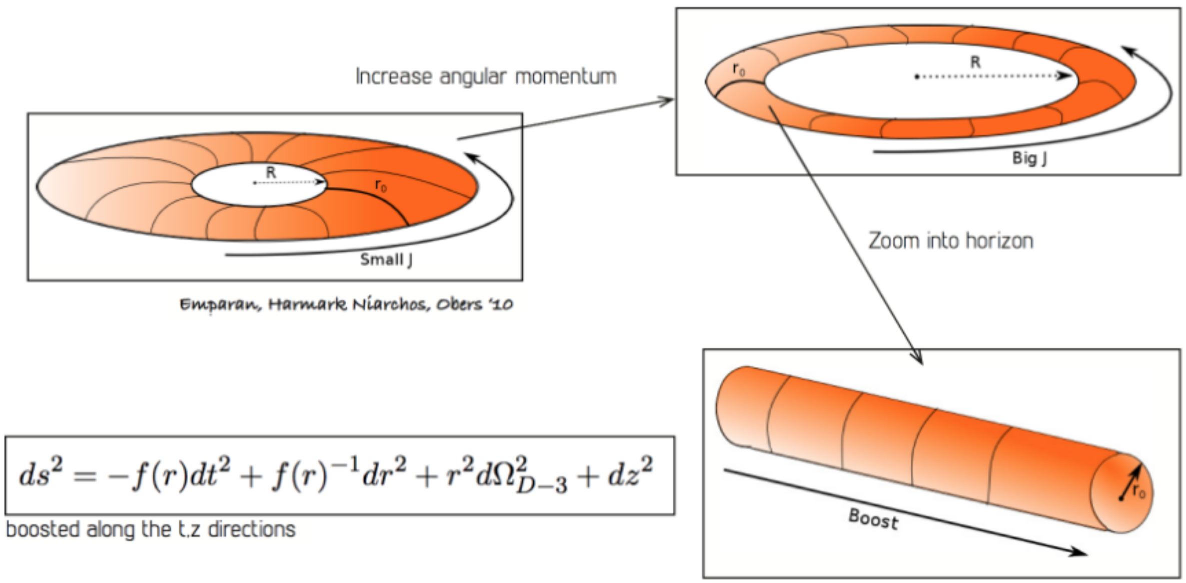
Myers-Perry (1986)

$S^1 X S^2$



Black Ring (2001)

Black Ring Example:



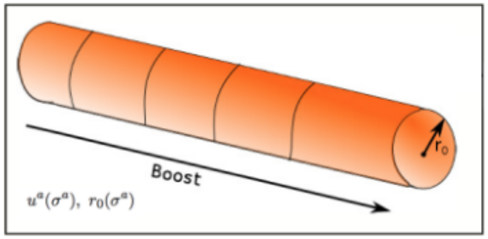
Emparan, Harmark Niarchos, Obers '10

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-3} + dz^2$$

boosted along the t,z directions

What is this effective theory?

Boosted black brane:



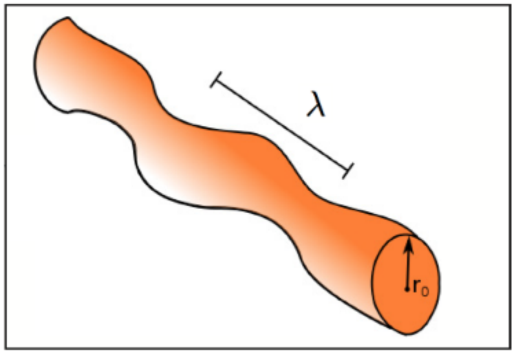
$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

Emparan, Harmark, Obers, Niarchos, 09

$$T^{ab} = P\eta^{ab} + (\epsilon + P)u^a u^b$$

$$\frac{\epsilon}{n+1} = -P = \frac{\Omega_{(n+1)} r_0^n}{16\pi G}$$

Hydrodynamic perturbation:



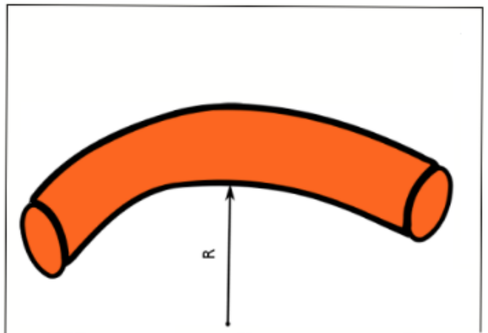
$$u_a = u_a(\sigma^c) \qquad \varepsilon = \frac{r_0}{\lambda} \ll 1$$
$$r_0 = r_0(\sigma^c)$$

Minwalla et al, 08
Emparan et al, 10

$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1} + h_{ab}(r) d\sigma^a d\sigma^b$$

$$G_{\mu\nu} = 0 \Rightarrow \nabla_a T^{ab} = 0$$

Elastic perturbation:



$$\eta_{ab} \to \gamma_{ab}(X^\mu(\sigma^c)) = \eta_{ab} - 2K_{ab}{}^{\hat{i}} r\cos\theta$$

$$\frac{r_0}{R} \ll 1$$

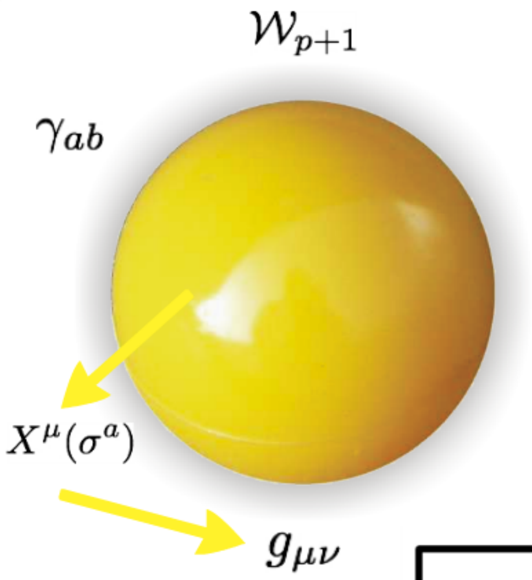
$$ds^2_{(1)} = \left(\eta_{ab} - 2K_{ab}{}^{\hat{i}} r\cos\theta + \frac{r_0^n}{r^n}u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\theta^2 + r^2\sin^2\theta d\Omega^2_{(n)}$$
$$+ h_{\mu\nu}(r,\theta)dx^\mu dx^\nu + \mathcal{O}\left(r^2/R^2\right) \quad .$$

$$h_{\mu\nu}(r,\theta) = \cos\theta \hat{h}_{\mu\nu}(r)$$

Emparan, Harmark, Niarchos, Obers, Rodriguez, 07
Emparan, Camps, 12 | JA, Gath, Obers, 12, 13

$$G_{\mu\nu} = 0 \Rightarrow T^{ab}K_{ab}{}^{i} = 0$$

Elastic matter (I):



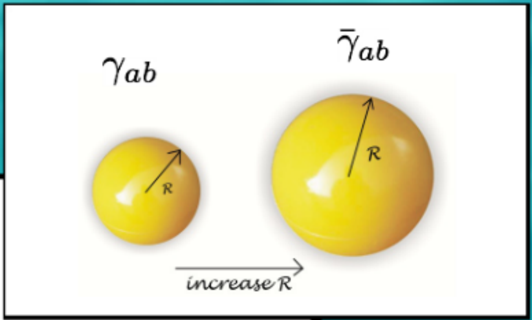
DBI-type action:

$$I[X^{\mu}] = \alpha \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}$$

$$T^{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \gamma_{ab}} = \alpha \gamma^{ab}$$

$$\nabla_a T^{ab} = 0$$
$$T^{ab} K_{ab}{}^{i} = 0$$

Elastic matter (II):



Lagrangian strain:

$$U_{ab} = -\frac{1}{2}\left(\gamma_{ab} - \bar{\gamma}_{ab}\right)$$

infinitesimal deformation →

$$dU_{ab} = K_{ab}{}^{i}\Phi_i$$

$$T^{ab}K_{ab}{}^{i} = 0$$

contract with orthogonal vector →

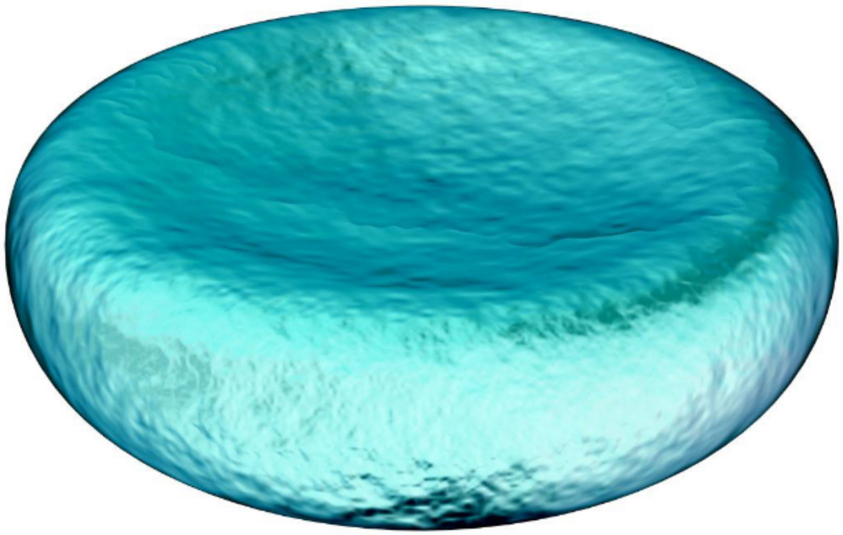
minimal surface

$$d\mathcal{V} = 0$$

$$d\mathcal{V} \equiv -\frac{1}{2}\gamma^{ab}dU_{ab}$$

Fluid branes (I):

Consider a fluid brane which is in stationary motion:



Fluid branes (II):

Stationarity implies: $$\mathbf{k}^a \partial_a = \partial_\tau + \Omega^{(a)} \partial_{\phi_{(a)}}$$

Therefore the action for non-extremal branes: $$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\lambda_0(\mathbf{k})$$

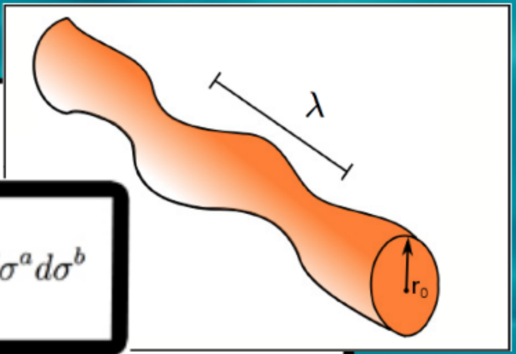
Emparan, Harmark, Obers, Niarchos, 09

and hence the stress-energy tensor:

$$T^{ab} = \lambda_0(\mathbf{k})\gamma^{ab} - \lambda_0'(\mathbf{k})\mathbf{k}u^a u^b \qquad u^a = \frac{\mathbf{k}^a}{\mathbf{k}}$$

identify: $$P = \lambda_0(\mathbf{k}) \ , \quad \epsilon + P = \mathcal{T}s = -\lambda_0'(\mathbf{k})\mathbf{k}$$

Hydrodynamic perturbation:



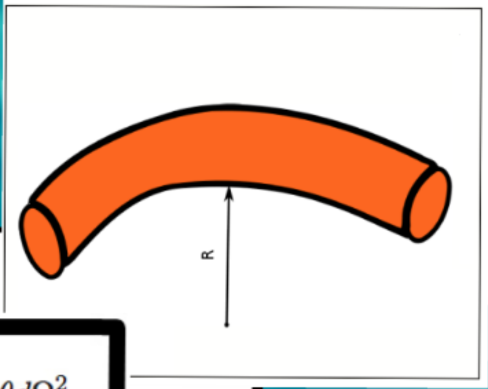
$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n}u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1} + h_{ab}(r) d\sigma^a d\sigma^b$$

$$T^{ab} = \left(\epsilon u^a u^b + P P^{ab} - 2\eta\sigma^{ab} - \zeta\vartheta P^{ab}\right)\delta^{n+2}(x^\rho - X^\rho)$$

Bhatacharyya, Hubeny, Minwalla, Rangamani, 08; Camps, Emparan, Hadad, 10

$$\eta = \frac{\Omega_{(n+1)} r_0^{n+1}}{16\pi G}, \ \zeta = 2\eta\left(\frac{1}{p} + \frac{1}{n+1}\right)$$

Elastic perturbation:



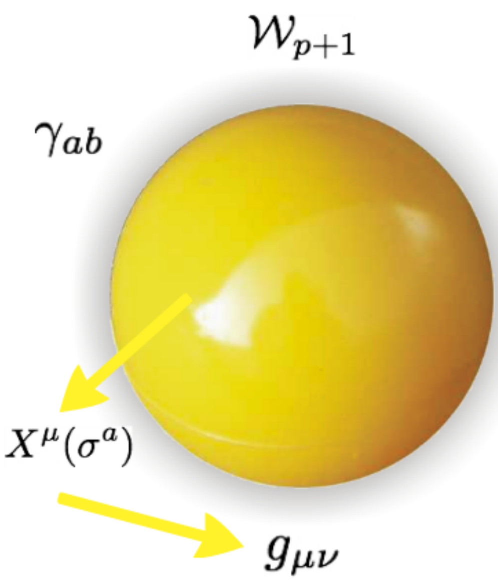
$$ds^2_{(1)} = \left(\eta_{ab} - 2K_{ab}{}^{i} r\cos\theta + \frac{r_0^n}{r^n}u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\theta^2 + r^2\sin^2\theta d\Omega^2_{(n)}$$
$$+ h_{\mu\nu}(r,\theta)dx^\mu dx^\nu + \mathcal{O}\left(r^2/R^2\right) \quad .$$

$$T^{ab} = T^{ab}_{(0)}\delta^{n+2}(x^\rho - X^\rho) - \mathcal{D}^{abi}\partial_i\delta^{n+2}(x^\rho - X^\rho)$$

JA, Camps, Harmark, Obers, 11; Emparan, Camps, 12 ; JA, Gath, Obers, 12, 13

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd}K_{cd}{}^{i}$$

The Elastic Expansion (I):



The generic action should be constructed from:

$$\gamma_{ab}\ ,\ \mathbf{k}^{a}\ ,\ \nabla_{a}\ ,\ K_{ab}{}^{i}\ ,\ \omega_{a}{}^{ij}\ ,\ \mathcal{R}_{abcd}\ ,\ R_{abcd}$$

hence consider:

$$I[X^{\mu}] = \int_{\mathcal{W}_{p+1}} \mathcal{L}\left(\sqrt{-\gamma}, \gamma_{ab}, \mathbf{k}^{a}, \nabla_{a}, K_{ab}{}^{i}, \omega_{a}{}^{ij}\right)$$

define the bending moment and spin current:

$$\mathcal{D}^{ab}{}_{i} = \frac{1}{\sqrt{-\gamma}}\frac{\delta\mathcal{L}}{\delta K_{ab}{}^{i}}\quad ,\quad \mathcal{S}^{a}{}_{ij} = \frac{1}{\sqrt{-\gamma}}\frac{\delta\mathcal{L}}{\delta\omega_{a}{}^{ij}}$$

$$K_{ab}{}^{i} = n^{i}{}_{\rho}\nabla_{a}u_{b}{}^{\rho}\quad ,\quad \omega_{a}{}^{ij} = -n^{j}{}_{\rho}\nabla_{a}n^{i\rho}$$

Elastic Expansion (II):

Multipole expansion of the stress-tensor:

$$T^{\mu\nu} = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(T^{\mu\nu}_{(0)}\delta^{n+2}(x^\alpha - X^\alpha) + T^{\mu\nu\rho}\partial_\rho \delta^{n+2}(x^\alpha - X^\alpha)\right)$$

$$T^{\mu\nu\rho} = -\mathcal{D}^{abi}{u_a}^\mu {u_b}^\nu {n_i}^\rho + 2\mathcal{S}^{aij}{u_a}^{(\mu}{n_i}^{\nu)}{n_i}^\rho$$

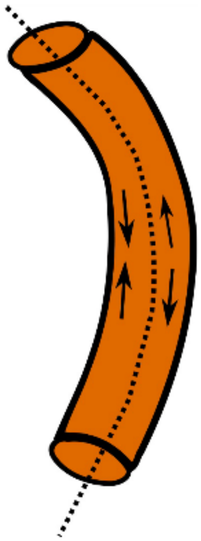
J.Armas, arXiv:1304.7773

the dipole moment is the bending moment:

$$D^{ab\rho} = \int_\Sigma d^{D-1}x\sqrt{-g}T^{\mu\nu}{u^a}_\mu {u^b}_\nu x^\rho = -\int_{\mathcal{B}_p}\sqrt{-\gamma}\mathcal{D}^{ab\rho}$$

the total spin is the integral over the current:

$$J^{\mu\nu}_\perp = \int_\Sigma d^{D-1}x\sqrt{-g}(T^{\mu 0}x^\nu - T^{\nu 0}x^\mu) = \int_{\mathcal{B}_p}\sqrt{-\gamma}\mathcal{S}^{0\mu\nu}$$



The Elastic Expansion (II):

The generic action is:

$$\mathcal{L} = \lambda_0 + v_1 \omega^{ab}\omega_{ab} + v_2 \mathcal{R} + v_3 u^a u^b \mathcal{R}_{ab} + \lambda_1 K^i K_i + \lambda_2 K^{abi} K_{abi} + \lambda_3 u^a u^b K_a{}^{ci} K_{bci} + \sum_{\hat{q}} \varpi_1^{(\hat{q})} u^a \omega_a^{(\hat{q})} + \sum_{\hat{q},\hat{q}'} \varpi_2^{(\hat{q},\hat{q}')} u^a \omega_a^{(\hat{q})} u^b \omega_b^{(\hat{q}')}$$

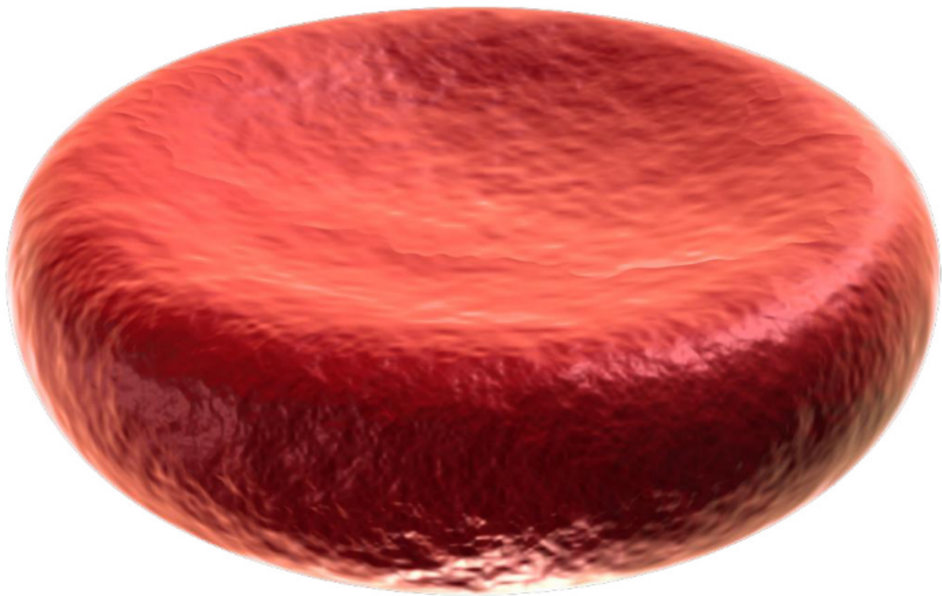
$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 u^{(a}\gamma^{b)(c}u^{d)}\right)$$

. The first 3 transport coefficients were found in hydrodynamics
Minwalla et al, 12; Son et al, 12

. The next two in the context of biophysical membranes
Helfrich, 69; Canham, 70; Polyakov, 86; Kleinert, 86; Capovilla, Guven; 00

. The others are new

Biophysical membranes(I):



red blood cell: erythrocyte

Biophysical membranes(II):

Helfrich-Canham propose in the 70's an additional piece:

$$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\,(\alpha + \lambda_1 K^i K_i)$$

This is enough to explain the biconcave shape of the cell, and many more! (see review by Seifert (1997))

- In the 80's Polyakov and Kleinert make the same proposal for an improved action of QCD.
- In the context of cosmic strings the most general elastic action to second order and codimension > 1 is written down:

$$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\,(\alpha + \lambda_1 K^i K_i + \lambda_2 K^{ab}{}_i K_{ab}{}^i + v_2 \mathcal{R})$$

The Elastic Expansion (I):

The hydrodynamic equations of motion are:

$$\nabla_a T^{ab} = 0$$
$$T^{ab} K_{ab}{}^{i} = 0$$

The elastic equations of motion are:

$$\nabla_a T^{ab} = u_\mu{}^b \nabla_a \nabla_c \mathcal{D}^{ac\mu} + \mathcal{D}^{aci} R^b{}_{aic}$$

$$T^{ab} K_{ab}{}^{i} = n^i{}_\rho \nabla_a \nabla_b \mathcal{D}^{ab\rho} + \mathcal{D}^{abj} R^i{}_{ajb}$$

The spin equations of motion are:

$$\nabla_a T^{ab} = \mathcal{S}^a{}_{ji} \Omega_a{}^{bij}$$

$$T^{ab} K_{ab}{}^{i} = 2n^i{}_\rho \nabla_b \left(\mathcal{S}_a{}^{j\rho} K^{ab}{}_j \right) + \mathcal{S}^{akj} R^i{}_{akj}$$

1954

$\sigma_{\alpha\beta}$; hence $\partial\Psi_2/\partial u_{\alpha\beta} = h\sigma_{\alpha\beta}$. Substituting also for $u_{\alpha\beta}$ the expression (14.1), we obtain

$$\delta\int\Psi_2\,\mathrm{d}f = h\int\sigma_{\alpha\beta}\delta u_{\alpha\beta}\,\mathrm{d}f$$

$$= \tfrac{1}{2}h\int\sigma_{\alpha\beta}\left\{\frac{\partial\delta u_\alpha}{\partial x_\beta}+\frac{\partial\delta u_\beta}{\partial x_\alpha}+\frac{\partial\zeta}{\partial x_\alpha}\frac{\partial\delta\zeta}{\partial x_\beta}+\frac{\partial\delta\zeta}{\partial x_\alpha}\frac{\partial\zeta}{\partial x_\beta}\right\}\mathrm{d}f,$$

or, by the symmetry of $\sigma_{\alpha\beta}$,

$$\delta\int\Psi_2\,\mathrm{d}f = h\int\sigma_{\alpha\beta}\left\{\frac{\partial\delta u_\alpha}{\partial x_\beta}+\frac{\partial\delta\zeta}{\partial x_\beta}\frac{\partial\zeta}{\partial x_\alpha}\right\}\mathrm{d}f.$$

Integrating by parts, we obtain

$$\delta\int\Psi_2\,\mathrm{d}f = -h\int\left\{\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta}\delta u_\alpha+\frac{\partial}{\partial x_\beta}\left(\sigma_{\alpha\beta}\frac{\partial\zeta}{\partial x_\alpha}\right)\delta\zeta\right\}\mathrm{d}f.$$

The contour integrals along the circumference of the plate are again omitted.

Collecting the above results, we have

$$\delta F_{\mathrm{pl}}+\delta U = \int\left[\left\{\frac{Eh^3}{12(1-\sigma^2)}\triangle^2\zeta-h\frac{\partial}{\partial x_\beta}\left(\sigma_{\alpha\beta}\frac{\partial\zeta}{\partial x_\alpha}\right)-P\right\}\delta\zeta-h\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta}\delta u_\alpha\right]\mathrm{d}f = 0.$$

In order that this relation should be satisfied identically, the coefficients of $\delta\zeta$ and δu_α must each be zero. Thus we obtain the equations

$$\frac{Eh^3}{12(1-\sigma^2)}\triangle^2\zeta-h\frac{\partial}{\partial x_\beta}\left(\sigma_{\alpha\beta}\frac{\partial\zeta}{\partial x_\alpha}\right) = P, \tag{14.4}$$

$$\partial\sigma_{\alpha\beta}/\partial x_\beta = 0. \tag{14.5}$$

The unknown functions here are the two components u_x, u_y of the vector $\mathbf{u}$ and the transverse displacement ζ. The solution of the equations gives both the form of the bent plate (i.e. the function $\zeta(x, y)$) and the extension resulting from the bending. Equations (14.4) and (14.5) can be somewhat simplified by introducing the function χ related to $\sigma_{\alpha\beta}$ by (13.7). Equation (14.4) then becomes

$$\frac{Eh^3}{12(1-\sigma^2)}\triangle^2\zeta-h\left(\frac{\partial^2\chi}{\partial y^2}\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\chi}{\partial x^2}\frac{\partial^2\zeta}{\partial y^2}-2\frac{\partial^2\chi}{\partial x\partial y}\frac{\partial^2\zeta}{\partial x\partial y}\right) = P. \tag{14.6}$$

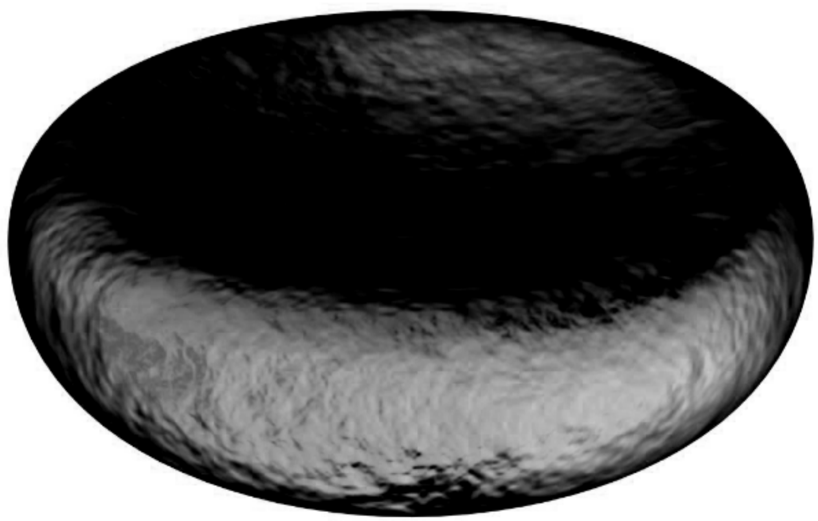
Equations (14.5) are satisfied automatically by the expressions (13.7). Hence another equation is needed; this can be obtained by eliminating u_α from the relations (13.7) and (13.2).

To do this, we proceed as follows. We express $u_{\alpha\beta}$ in terms of $\sigma_{\alpha\beta}$, obtaining from (13.2)

$$u_{xx} = (\sigma_{xx}-\sigma\sigma_{yy})/E, \qquad u_{yy} = (\sigma_{yy}-\sigma\sigma_{xx})/E, \qquad u_{xy} = (1+\sigma)\sigma_{xy}/E.$$

Constructing black holes (I):

The fluid becomes black:



Constructing black holes (II):

The dipole moment takes the form:

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd} K_{cd}{}^{i}$$

The Young modulus is:

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 u^{(a}\gamma^{b)(c}u^{d)}\right)$$

J.Armas, arXiv:1304.7773, J.Armas, arXiv: 1312.0587

$$\lambda_1(\mathbf{k},T) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}\mathbf{k}^{n+2}\left(1-\frac{\Phi_H^2}{\mathbf{k}^2}\right)^{\frac{n}{2}}\left(\frac{3n+4}{2n^2(n+2)} - \bar{k}\right)$$

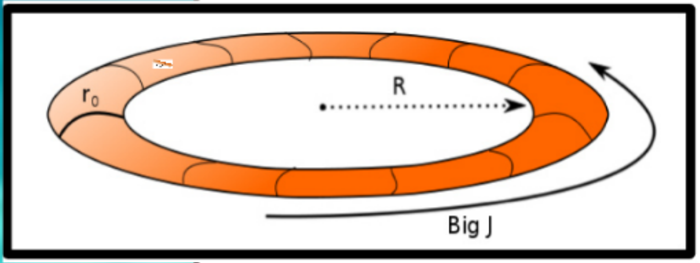
$$\lambda_2(\mathbf{k},T) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}\mathbf{k}^{n+2}\left(1-\frac{\Phi_H^2}{\mathbf{k}^2}\right)^{\frac{n}{2}+1}\frac{1}{2(n+2)}~,$$

$$\lambda_3(\mathbf{k},T) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}\mathbf{k}^{n}\left(1-\frac{\Phi_H^2}{\mathbf{k}^2}\right)^{\frac{n}{2}}~.$$

Constructing black holes (III):

A ring embedded in flat space:

$$ds^2 = -d\tau^2 + R^2 d\phi^2 \quad , \quad \mathbf{k}^a \partial_a = \partial_\tau + \Omega \partial_\phi$$



The action is:

$$\mathcal{F}[R] = -2\pi R \left(P + \tilde{\lambda}_1 K^i K_i \right)$$

J.Armas, arXiv:1304.7773

The solution is:

$$\Omega = \Omega_{(0)} + \Omega_{(2)}$$

$$\Omega_{(0)} = \frac{1}{R} \frac{1}{\sqrt{n+1}}$$

$$\Omega_{(2)} = \frac{(n-4)\sqrt{n+1}}{2n^2(n+2)R} \xi(n) \frac{r_0^2}{R^2}$$

All other charges can be predicted!

Constructing black holes (IV):

Phase diagram for black rings in D>=7:

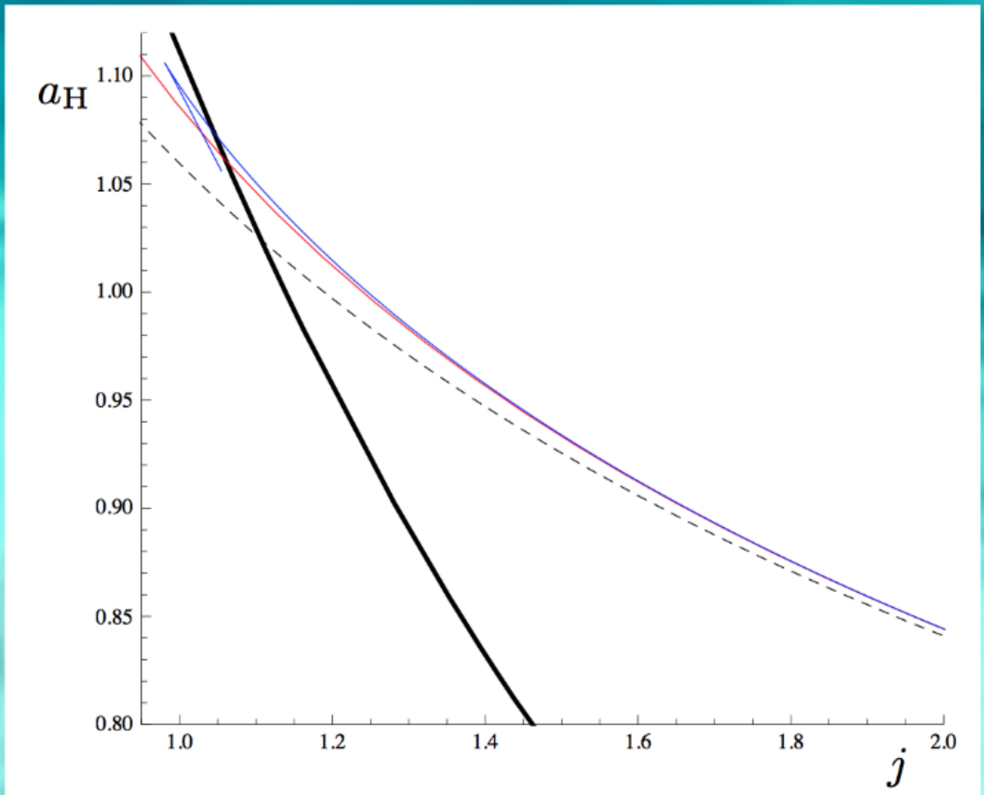
$$a_{\rm H}(j) = \frac{2^{\frac{n-2}{n(n+1)}}}{j^{\frac{1}{n}}}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{3n+4}{n}}n^3(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

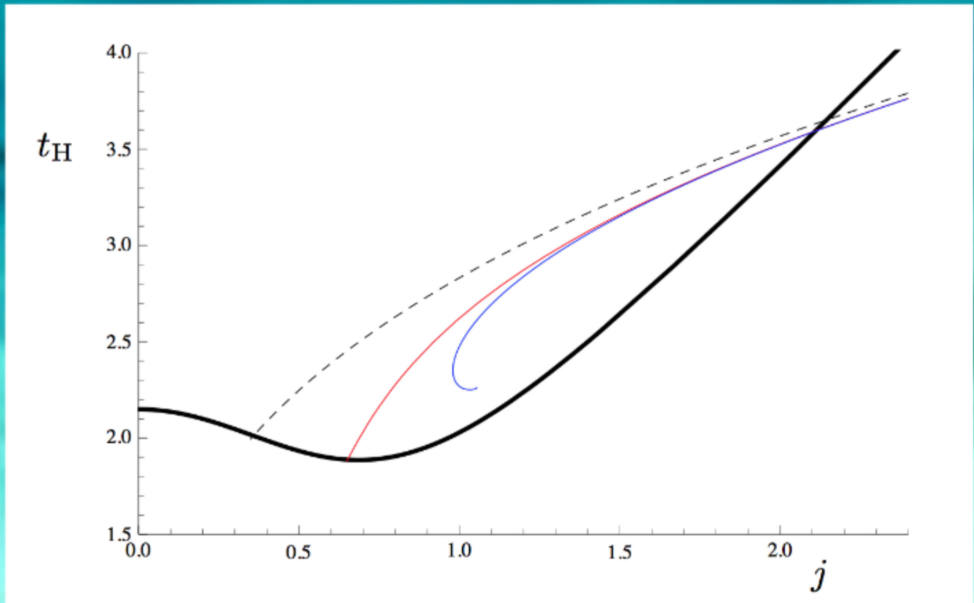
$$\omega_{\rm H}(j) = \frac{1}{2j}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{2(n+2)}{n}}n^2(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right) ,$$

$$t_{\rm H}(j) = \frac{nj^{\frac{1}{n}}}{2^{\frac{n-2}{n(n+1)}}}\left(1 - \frac{3(n+1)(3n+4)}{2^{\frac{3n+4}{n}}n^3(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

JA & T. Harmark, arXiv:1402.6330

Constructing black holes (V):





JA & T. Harmark, arXiv:1402.6330

Numerics by: O. Dias, J. Santos & B.Way, arXiv:1402.6345
Leading order by: R.Emparan, T.Harmark, N.Obers, V.Niarchos, M.Rodriguez, 07

Constructing black holes (VI):

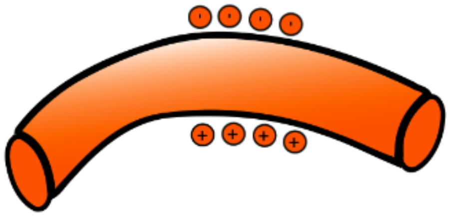
The same can be done for charged branes:

$$J^{\mu_1\cdots\mu_{q+1}}(x^\alpha) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}\left[J_{(0)}^{\mu_1\cdots\mu_{q+1}}\frac{\delta^D(x^\alpha - X^\alpha)}{\sqrt{-g}} - \nabla_\rho\left(J_{(1)}^{\mu_1\cdots\mu_{q+1}\rho}\frac{\delta^D(x^\alpha - X^\alpha)}{\sqrt{-g}}\right) + \ldots\right]$$

JA, Gath, Obers, arXiv:1209.5197, arXiv:1307.504

Decompose the dipole correction as:

$$J_{(1)}^{\mu\nu} = m^{\mu\nu} + u_a^\mu p^{a\nu} + J_{(1)}^{\mu a}u_a^\nu$$



Split the gauge field as:

$$A_\mu = A_\mu^{(M)} + A_\mu^{(D)} + \mathcal{O}\left(r^{-n-2}\right) \rightarrow \nabla_\perp^2 A_\nu^{(D)} = 16\pi G p_\nu{}^{r_\perp}\partial_{r_\perp}\delta^{(n+2)}(r)$$

Constructing black holes (VII):

The electric dipole moment is of the form:

$$p^{a\rho} = \tilde{\kappa}^{abc} K_{bc}{}^{\rho}$$

for charged dilatonic branes from KK reduction:

$$\tilde{\kappa}_a{}^{bc} = -\xi_2(n) r_0^2 \left(\frac{Q}{n} \delta_a{}^{(b} u^{c)} + \bar{k} J_a^{(0)} \eta^{bc} \right)$$

JA, Gath, Obers, arXiv:1209.5197, arXiv:1307.504

Constructing black holes (VIII):

Phase diagram for charged black rings in D>=7:

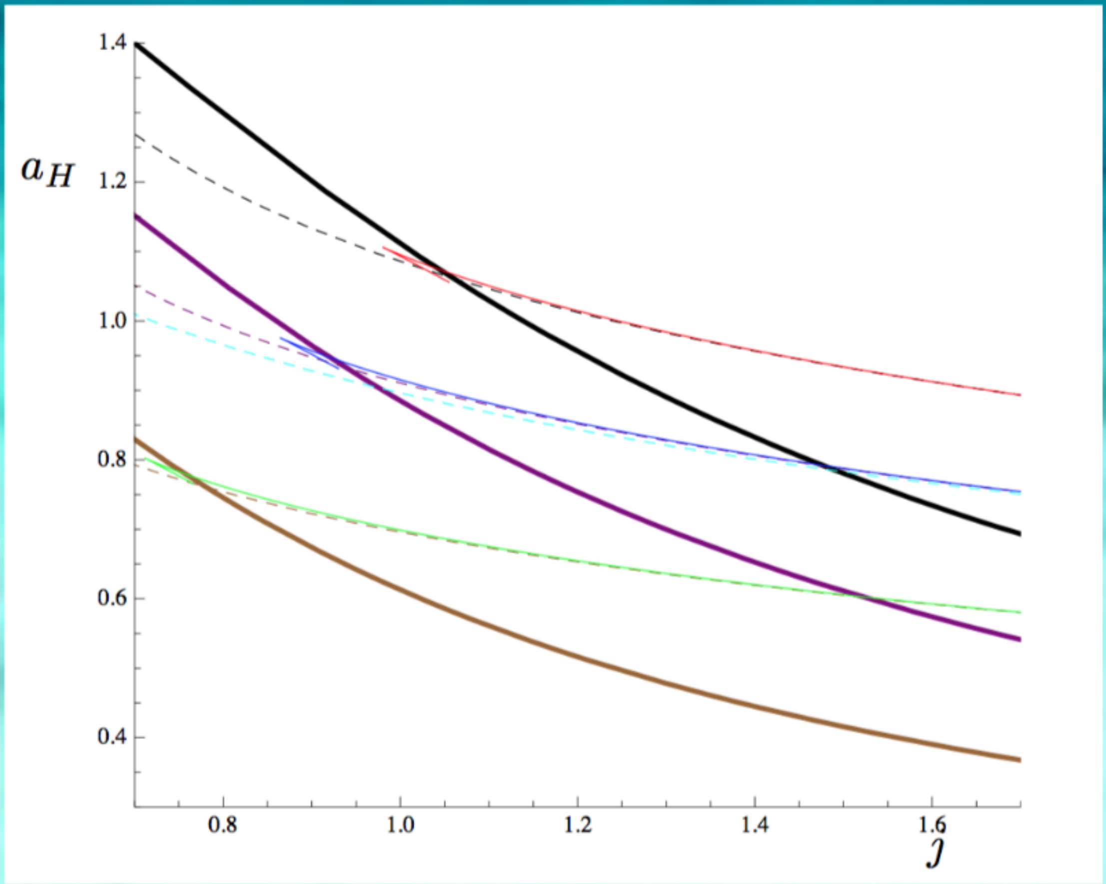
$$a_H(j,\Phi_H) = \frac{2^{\frac{n-2}{n(n+1)}}}{j^{\frac{1}{n}}} \frac{(1-\Phi_H^2)^{\frac{n+3}{2n}}}{(n+2-\Phi_H^2)^{\frac{n+2}{n}}} (n+2)^{\frac{n+2}{n}} \left(1 + \frac{(n+1)(3n+4)}{2^{\frac{3n+4}{n}} n^4} \frac{(1-\Phi_H^2)^{\frac{3}{n}}}{(n+2-\Phi_H^2)^{\frac{3n+4}{n}}} \frac{f_a(n,\Phi_H)\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

$$t_{\rm H} = \frac{2^{\frac{2-n}{n(n+1)}}}{(n+2)^{\frac{2}{n}}} n j^{\frac{1}{n}} \frac{(n+2-\Phi_H^2)^{\frac{2}{n}}}{(1-\Phi_H^2)^{\frac{3}{2n}}} \left(1 + \frac{(n+1)(3n+4)}{2^{\frac{3n+4}{n}} n^4} \frac{(1-\Phi_H^2)^{\frac{3}{n}}}{(n+2-\Phi_H^2)^{\frac{3n+4}{n}}} f_t(n,\Phi_H) \frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

$$\omega_{\rm H} = \frac{(n+2)}{2j} \frac{1-\Phi_H^2}{(n+2-\Phi_H^2)} \left(1 + \frac{(n+1)(3n+4)}{2^{\frac{2(n+2)}{n}} n^3} \frac{(1-\Phi_H^2)^{\frac{6-n}{2n}}}{(n+2-\Phi_H^2)^{\frac{3n+4}{n}}} f_\omega(n,\Phi_H) \frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

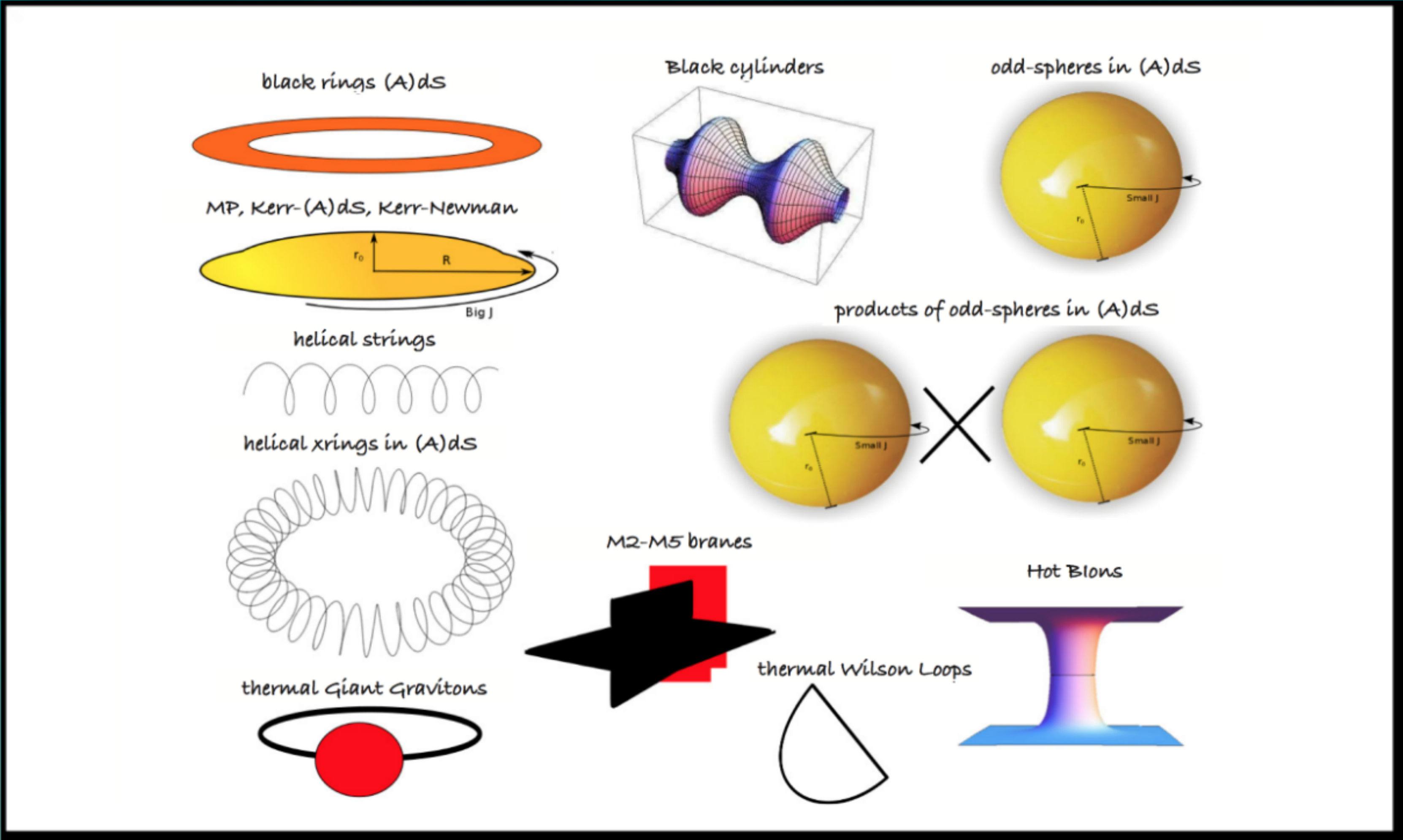
JA & T.Harmark, arXiv: 1406.7813

Constructing black holes (IX):



JA & T.Harmark, arXiv: 1406.7813

Conclusions (I):



Conclusions (II):

A summary of the results:

- Perturbative method for constructing black holes in any theory of gravity with p-branes

Future directions:

- Applications to biophysical membranes, cosmic strings, long strings, entanglement entropy, etc.
  Camps; Dong; Castro, Detournay, Iqbal, Perlmutter; Astaneh, Gibbons, Solodukhin; Myers, Smolkin; Gregory; Capovilla, Guven; Aharony, Komargodski.
- Effective theory for charged fluids coupled to external fields
  J. A. , J.Gath, N. A. Obers, V. Niarchos, A. V. Pedersen (to appear)
- AdS/CFT interpretation of the Young modulus | bending D3-brane
- Anomalous couplings, Chern-Simons terms
- Universality of transport coefficients
- Full dissipative theory and non-relativistic theory.
- Spinning actions and thermodynamics to all orders
  J. Armas, Troels Harmark (to appear)