SCRAMBLING WITH MATRIX BLACK HOLES

based on arXiv:1205.3847 with Riggins, 1306.5200 with Brady, 1409.???? with Pramodh (work in progress)

SCRAMBLING

Scrambling time: amount of time required for any subset of a system — up to 1/2 of the total number of d.o.f. — to attain maximal entanglement entropy

For a self-respecting theory

$$t_{scr} \geq K \frac{S^{1/d}}{T}$$

For black holes, conjecture suggests

$$t_{scr} \sim \frac{\ln S}{T}$$

Sekino, Susskind 0808.2096, Susskind 1101.6048, Page gr-qc/9305007

OUTLINE

- Matrix black holes
- Black hole qubits
- Ripples near the horizon
- High seas

MATRIX BLACK HOLES

DLCQ M-theory $\equiv$ D0 Matrix mechanics

Banks, Fischler, Shenker, Susskind 9610043, Susskind 9704080, Seiberg 9710009

M-theory Schwarzschild black hole $\sim$ D0 gas

Horowitz, Martinec 9710217, Banks, Fischler, Klebanov, Susskind 9709091, 9711005

DLCQ M-theory on PP wave background $\equiv$ BMN Matrix theory

Berenstein, Maldacena, Nastase 0202021

Bubbling black giant gravitons $\sim$ Fuzzy D2 branes

Asano, Ishiki, Okada, Shimasaki1401.5079; Lin, Lunin, Maldacena 0409174; Lin, Maldacena 0509235

BLACK HOLE QUBITS

Lagrangian

$$L = \frac{1}{2} R \,\mathrm{Tr} \Bigg[\frac{1}{R^2} (D_t X_i)^2 + \frac{1}{R^2} (D_t Y_a)^2 + \frac{1}{2} [X_i, X_j]^2 + [X_i, Y_a]^2 + \frac{1}{2} [Y_a, Y_b]^2$$

$$- \left(\frac{\mu}{3R}\right)^2 X_i^2 - \left(\frac{\mu}{6R}\right)^2 Y_a^2 - \frac{i}{3} \frac{\mu}{R} \epsilon_{ijk} [X_i, X_j] X_k$$

$$+ \frac{1}{R} \Psi D_t \Psi + \Psi \gamma_i [X_i, \Psi] + \Psi \gamma_a [Y_a, \Psi] - \frac{i}{4} \frac{\mu}{R} \Psi \gamma_{123} \Psi \Bigg]$$

Spherical configurations + deformations

$$X_i = \nu \tau_i + x_i \quad , \quad Y_a = y_a$$

$$r = \sqrt{\frac{\mathrm{Tr} X_i^2}{N}} = \nu \frac{\sqrt{N^2 - 1}}{\sqrt{3}} \sim \nu N \quad \text{for large } N \gg 1 \,.$$

BPS case in BMN

$$\nu = \frac{\mu}{6\,R}$$

Dasgupta, Sheikh-Jabbari, Van Raamsdonk 0205185

BLACK HOLE QUBITS

Spherical harmonic matrix decomposition

$$\Psi_\alpha = \psi^j_{m\,\alpha} Y^j_m \quad , \quad x_i = x^j_{m\,i} Y^j_m \ ,$$

$$j_{max} = N - 1$$

Algebra

$$\left[Y^j_m, Y^{j'}_{m'} \right] = f_{jm,j'm',j''m''} (-1)^{m''} Y^{j''}_{-m''}$$

$$f_{jm,j'm',j''m''} = \frac{2}{N} (-1)^N N^{3/2} \sqrt{(2j+1)(2j'+1)(2j''+1)} \times \begin{pmatrix} j & j' & j'' \\ m & m' & m'' \end{pmatrix} \times \begin{Bmatrix} j & j' & j'' \\ \frac{N-1}{2} & \frac{N-1}{2} & \frac{N-1}{2} \end{Bmatrix}$$

Hoppe, PhD Thesis 1982, Mohazab 2004

Hermiticity conditions

$$(x^j_{m\,i})^\dagger = (-1)^m x^j_{-m\,i} \ . \qquad (\psi^j_{m\,\alpha})^\dagger = (-1)^m \psi^j_{-m\,\alpha} \ .$$

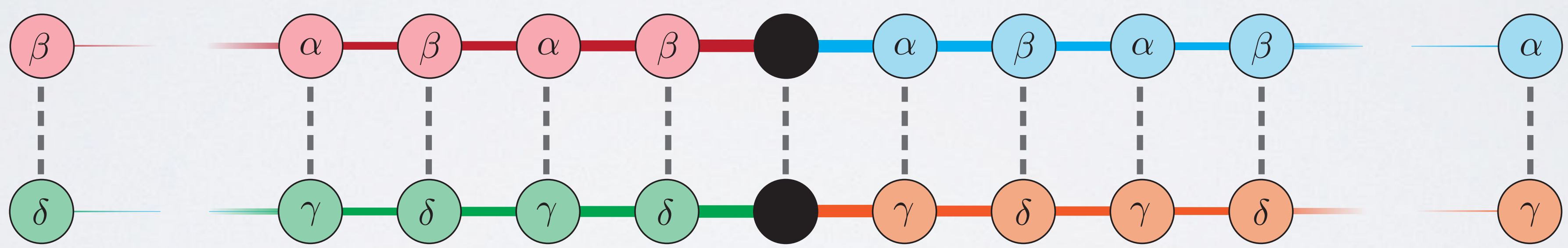
RIPPLES

Decoupled gas regime

$$L = L_{BB} + \boxed{L_{FF}} + L_{BBBB} + L_{FFB}$$

$$\left(\frac{R}{\mu N}\right)^{3} \qquad \left(\frac{R}{\mu N}\right)^{3/2}$$

$8\,N^2$ qubits with nearest neighbor interactions



N plays the role of a UV cutoff

No fast scrambling

Brady, VS 1306.5200

HIGH SEAS

Membrane-Qubits coupling

$$L = L_{BB} + L_{FF} + L_{BBBB} + L_{FFB}$$

bath probe bath coupling

$$H_c = \frac{1}{2} N g_s f_{jm,j'm',j''m''} x^j_{m\,i} \psi^{j''}_{m''} \gamma^i \psi^{j'}_{m'}$$

Heat bath & Markovian thermalization

$$\text{Prob}[x] \sim e^{-\frac{x^2}{2\sigma_x^2}} e^{-\beta \delta E_{x,\psi}}$$

Back-reaction effect

$$\langle x^j_{m\,i}(t) \rangle_{th} \sim N g_s \beta \delta t_0 \sigma_x^2 \langle \left[H_\psi, \mathcal{O}^j_{m\,i} \right] \rangle$$

$$\mathcal{O}^j_{m\,i} \equiv f_{jm,j'm',j''m''} \psi^{j''}_{m''} \gamma^i \psi^{j'}_{m'}$$

HIGH SEAS

Setup

Focus on BMN case only

Compactify to 3 dimensions $(y^a = 0)$

Use bosonic sector (membrane) as a heat bath

Consider large **N** regime

Consider large **r** regime

$$L = L_{BB} + L_{FF} + L_{BBBB} + L_{FFB}$$

$$\left(\frac{R}{\mu N}\right)^3 \qquad \left(\frac{R}{\mu N}\right)^{3/2}$$

HIGH SEAS

Diagonalization

$$x \to \alpha_{jm}, \beta_{jm} \qquad \Psi_\alpha \to {\eta_{jm}}_a^I, {\chi_{jm}}_a^I$$

Lagrangian

$$L_{BB} \begin{cases} \sum_{j=1}^{N-2} \sum_{m=1}^{j} |\dot{\alpha}_{jm}|^2 - \left(\frac{j+1}{3}\right)^2 |\alpha_{jm}|^2 + \sum_{j=0}^{N-2} \frac{1}{2}\dot{\alpha}_{j0}^2 - \frac{1}{2}\left(\frac{j+1}{3}\right)^2 {\alpha_{j0}}^2 \\ + \sum_{j=1}^{N} \sum_{m=1}^{j} \left|\dot{\beta}_{jm}\right|^2 - \left(\frac{j}{3}\right)^2 |\beta_{jm}|^2 + \sum_{j=1}^{N} \frac{1}{2}\dot{\beta}_{j0}^2 - \frac{1}{2}\left(\frac{j}{3}\right)^2 {\beta_{j0}}^2 \end{cases}$$

$$L_{FF} \begin{cases} + \sum_{j=1/2}^{N-3/2} \sum_{m=-j}^{j-1} i\overline{\chi}_{jm}^{aI}\dot{\chi}_{jm}^{aI} - \frac{1}{3}\left(j+\frac{3}{4}\right)\overline{\chi}_{jm}^{aI}\chi_{jm}^{aI} \\ + \sum_{j=1/2}^{N-1/2} \sum_{m=-j-1}^{j} i\,\overline{\eta}_{jm}^{aI}\dot{\eta}_{jm}^{aI} - \frac{1}{3}\left(j+\frac{1}{4}\right)\overline{\eta}_{jm}^{aI}\eta_{jm}^{aI} \end{cases}$$

$$L_{FFB} \quad + \frac{1}{r^{3/2}}[\mathrm{F\ F\ B}]$$

Dasgupta, Sheikh-Jabbari, Van Raamsdonk 0205185

HIGH SEAS

Density matrix & Feynman-Vernon technique

$$\rho(F_f, F'_f, t) = \int dB_f \langle F_f B_f | \hat{\rho}(t) | F'_f B_f \rangle$$

$$= \int d\overline{F}_i dF_i d\overline{F}'_i dF'_i \, e^{-\overline{F}_i F_i} e^{-\overline{F}'_i F'_i} J_{FV}(F_f, F'_f, t; F_i, F'_i, 0) \rho(F_i, F'_i, 0)$$

$$J_{FV}(F_f, F'_f, t; F_i, F'_i, 0) = \int_{F_i}^{\overline{F}_f} \mathcal{D}\overline{F}\mathcal{D}F \int_{F'_i}^{\overline{F}'_f} \mathcal{D}\overline{F}'\mathcal{D}F'$$

$$\times \exp\left[\overline{F}_f F(t) + i\int_0^t \left(i\overline{F}(t')\dot{F}(t') - H_{FF}(\overline{F}(t'), F(t'))\right) dt'\right.$$

$$\left. + \overline{F}'(t) F'_f - i\int_0^t \left(-i\dot{\overline{F}}'(t') F'(t') - H_{FF}(\overline{F}(t'), F(t'))\right) dt'\right] \times \mathcal{G}_{FV}(F, F')$$

$$\mathcal{G}_{FV}(F, F') = \int dB_f dB_i dB'_i \int_{B_i}^{B_f} \mathcal{D}B \int_{B'_i}^{B_f} \mathcal{D}B' \exp\left[iS_{FFB}(B, F) - iS_{FFB}(B', F')\right]$$

$$\times \langle B_i | Z_R^{-1} e^{-\beta \hat{H}_{BB}} | B'_i \rangle$$

Pramodh, VS 1409.????

HIGH SEAS

Density matrix & Feynman-Vernon (cont.)

Initial condition

$$\rho(F_i, F_i', 0) = 1$$

$$\mathcal{G}_{FV}(F, F') \equiv \exp\left[-S_{FV}(F, F')\right]$$

$$S_{FV}(F, F') = r^{-3} \sum_{j=0}^{N-2} (2j+1) K((j+1)^2/9) + r^{-3} \sum_{j=1}^{N} (2j+1) K(j^2/9)$$

HIGH SEAS

Density matrix & Feynman-Vernon (cont.)

$$
\begin{aligned}
K(\lambda) = \int_0^t dt' \int_0^t dt'' & - \frac{\coth \beta\lambda/2}{4\,\lambda} \left(\cos(\lambda t') \cos(\lambda t'')\omega(t')\omega(t'') + \sin(\lambda t') \sin(\lambda t'')\omega(t')\omega(t'')\right) \\
& - \frac{\coth \beta\lambda/2}{4\,\lambda} \left(\cos(\lambda t') \cos(\lambda t'')\omega'(t')\omega'(t'') + \sin(\lambda t') \sin(\lambda t'')\omega'(t')\omega'(t'')\right) \\
& + \frac{\coth \beta\lambda/2}{2\,\lambda} \left(\cos(\lambda t') \cos(\lambda t'')\omega(t')\omega'(t'') + \sin(\lambda t') \sin(\lambda t'')\omega(t')\omega'(t'')\right) \\
& + \frac{i}{2\,\lambda} \left(\sin(\lambda t') \cos(\lambda t'')\omega(t')\omega'(t'') - \cos(\lambda t') \sin(\lambda t'')\omega(t')\omega'(t'')\right)
\end{aligned}
$$

Quartic in fermions

Contains couplings between
all angular momentum modes

Non-local in time, one-loop result

HIGH SEAS

Density matrix & Feynman-Vernon (cont.)

$$\exp\left[-S_{FV}(F,F')\right] \simeq 1 - S_{FV}(F,F') \to 1 - S_{FV}(\delta/\delta J, \delta/\delta J')$$

⬇

$$\rho(\eta_f, \eta'_f, \chi_f, \chi'_f)$$

Reduced density matrix

$$\mathrm{Tr}_q \hat{\rho} = \int d\eta d\eta' e^{-\overline{\eta}\eta} e^{-\overline{\eta}'\eta'} \left(\rho + \eta\rho\overline{\eta}'\right) \quad \Rightarrow \quad \rho' = 1 + r^{-3} R(\chi_f, \chi'_f)$$

Entropy

$$S(t) = -\mathrm{Tr}\hat{\rho}' \ln \hat{\rho}' \simeq -r^{-3} \mathrm{Tr}\, R(\chi_f, \chi'_f)$$

HIGH SEAS

When the dust settles

$$S(t) = r^{-3}\sum_{j=0}^{N-2}(2j+1)\mathcal{S}((j+1)^2/9) + r^{-3}\sum_{j=1}^{N}(2j+1)\mathcal{S}(j^2/9)$$
$$+\ \text{time independent part}$$

$$\mathcal{S}(\lambda) = \frac{32\coth\beta\lambda/2}{\lambda^3}\sin^2\frac{\lambda t}{2}\Big(-2W3^{k_2,n_2}_{j_1,m_1}W4^{k_2,n_2}_{j_1,m_1} - 9(W1^{j_1,m_1}_{j_1,m_1})^2 + W1^{j_2,m_2}_{j_1,m_1}W1^{j_1,m_1}_{j_2,m_2} - 9(W2^{k_1,n_1}_{k_1,n_1})^2 + W2^{k_2,n_2}_{k_1,n_1}W2$$
$$+\frac{4\,\mathrm{csch}\beta\lambda/2}{\lambda}\frac{W3^{k_1,n_1}_{j_1,m_1}W4^{k_1,n_1}_{j_2,m_2}}{(-j_1+\lambda-k_1)(j_1+\lambda+k_1)(j_2-\lambda+k_1)(j_2+\lambda+k_1)}$$
$$\times\left[\left(-\cos(\lambda t)\left(\cos(j_1+k_1)t+\cos(j_2+k_1)t\right)+\cos(j_1-j_2)t+1\right)\right.$$
$$\times\left(\lambda(j_1+j_2+2k_1)\sinh\left(\frac{\beta\lambda}{2}\right)-\cosh\left(\frac{\beta\lambda}{2}\right)\left((j_1+k_1)(j_2+k_1)+\lambda^2\right)\right)$$
$$-\sin(\lambda t)\left(\sin(j_1+k_1)t+\sin(j_2+k_1)t\right)$$
$$\left.\left(\sinh\left(\frac{\beta\lambda}{2}\right)\left((j_1+k_1)(j_2+k_1)+\lambda^2\right)-\lambda(j_1+j_2+2k_1)\cosh\left(\frac{\beta\lambda}{2}\right)\right)\right]$$

CONCLUSIONS

We have quantum informational playground for black hole problems

We can determine whether Matrix black holes are fast scramblers

Pramodh, VS, to appear soon

Matrix theory case may require numerical techniques

Cole, VS, work in progress

We can start modeling black hole evaporation via Matrix qubits

Giddings, Mathur, Avery

We can perhaps address firewall dilemma directly