Effective Theory for the Dynamics of Black Branes in Supergravity and Fluids with q-brane charge

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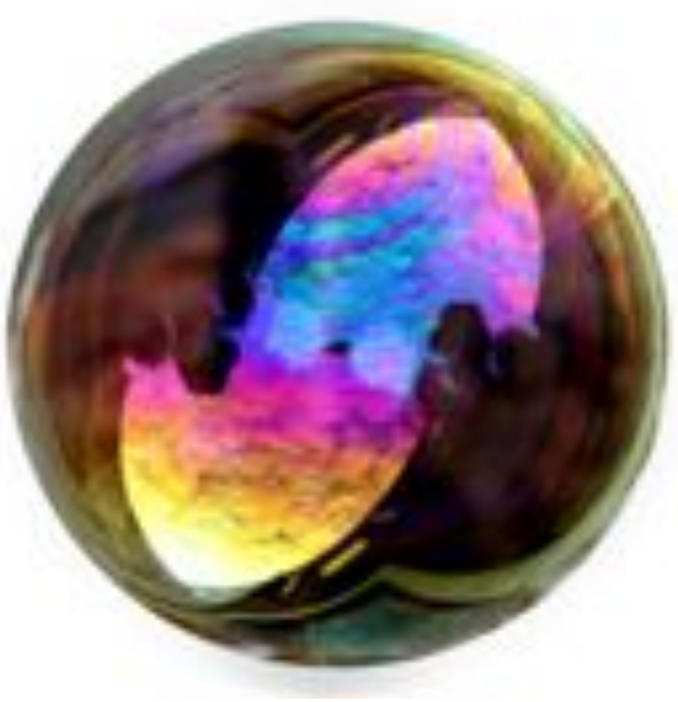


Based on:
arXiv:170X.XXXX by JA, J.Gath, A.V. Perdersen
arXiv:1606.09644 by JA, J.Gath, N.Obers, V. Niarchos, A.V. Perdersen
arXiv:1503.08834 and 1504.01393 by J.A. and M. Blau

Motivations

- Construct new perturbative black hole solutions in any gravity theory with black branes
- Understand the dynamics of black branes and string theory bound states
- Probe spacetimes at finite temperature by means of non-extremal probes

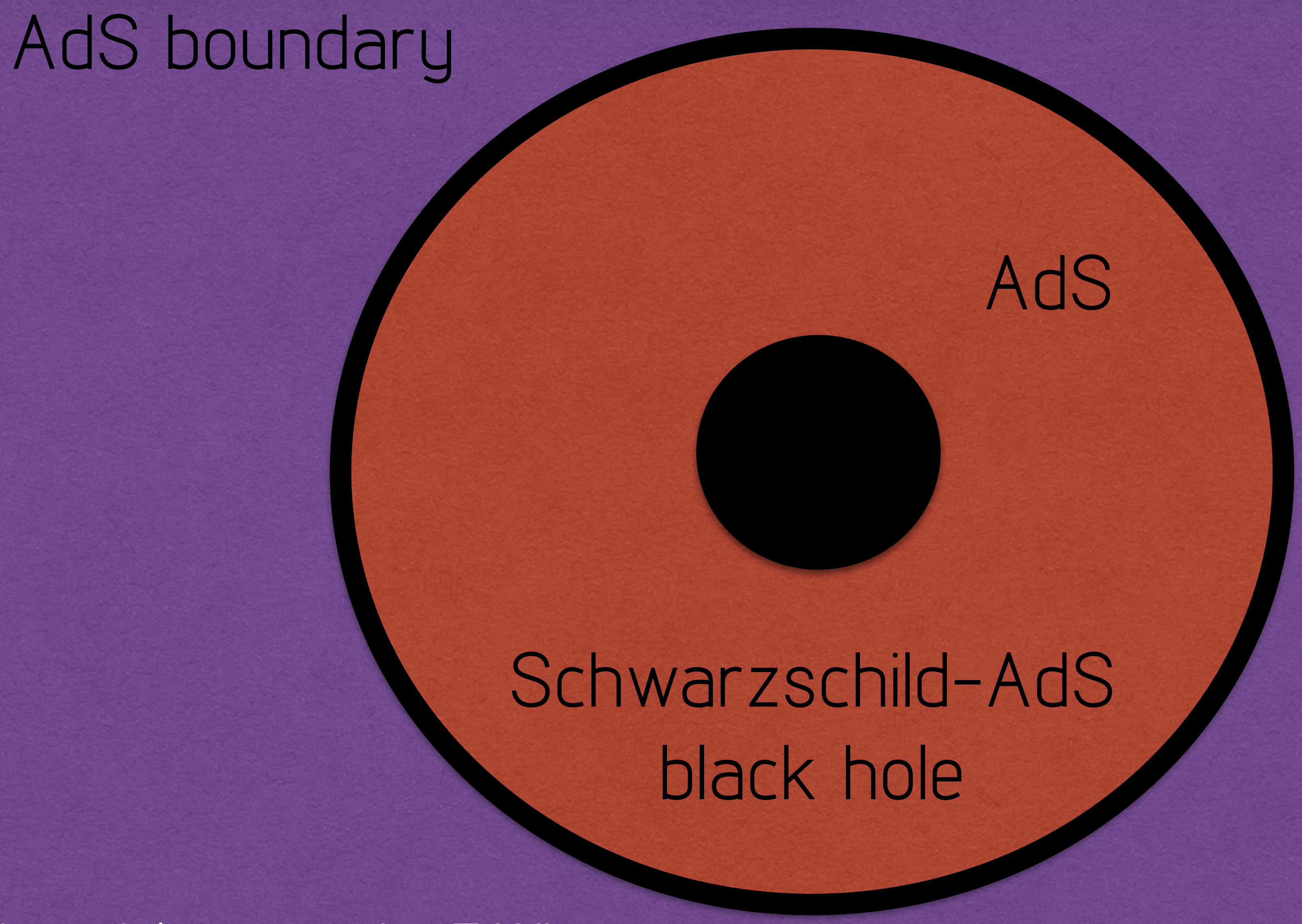
Plan

- Context (with a brief non-linear history)
- Effective theory in pure gravity
- New black hole horizons
- Generating further effective theories
- Effective theory for supergravity
- Fluids with q-brane charge

Context

PART I

Context



arXiv:hep-th/9803131 by E.Witten

Context

$$ds^2 = -\left(1 + \frac{r^2}{L^2} - \frac{r_0^{D-3}}{r^{D-3}}\right) dt^2 + \left(1 + \frac{r^2}{L^2} - \frac{r_0^{D-3}}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega^2_{(D-2)}$$

$$r = \left(\frac{r_0}{L}\right)^{\frac{D-3}{D-1}} \quad , \quad t = \left(\frac{r_0}{L}\right)^{-\frac{D-3}{D-1}} \tau$$

(scaling)

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{L^{D-3}}{\rho^{D-3}}\right) d\tau^2 + \left(\frac{\rho^2}{L^2} - \frac{L^{D-3}}{\rho^{D-3}}\right)^{-1} d\rho^2 + \left(\frac{r_0}{L}\right)^{2\frac{D-3}{(D-1)}} \rho^2 d\Omega^2_{(D-2)}$$

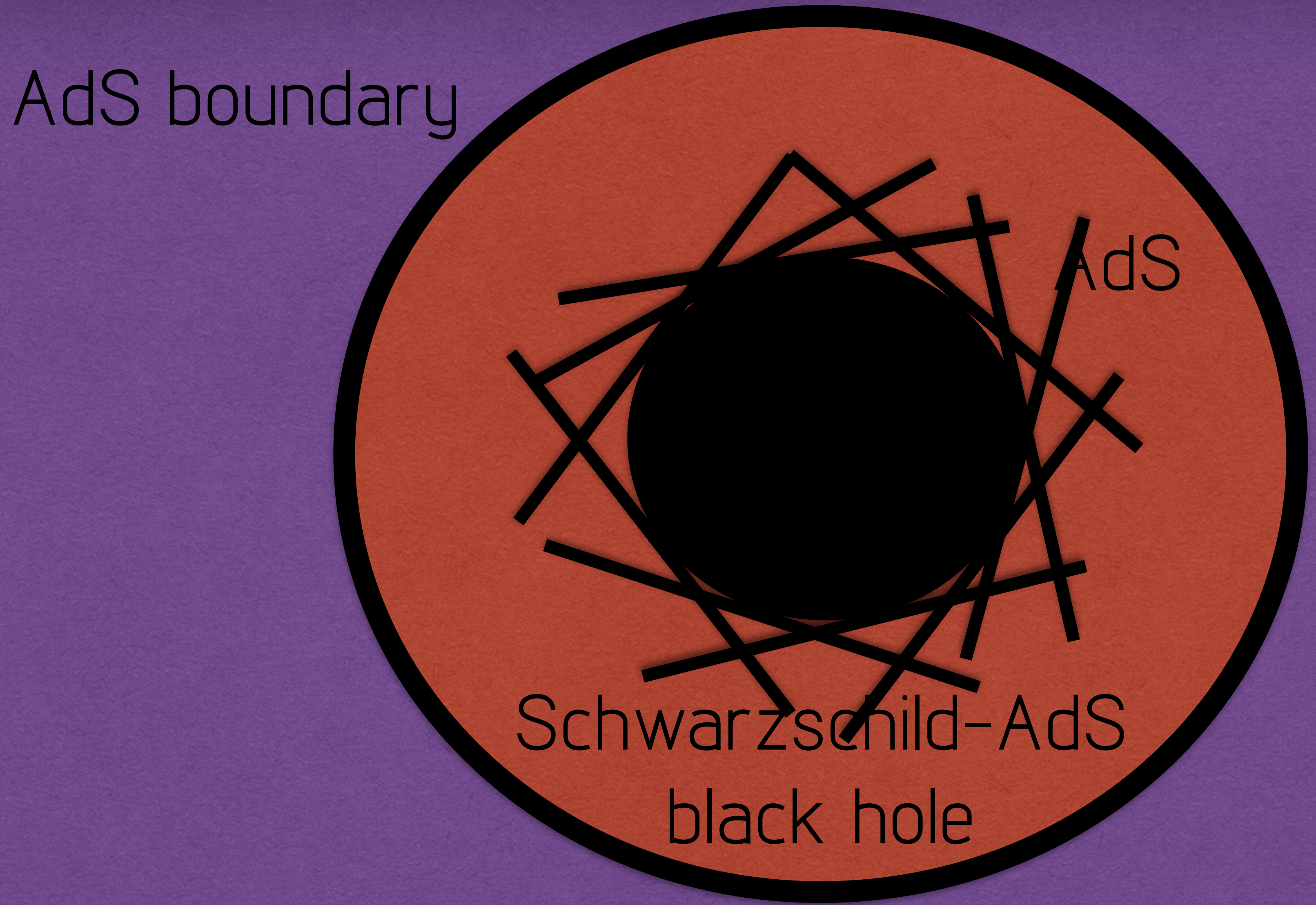
(limit)

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{L^{D-3}}{\rho^{D-3}}\right) d\tau^2 + \left(\frac{\rho^2}{L^2} - \frac{L^{D-3}}{\rho^{D-3}}\right)^{-1} d\rho^2 + \rho^2 \sum_{i=1}^{D-2} dy_i^2$$

(locally)

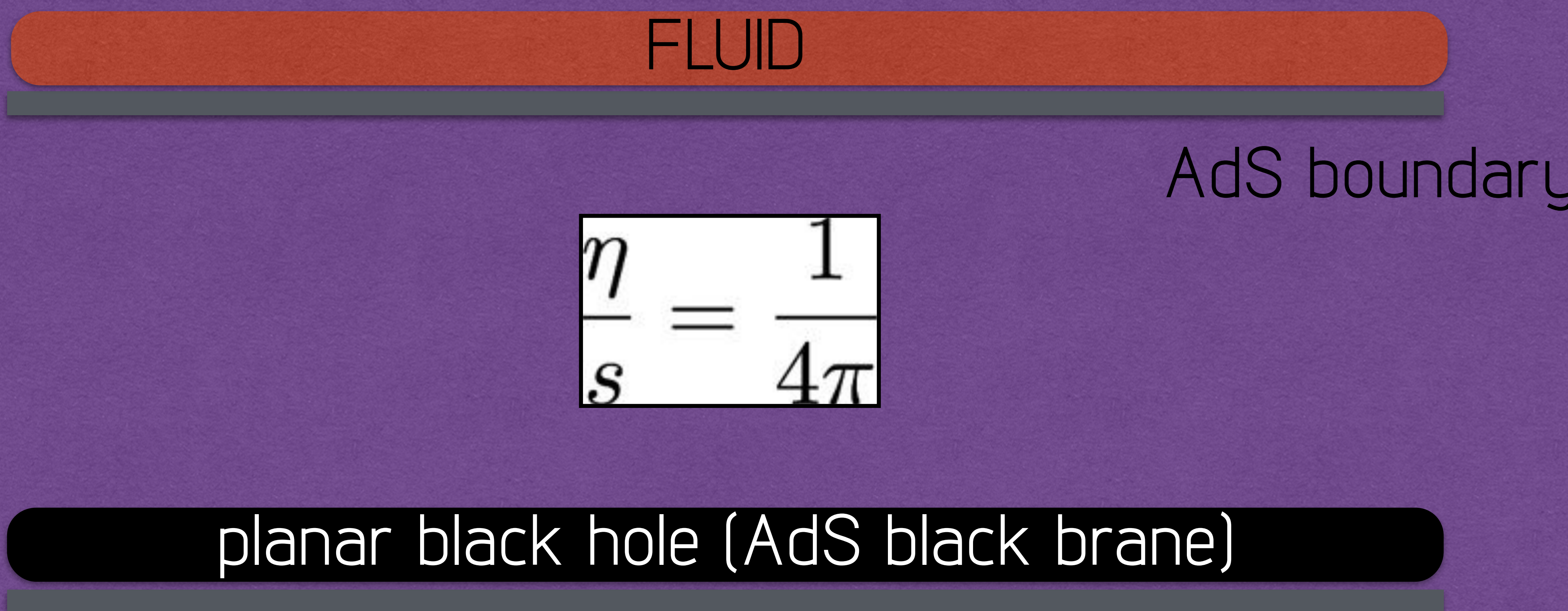
arXiv:hep-th/9803131 by E.Witten

Context



arXiv:hep-th/9803131 by E.Witten

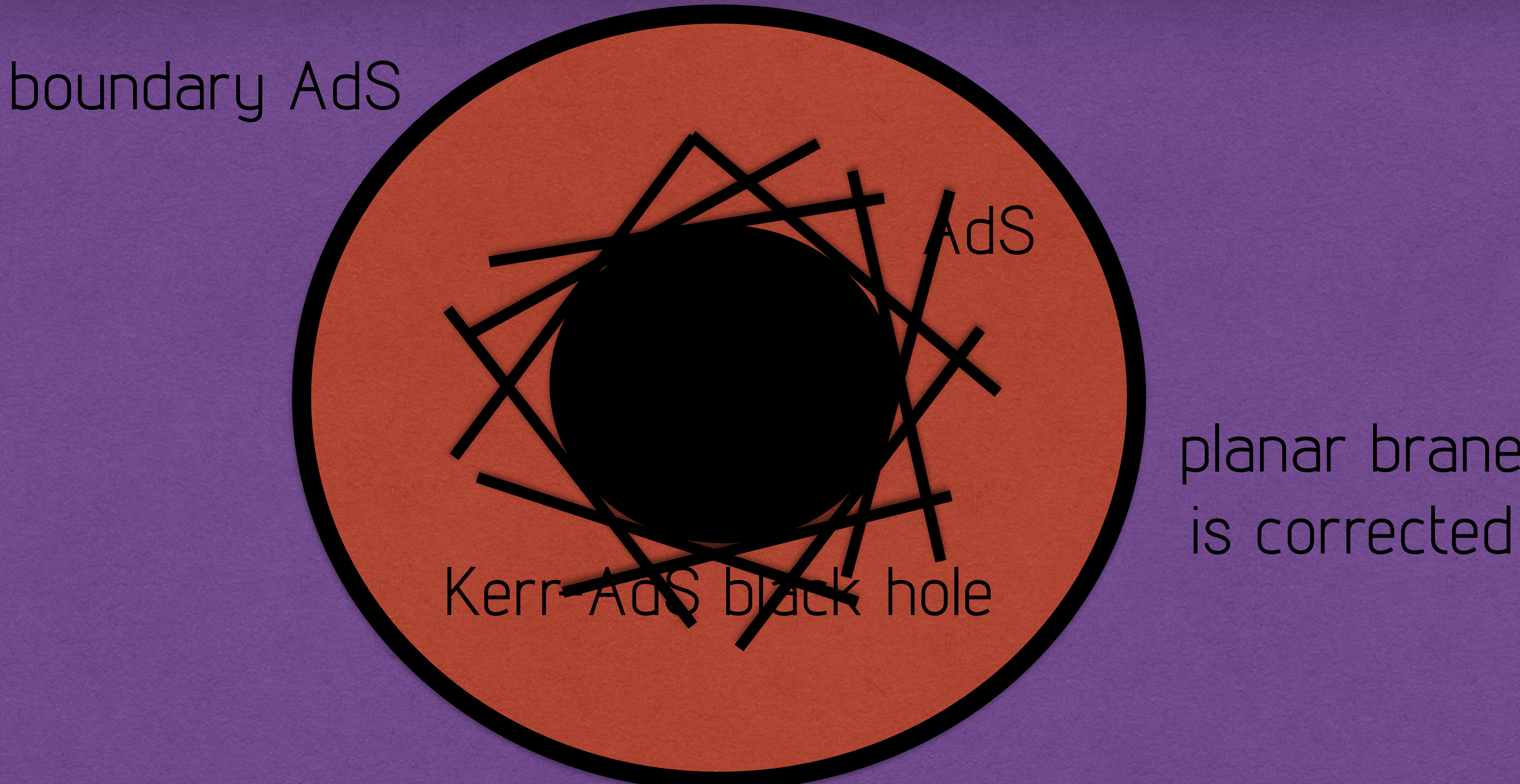
Context



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

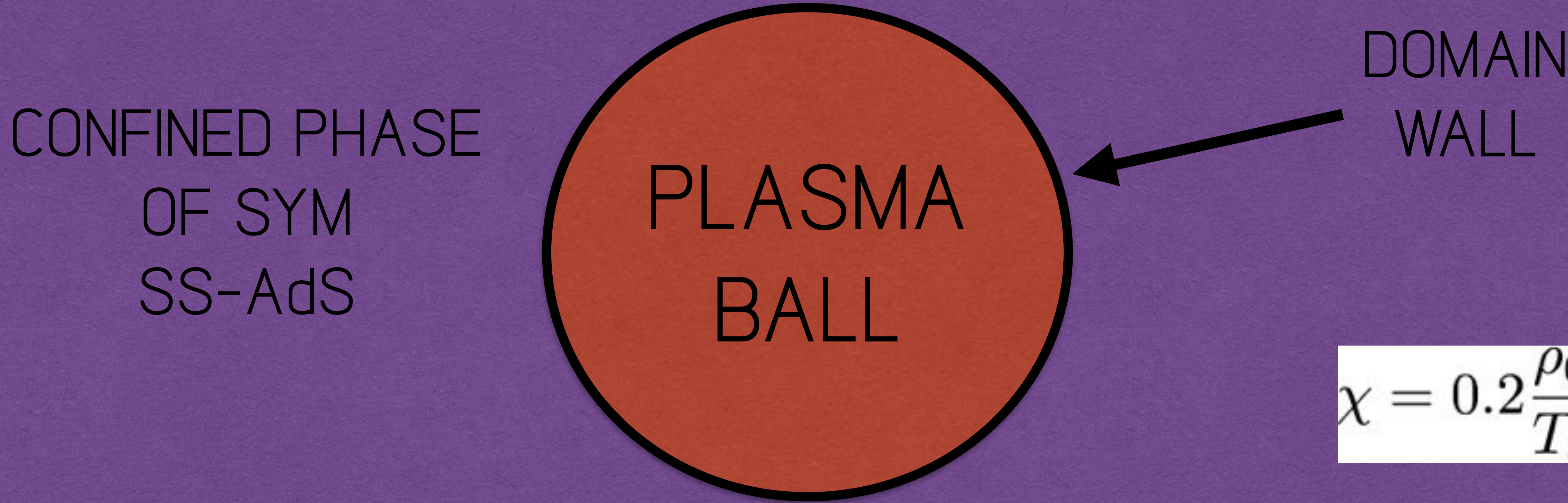
arXiv:hep-th/0104066 by Policastro, Son and Starinets.

Context



arXiv:0712.2456 by S. Bhattacharyya, V.E.Hubeny, S. Minwalla and M. Rangamani
arXiv:0708.1770 by S. Bhattacharyya, S. Lahiri, R. Loganayagam & S. Minwalla

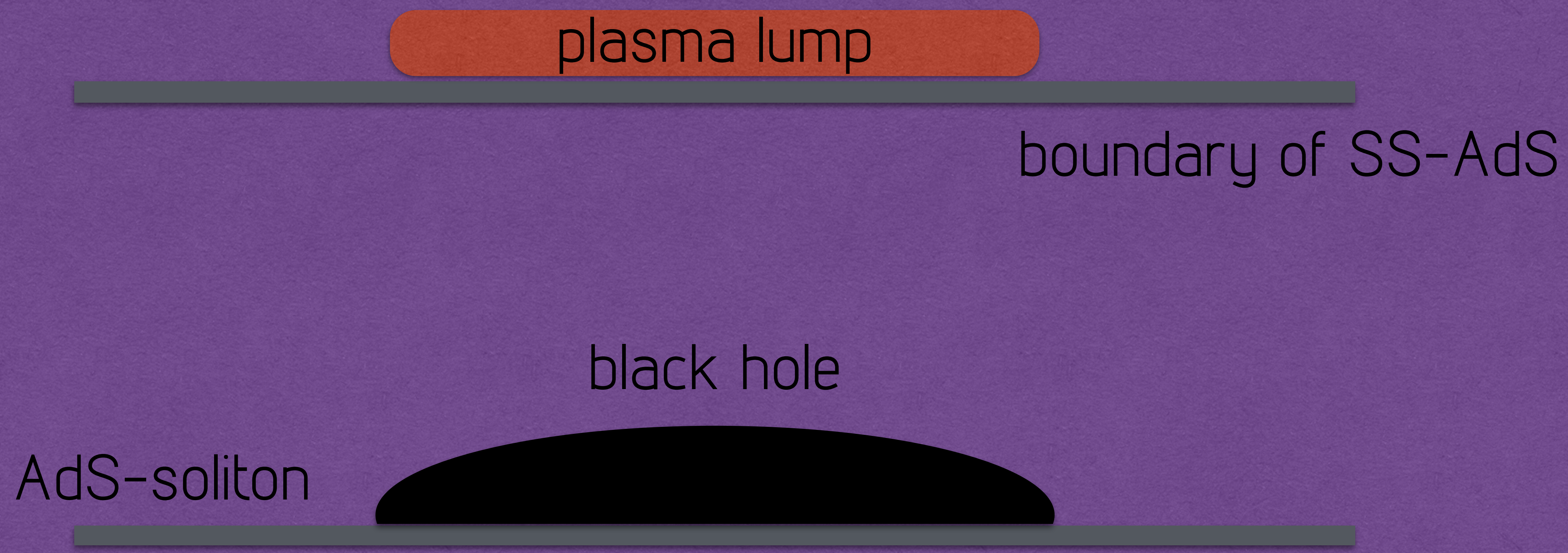
Context



$$\chi = 0.2\frac{\rho_0}{T_c}$$

arXiv:0507219 by O. Aharony, S. Minwalla & T. Wiseman

Context

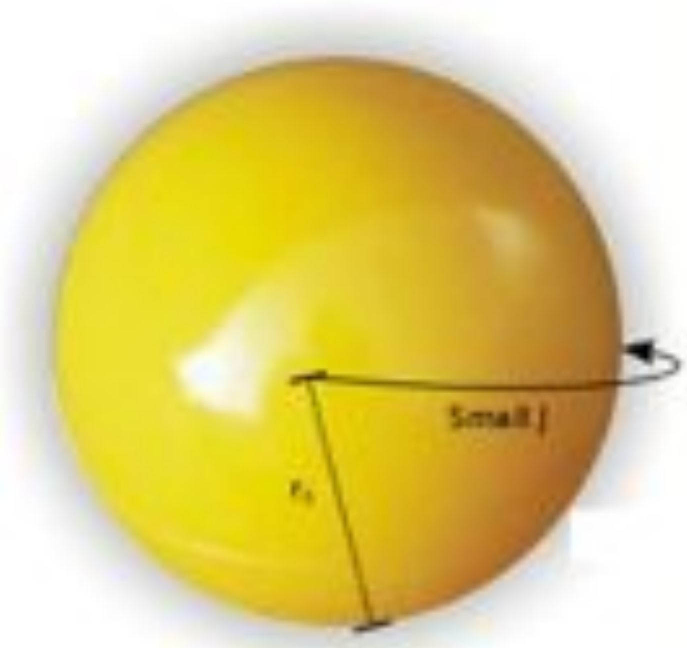


arXiv:0507219 by O. Aharony, S. Minwalla & T. Wiseman

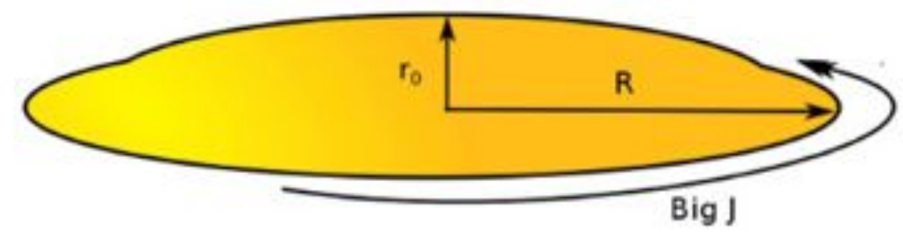
Context

These are fluids with a boundary

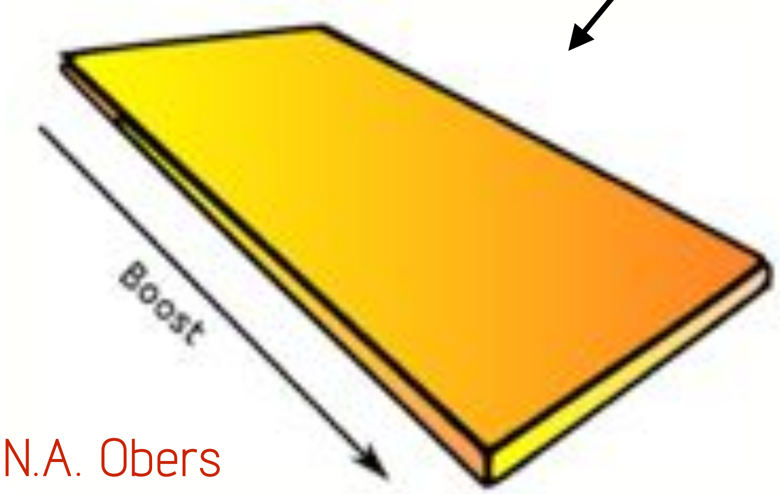
Context



increase J

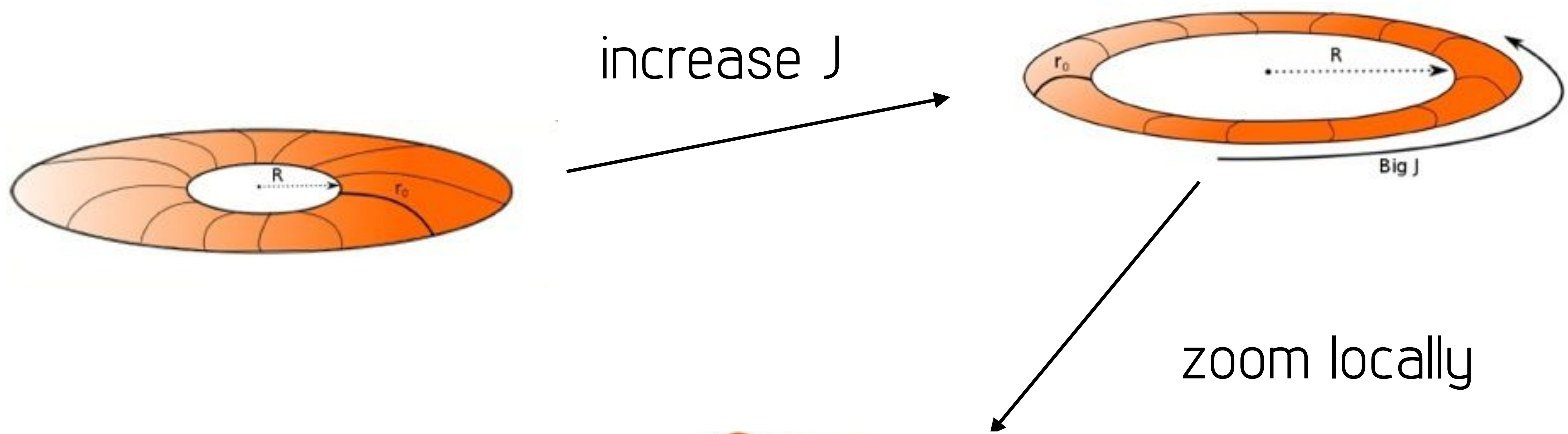


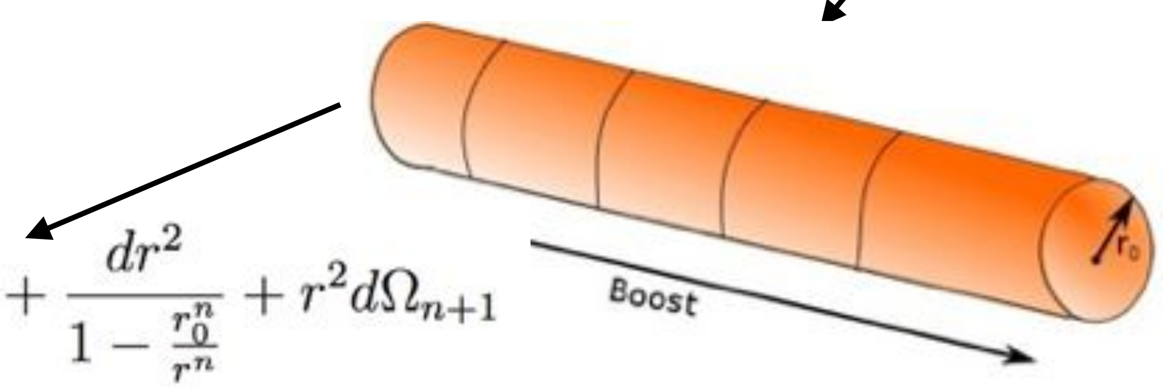
zoom locally



arXiv:0308056 by E. Emparan & R. Myers
arXiv:1110.4835 by J.A. J.Camps, T.Harmark & N.A. Obers

Context





$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

arXiv:0708.2181 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers & M.Rodriguez

Context

DYNAMICS OF
(INTERSECTING)
BRANES

arXiv:1106.4428 by R. Emparan, T. Harmark, V. Niarchos and N.A.Obers

Context

$$K\big|_{\Sigma_B} = \frac{n}{r_0(z)}$$

$$ds^2 = r_0^2(z)\left(-4\tilde{\kappa}^2\tanh^2(\rho/2)dt^2 + \frac{d\rho^2}{n^2}\right) + \left(\gamma_{ab}(z) + \frac{4}{n}r_0^2(z)f_{ab}(z)\ln\cosh(\rho/2)\right)dz^a dz^b$$
$$+\mathcal{R}^2(z)\left(\cosh(\rho/2)\right)^{\frac{4}{n}} d\Omega_{n+1}\,.$$

arXiv:1504.06489 by R. Emparan, T. Shiromizu, R. Suzuki, K. Tanabe & T. Tanaka
arXiv:1504.06613 by S. Bhattacharyya, A. De, S. Minwalla, R. Mohan & A. Saha

The effective theory

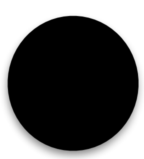
PART II

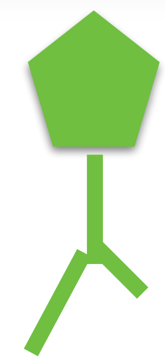
The effective theory

$$G_{\mu\nu} = 0$$

The effective theory

Schwarzschild at infinity





$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T^{tt} = r_0 \delta(r)$$

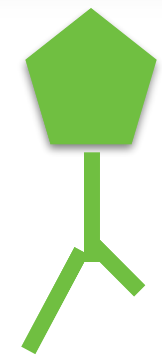
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{(2)}$$

$$f(r) = 1 - \frac{r_0}{r} \longrightarrow f(r)^{-1} = 1 + \frac{r0}{r}$$

Infinity

The effective theory

Black string at infinity



$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T^{tt} = r_0 \delta(r)$$

$$T^{zz} = -r_0 \delta(r)$$

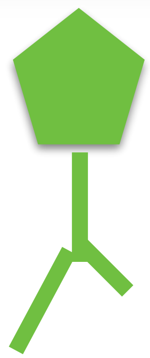
$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{(2)} + dz^2$$

$$f(r) = 1 - \frac{r_0}{r} \longrightarrow f(r)^{-1} = 1 + \frac{r0}{r}$$

Infinity

The effective theory

Boosted black string at infinity



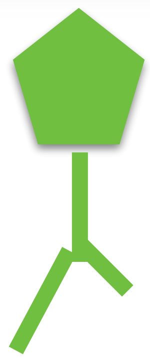
Infinity

$$T^{ab} = (P\eta^{ab} + (\epsilon + P)u^a u^b)\delta(r)$$

$$ds^2 = (\eta_{ab} + (1-f)f u_a u_b)d\sigma^a d\sigma^b + f^{-1}dr^2 + r^2 d\Omega_{(2)}$$

The effective theory

Boosted black string at infinity



Infinity

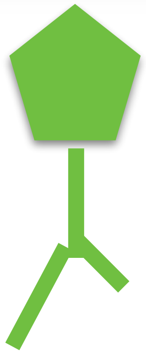
$$T^{ab} = (P\eta^{ab} + (\epsilon + P)u^a u^b)\delta(r)$$
$$-(\eta\Theta P^{ab} + \xi\sigma^{ab})\delta(r)$$

$$h_{\mu\nu} \propto \partial r_0, \partial u_a$$

$$G_{\mu\nu} = 0$$

The effective theory

Boosted black string at infinity

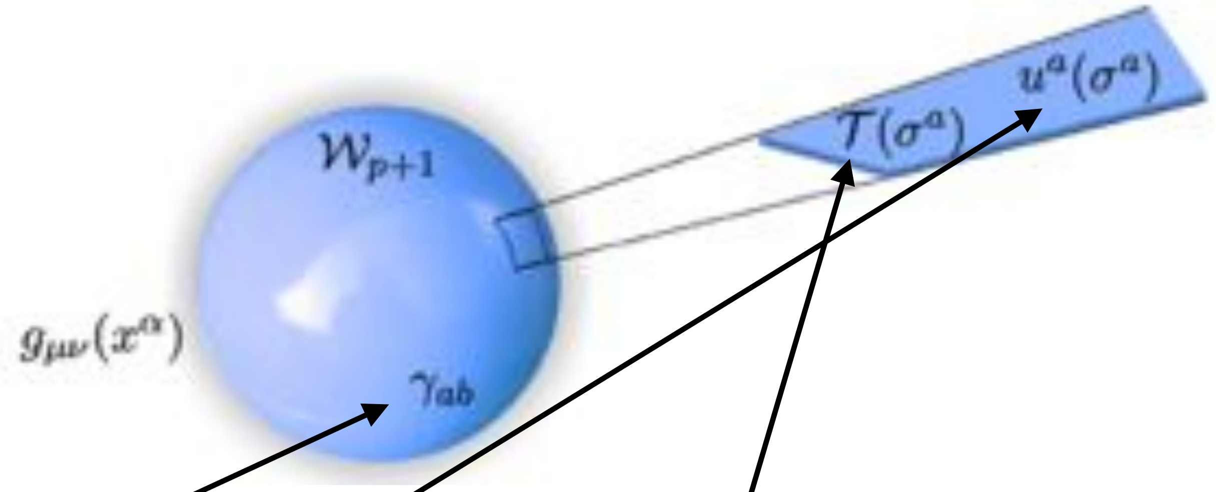


$$h_{\mu\nu} \propto K \ , \ K^{i}_{ab} \ , \ \partial r_0 \ , \ \partial u_a$$

$$T^{ab} = (P\gamma^{ab} + (\epsilon + P)u^a u^b \delta(r) + Y^{abcd} K_{cd}{}^{i} \partial_i \delta(r)$$

The effective theory

Neutral black brane:



$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1} + \ldots$$

The effective theory

Read off the stress tensor from the metric:

$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

$$T^{ab} = P\eta^{ab} + (\epsilon + P) u^a u^b$$

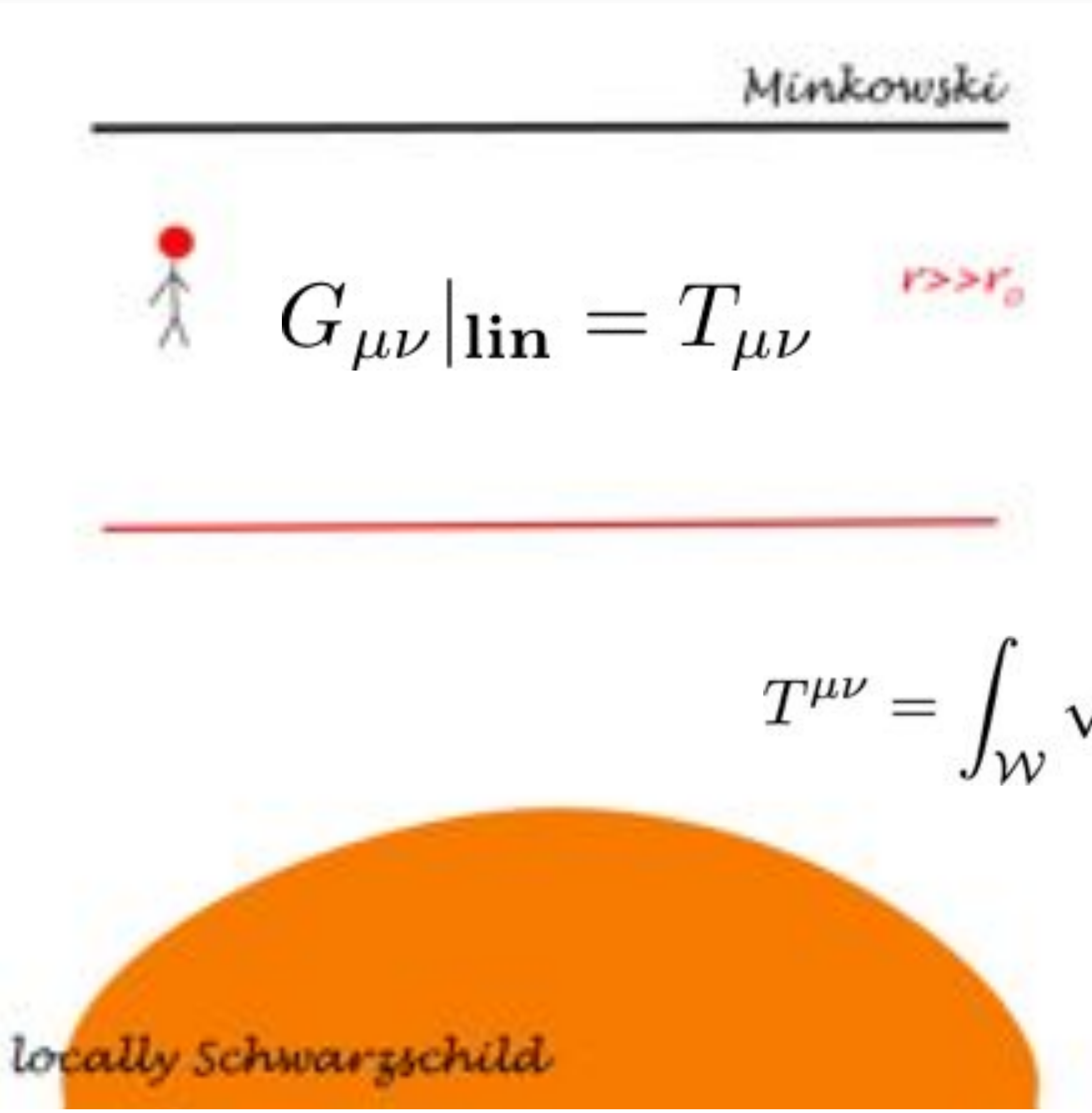
$$T^{\mu\nu} = \int_{\mathcal{W}} \sqrt{-\gamma} T^{ab} u_a^\mu u_b^\nu \delta(x^\alpha - X^\alpha)$$

$$\boxed{\frac{\epsilon}{n+1} = -P = \frac{\Omega_{(n+1)} r_0^n}{16\pi G}}$$

arXiv::0910.1601 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers

The effective theory

The effective theory



$$G_{\mu\nu}|_{\mathbf{lin}} = T_{\mu\nu}$$

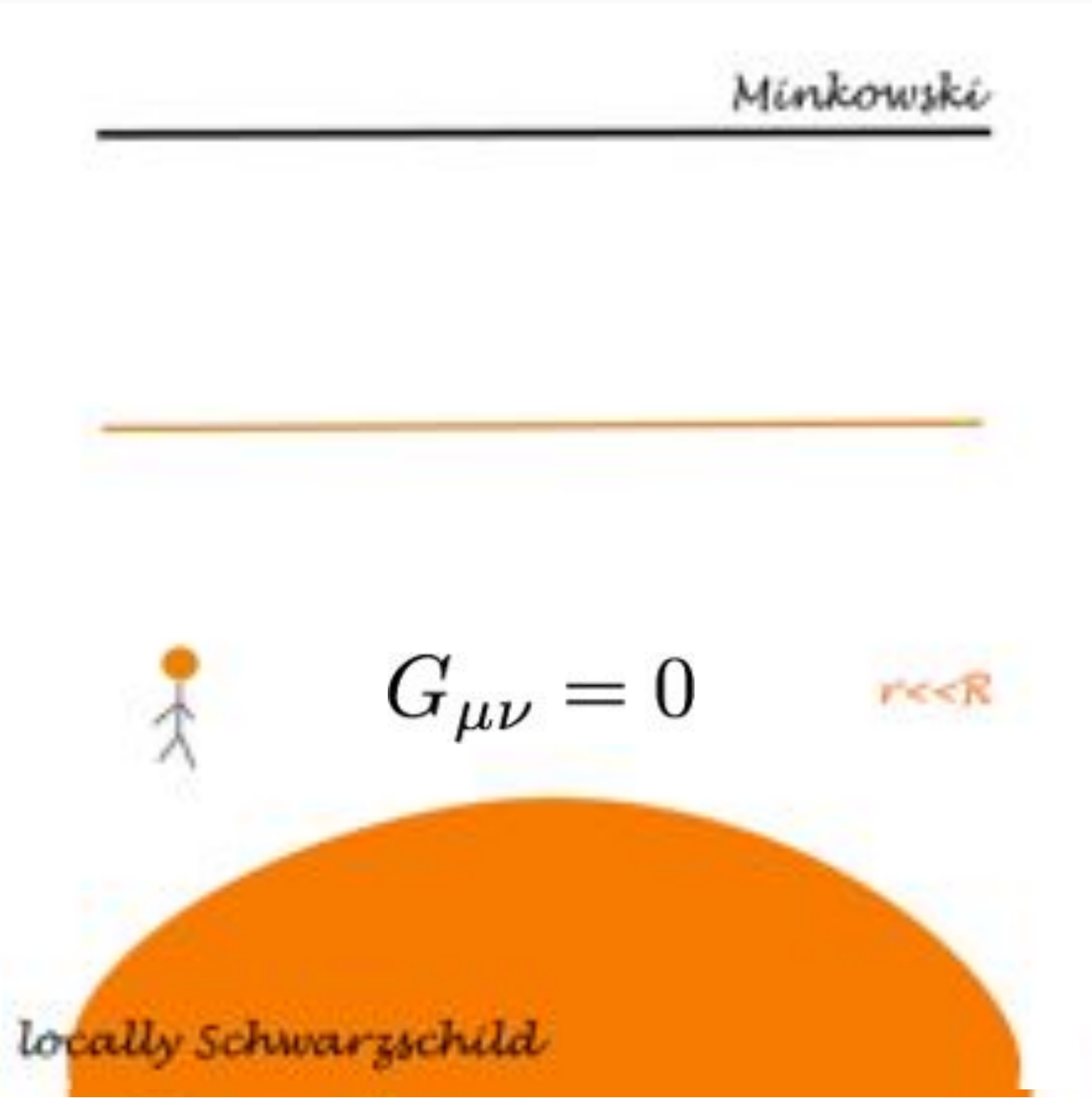
$$T^{\mu\nu} = \int_{\mathcal{W}} \sqrt{-\gamma} T^{ab} u_a^{\mu} u_b^{\nu} \delta(x^{\alpha} - X^{\alpha})$$

The effective theory

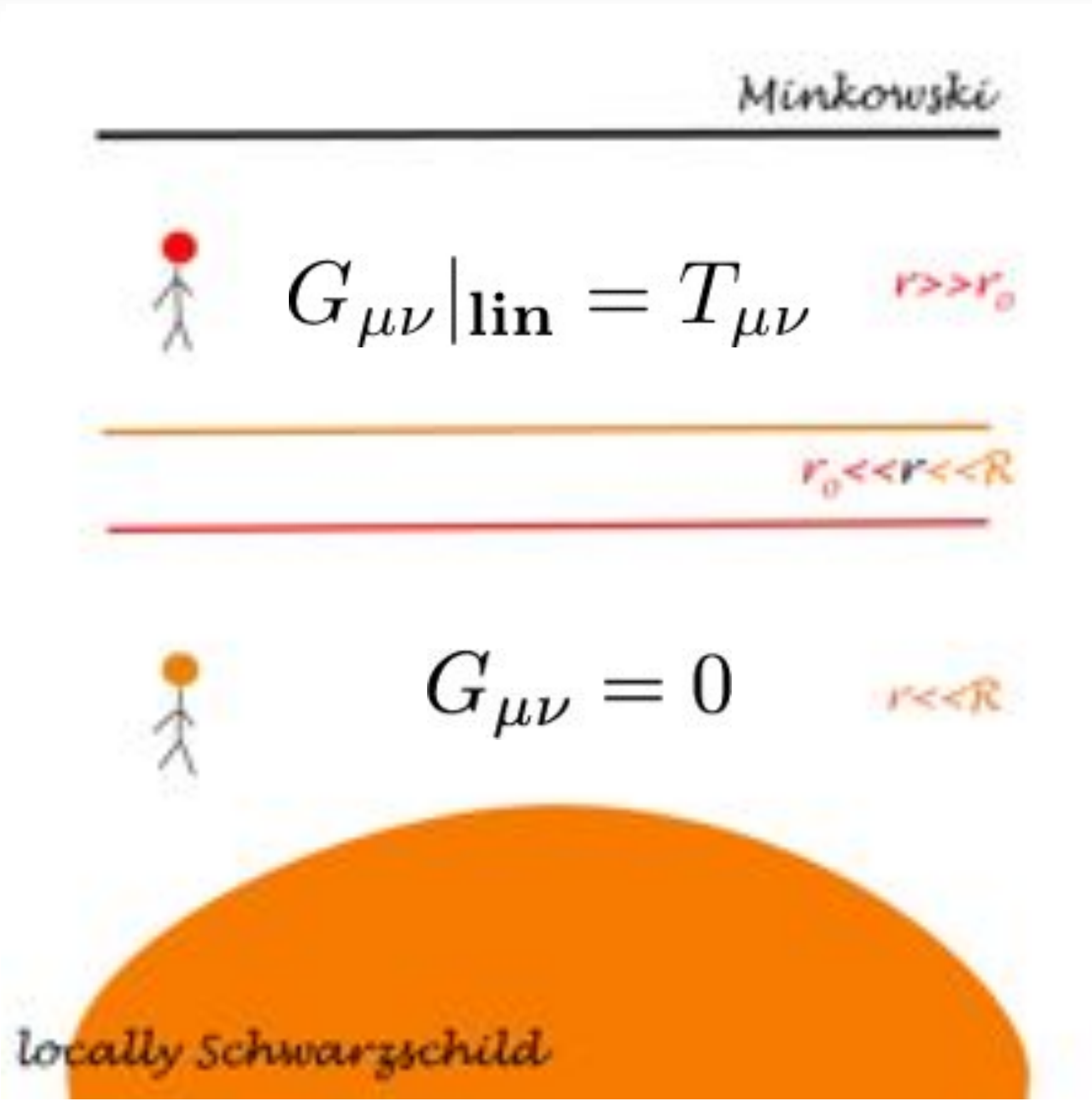
$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

$$\nabla_a T^{ab} = 0 \qquad T^{ab} K_{ab}{}^{i} = 0$$

The effective theory



The effective theory



The effective theory

Perturbing the brane:

$$ds^2_{(1)} = \left(\eta_{ab} - 2K_{ab}{}^{i} r\cos\theta + \frac{r_0^n}{r^n}u_a u_b\right)d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\theta^2 + r^2\sin^2\theta d\Omega^2_{(n)}$$
$$+ h_{\mu\nu}(r,\theta)dx^\mu dx^\nu + \mathcal{O}\left(r^2/R^2\right) \quad .$$

$$T^{\mu\nu} = \int_{\partial\mathcal{W}} \sqrt{-\gamma}\left(T^{ab}u_a^\mu u_b^\nu \delta(x) + \mathcal{D}^{abi}u_a^\mu u_b^\nu \partial_i\delta(x) + u_a^{(\mu}\mathcal{S}^{a\nu)i}\partial_i\delta(x)\right)$$

arXiv:0708.2181 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers & M.Rodriguez
arXiv:1110.4835 by JA, J.Camps, T.Harmark & N.A. Obers
arXiv:1201.3506 by J. Camps & R. Emparan

The effective theory

To find the effective action in equilibrium:

(1) Classification and comparison

(2) Black hole partition function (+ global condition)

$$F = \int \sqrt{-g}R + \int_\partial \sqrt{-h}K$$

The effective theory

<u>Effective action:</u>

arXiv:1203.3544 by Bhattacharya et al.

$$I[X^i] = \int_{\mathcal{W}} \sqrt{-\gamma}\Big(P + \boxed{\upsilon_1 \mathfrak{a}^2 + \upsilon_2 \omega^2 + \upsilon_3 \mathcal{R}}$$

$$+\lambda_1 K^i K_i + \lambda_2 K^{abi} K_{abi} + \lambda_3 u^a u^b K_a{}^{ci} K_{bci}$$

$$+\varpi_1 u^a \omega_a + \varpi_2 u^a \omega_a u^b \omega_b\Big)$$

$$\nabla_a\left(e^b u^\mu{}_b\right) = u^{\mu b}\gamma_{ab}{}^c e_c + n^\nu{}_i K_{ab}{}^i e^a \ , \qquad \omega_a = \epsilon_{ij}\omega_a{}^{ij}$$

$$\nabla_a\left(n^i n^\mu{}_i\right) = -u^{\mu b} K_{ab}{}^i n_i - n^\mu{}_j \omega_a{}^{ij} n_i$$

arXiv:1304.7773 and arXiv:1312.0597 by JA
arXiv:1406.7813 by JA & T.Harmark

The effective theory

From here one can extract the bending moment and the spin current:

$$\mathcal{D}^{ab}{}_{i} = \frac{\partial I}{K_{ab}{}^{i}} = \mathcal{Y}^{abcd} K_{cdi} \qquad \mathcal{S}^{a}{}_{ij} = \frac{\partial I}{\omega_{a}{}^{ij}}$$

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 u^{(a}\gamma^{b)(c}u^{d)}\right)$$

$$\mathcal{S}^{aij} = \varpi_1 u^a \epsilon^{ij} + \varpi_2 u^b \omega_b u^a \epsilon^{ij}$$

arXiv:1304.7773 by JA
arXiv:1312.0597 by JA

The effective theory

The equations of motion are modified to:

$$\nabla_a T^{ab} - u_\mu{}^b \nabla_a \nabla_c \mathcal{D}^{ac\mu} + 2\,\mathcal{S}^a{}_{ij} K_{ac}{}^i K^{bcj} = \mathcal{D}^{aci} R^b{}_{aic} + \mathcal{S}^{aij} R^b{}_{aij} - \omega^{bij} \nabla_a \mathcal{S}^a{}_{ij}$$

$$T^{ab} K_{ab}{}^i = n^i{}_\rho \nabla_a \nabla_b \mathcal{D}^{ab\rho} - 2\, n^i{}_\rho \nabla_b \left(\mathcal{S}_a{}^{\rho j} K^{ab}{}_j \right) + \mathcal{D}^{abj} R^i{}_{ajb} + \mathcal{S}^{akj} R^i{}_{akj}$$

$$T^{\mu\nu} = \int_{\partial\mathcal{W}} \sqrt{-\gamma} \left(T^{ab} u_a^\mu u_b^\nu \delta(x) + \mathcal{D}^{abi} u_a^\mu u_b^\nu \partial_i \delta(x) + u_a^{(\mu} \mathcal{S}^{a\nu)i} \partial_i \delta(x) \right)$$

arXiv:1304.7773 by JA
arXiv:1312.0597 by JA

The effective theory

Contrast this with the action for a biophysical membrane:

$$I[X^i] = \int_{\mathcal{W}} \left(\alpha + \alpha_1 K + \alpha_2 K^2\right)$$

Helfrich-Canham 70's
Polyakov, 86, Kleinert 86

Black Hole Horizons

PART III

Black Hole Horizons

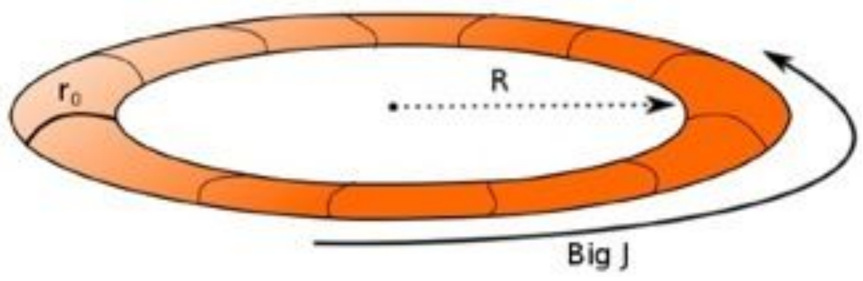
The stationary sector: $u^a = \frac{\mathbf{k}^a}{\mathbf{k}}$ $\quad T = \mathcal{T}\mathbf{k}$

$$I = \int_{\mathcal{W}} \sqrt{-\gamma} P$$

Equations of motion: $K^i = n u^a u^b K_{ab}{}^i$ $\quad \mathbf{k}|_{\partial \mathcal{W}_{p+1}} = 0$

Black Hole Horizons

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dx_i^2$$



$$t = \tau \ , \ r = R, \phi = \phi \ , \ x_i = 0$$

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + R^2 d\phi^2$$

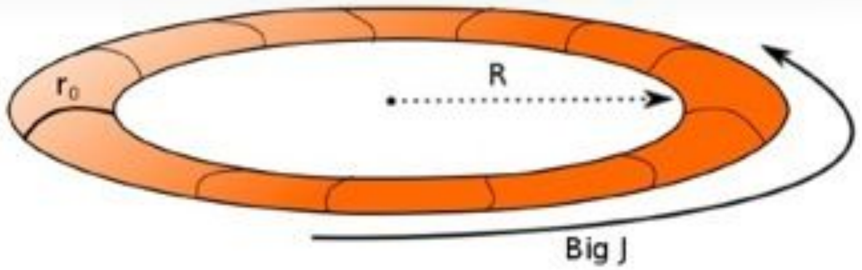
$$\mathbf{k}^2 = 1 - \Omega^2 R^2$$

$$I[R] = \frac{\Omega_{(n+1)}}{16\pi G} \left(\frac{n}{4\pi T}\right)^n R\, \mathbf{k}^n \longrightarrow \Omega R = \frac{1}{\sqrt{n+1}}$$

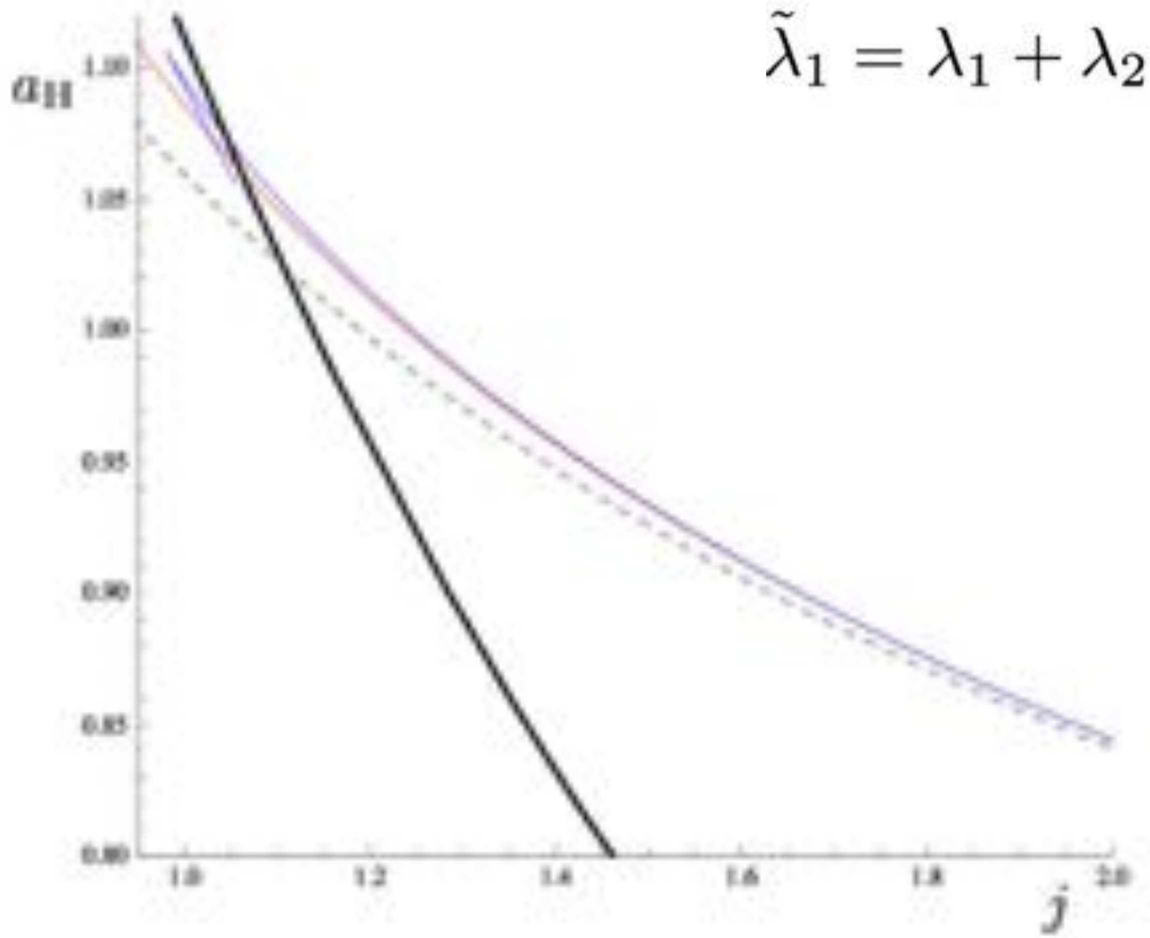
arXiv:0912.2352 by R. Emparan, T. Harmark, V. Niarchos, N.A. Obers

Black Hole Horizons

$$\mathcal{F}[R] = -2\pi R\left(P + \tilde{\lambda}_1 K^i K_i\right)$$



$$\tilde{\lambda}_1 = \lambda_1 + \lambda_2 + \frac{\lambda_3}{n}$$



$$a_H(j) = \frac{2^{\frac{n-2}{n(n+1)}}}{j^{\frac{1}{n}}}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{3n+4}{n}}n^3(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

$$\omega_H(j) = \frac{1}{2j}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{2(n+2)}{n}}n^2(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right),$$

$$t_H(j) = \frac{n j^{\frac{1}{n}}}{2^{\frac{n-2}{n(n+1)}}}\left(1 - \frac{3(n+1)(3n+4)}{2^{\frac{3n+4}{n}}n^3(n+2)}\frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

arXiv:1402.6330 by JA & T. Harmark
arXiv:1402.6345 by O.J.C. Dias, J.E.Santos & B. Way

Black Hole Horizons

How many solutions to the equations below?

$$K^i = n u^a u^b K_{ab}{}^i \qquad \mathbf{k}|_{\partial \mathcal{W}_{p+1}} = 0$$

One possibility is to look for minimal surfaces which also satisfy this.

arXiv:1503.08834 by JA & M. Blau

Black Hole Horizons

- Plane
- Helicoid
- Catenoid
- Riemann surfaces
- Schwarz minimal surfaces
- Enneper surface
- Henneberg surface
- Bour surface
- Catalan surface
- Costa surface
- Chen-Gackstatter
- Barbosa-Dajczer-Jorge helicoids
- Scherk surfaces
- Meeks mobius minimal surface
- etc

These are all non-compact.

Black Hole Horizons

$$K^i = n u^a u^b K_{ab}{}^i \qquad \mathbf{k}|_{\partial\mathcal{W}_{p+1}} = 0$$

In two dimensions, only the plane and the helicoid solve these equations.

$$K_{ab}{}^i = 0$$

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + d\rho^2 + \rho^2 d\phi^2$$

$$\mathbf{k}^2 = 1 - \Omega^2 \rho^2$$

arXiv:0912.2352 by R. Emparan, T. Harmark, V. Niarchos & N. Obers
arXiv:1503.08834 by JA & M. Blau

Black Hole Horizons

The helicoid:

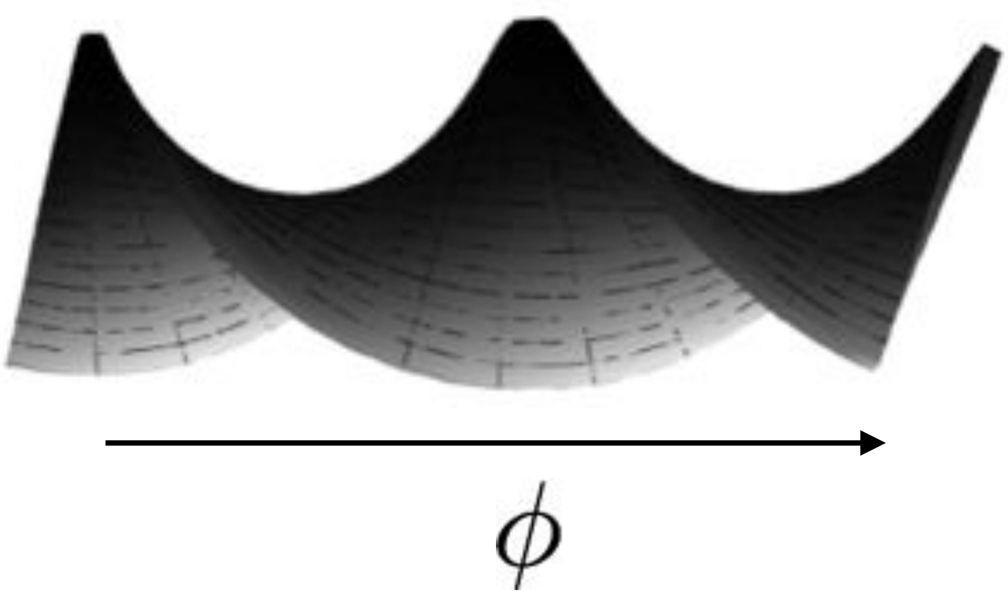
$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + d\rho^2 + (\lambda^2 + \rho^2) d\phi^2$$

$$\mathbf{k}^2 = 1 - \Omega^2(\lambda^2 + \rho^2)$$

$$\mathcal{F}[R] = \frac{\Omega_{(n+1)}}{16\sqrt{\pi}G}\frac{r_+^n}{a\Omega}\int d\phi\lambda\Gamma\left(1+\frac{n}{2}\right)\left(1-\lambda^2\Omega^2\right)^{\frac{n+1}{2}} {}_2\tilde{F}_1\left(-\frac{1}{2},\frac{1}{2};\frac{n+3}{2};1-\frac{1}{\lambda^2\Omega^2}\right)$$

Topology: $\mathbb{R} \times \mathbb{S}^{D-3}$

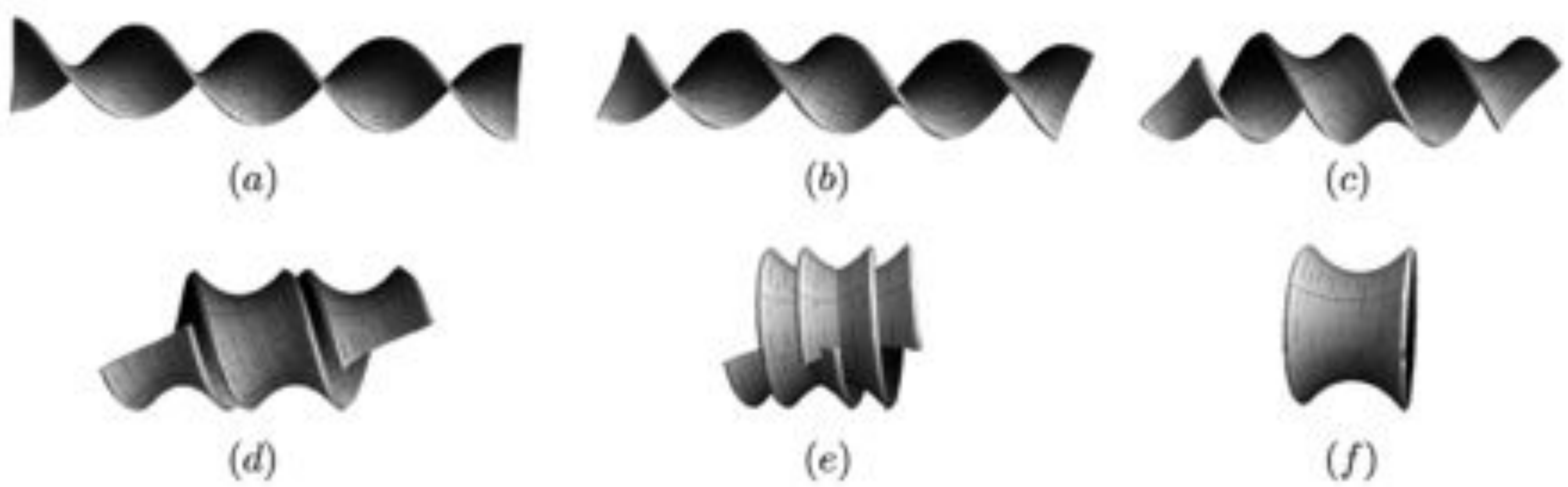
arXiv:1503.08834 by JA & M. Blau



Black Hole Horizons

Black Scherk Surfaces/Catenoids in Plane Waves:

$$ds^2 = -(1 + A(x^q))dt^2 + (1 - A(x^q))dy^2 - 2A(x^q)dtdy + d\mathbb{E}^2_{(D-2)}(x^q)$$

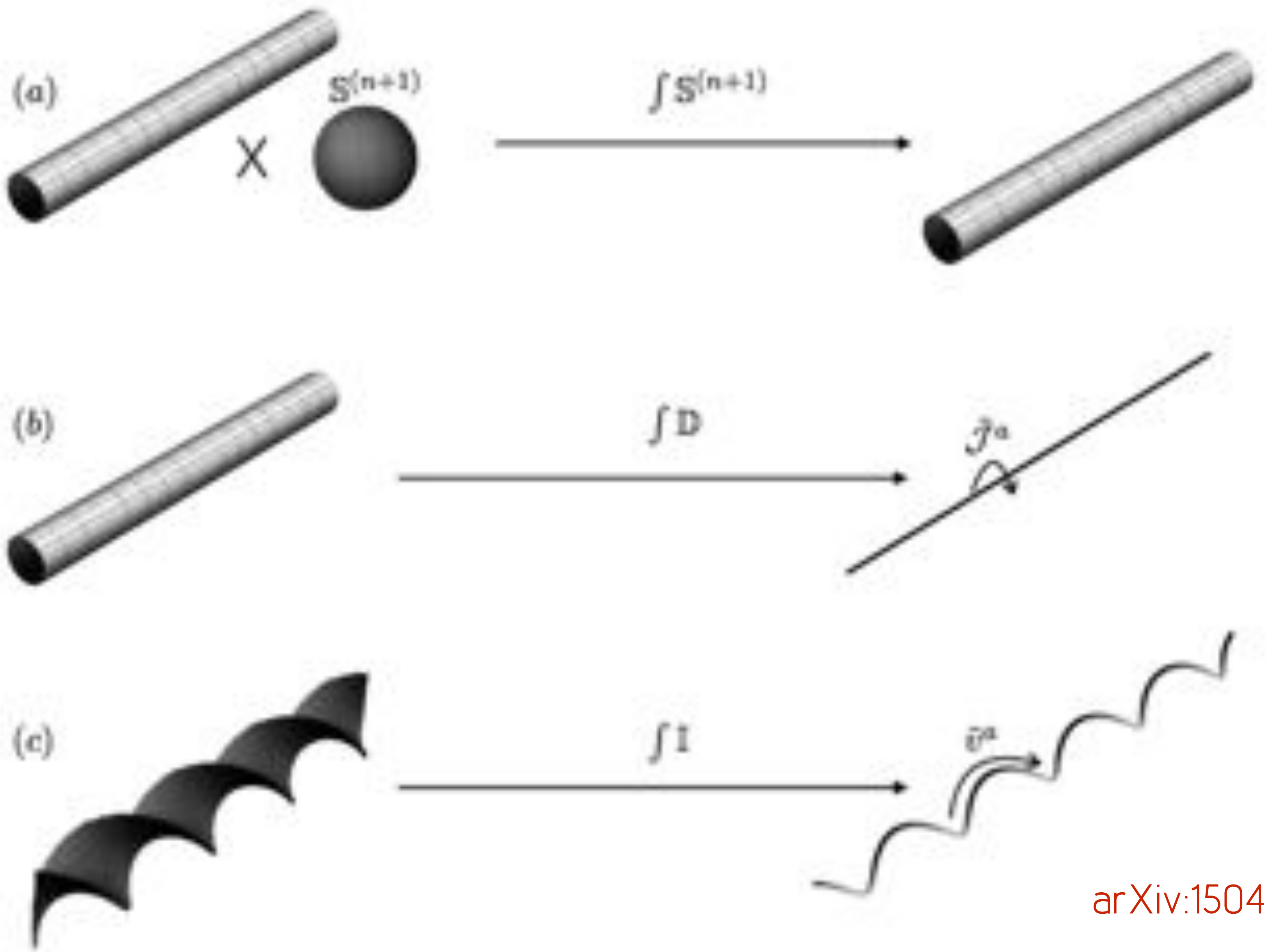


arXiv:1503.08834 by JA & M. Blau

Integrating out further scales

PART IV

Integrating out further scales



arXiv:1504.01393 by JA & M. Blau

Integrating out further scales

Minimal surfaces decouple dynamics in parts of the worldvolume.

$$T^{\hat{a}\hat{b}}K_{\hat{a}\hat{b}}{}^{i}=0$$

$$\tilde{\mathcal{F}}[\tilde{X}^i]=-\int_{\tilde{\mathcal{B}}_{\tilde{p}}}\tilde{R}_0 d\tilde{V}_{(\tilde{p})}\Big(\tilde{P}+\tilde{\upsilon}_1\tilde{\mathfrak{a}}^c\tilde{\mathfrak{a}}_c+\tilde{\upsilon}_2\tilde{\mathcal{R}}+\tilde{\upsilon}_3\tilde{u}^a\tilde{u}^b\tilde{\mathcal{R}}_{ab}$$
$$+\tilde{\lambda}_1\tilde{K}^i\tilde{K}_i+\tilde{\lambda}_2\tilde{K}^{abi}\tilde{K}_{abi}+\tilde{\lambda}_3\tilde{u}^a\tilde{u}^b\tilde{K}_a^{ci}\tilde{K}_{bci}+\dots\Big)$$

arXiv:1504.01393 by JA & M. Blau

Integrating out further scales

Helicoidal string:

$$\tilde{\mathcal{F}}[\tilde{X}^i] = \frac{V_{(\tilde{n}+1)}}{16\pi G}\frac{\tilde{r}_+^{\tilde{n}-k}}{\prod_{a=1}^{k}\tilde{\Omega}_a}\int_{\mathcal{B}_{\tilde{p}}}\sqrt{-\tilde{\gamma}}\,\tilde{\mathbf{k}}^{\tilde{n}}\,{}_2\tilde{F}_1\left(-\frac{1}{2},\frac{k}{2};\frac{\tilde{n}+2}{2};-\frac{\tilde{\mathbf{k}}^2}{\tilde{\mathbf{v}}^2}\right)$$

$$\tilde{T}^{ab} = \tilde{P}\tilde{\gamma}^{ab} + \left(\tilde{\mathcal{T}}\tilde{s} + \sum_{a=1}^{k}\tilde{\omega}_a\tilde{\mathcal{J}}_{(a)} + \Xi\tilde{\mathcal{P}}_\Xi\right)\tilde{u}^a\tilde{u}^b - \Xi\tilde{\mathcal{P}}_\Xi\tilde{v}^a\tilde{v}^b$$

arXiv:1504.01393 by JA & M. Blau

Integrating out further scales

Helicoidal string:

$$\tilde{s} = \frac{(\tilde{n}-k)}{\tilde{\mathcal{T}}}\tilde{\mathcal{G}} \quad , \quad \tilde{\mathcal{J}}_{(a)} = \frac{1}{\tilde{\omega}_a}\tilde{\mathcal{G}} \quad , \quad \tilde{\mathcal{P}}_{\Xi} = \frac{k}{2\Xi^3}\frac{f(\frac{1}{2}, k+2, \tilde{n}+2, \Xi)}{f(-\frac{1}{2}, k, \tilde{n}, \Xi)}\tilde{\mathcal{G}}$$

$$(\tilde{\epsilon} + \tilde{P}) = \tilde{\mathcal{T}}\tilde{s} + \sum_{a=1}^{k}\tilde{\omega}_a\tilde{\mathcal{J}}_{(a)} + \Xi\tilde{\mathcal{P}}_{\Xi}$$

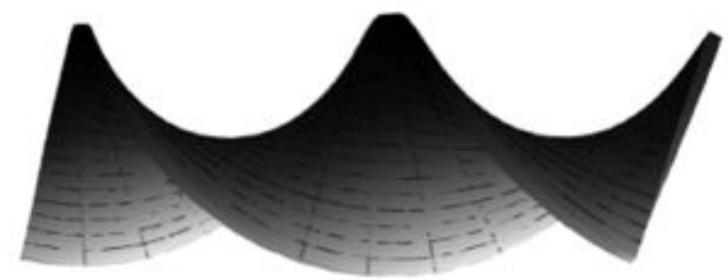
$$d\tilde{P} = \tilde{s}d\tilde{\mathcal{T}} + \sum_{a=1}^{k}\tilde{\mathcal{J}}_{(a)}d\tilde{\omega}_a + \tilde{\mathcal{P}}_{\Xi}d\Xi \quad , \quad d\tilde{\epsilon} = \tilde{\mathcal{T}}d\tilde{s} + \sum_{a=1}^{k}\tilde{\omega}_a d\tilde{\mathcal{J}}_{(a)} + \Xi d\tilde{\mathcal{P}}_{\Xi}$$

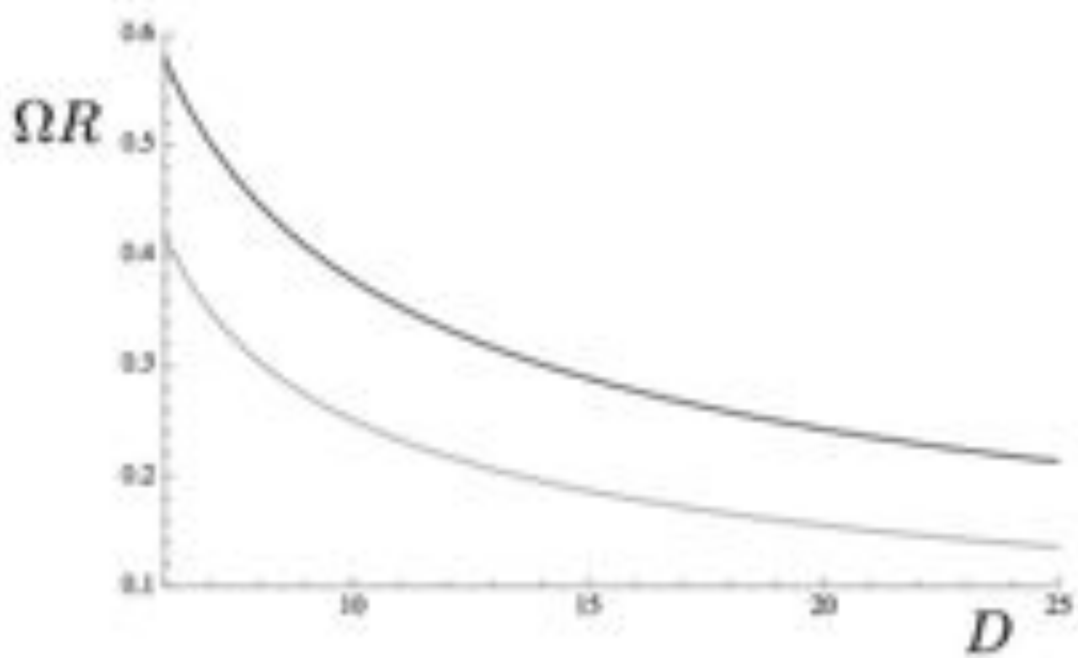
arXiv:1504.01393 by JA & M. Blau

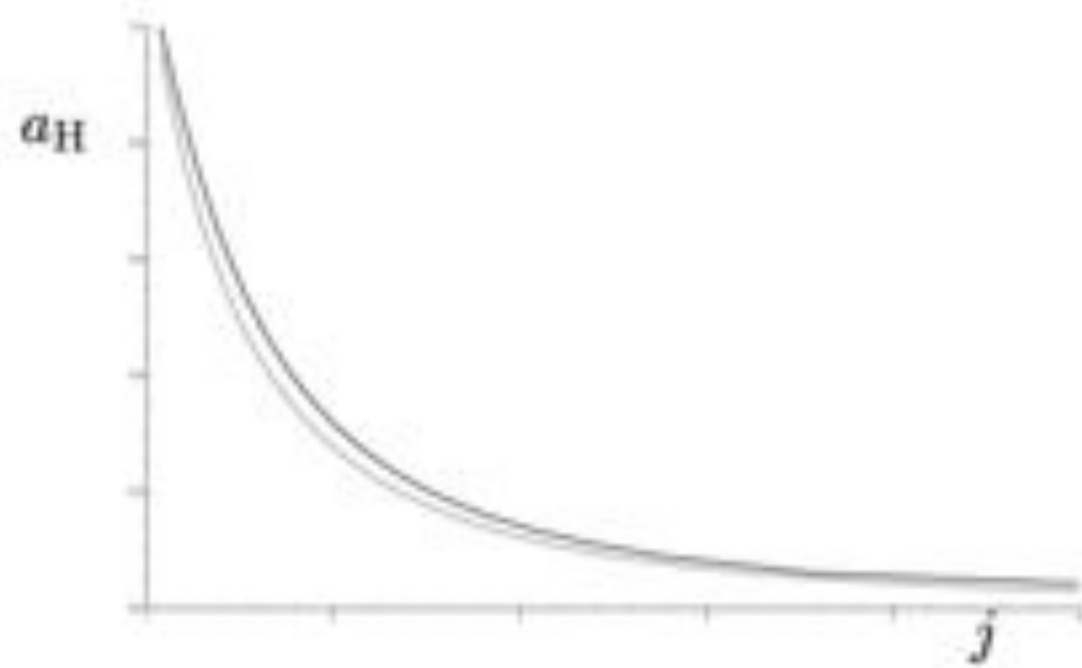
Integrating out further scales

Helicoidal black ring in D>5:

$$\frac{1-(\tilde{n}+1)\Omega^2R^2}{(1-\Omega^2R^2)} - \frac{{}_2\tilde{F}_1\left(\frac{1}{2},\frac{3}{2};\frac{\tilde{n}+4}{2};-\frac{1-\Omega^2R^2}{\Omega^2R^2}\right)}{2\Omega^2R^2\,{}_2\tilde{F}_1\left(-\frac{1}{2},\frac{1}{2};\frac{\tilde{n}+2}{2};-\frac{1-\Omega^2R^2}{\Omega^2R^2}\right)} = 0$$







arXiv:1504.01393 by JA & M. Blau

Integrating out further scales

Helicoidal black tori in D=7:

$$1-\frac{n\tilde{\mathbf{v}}^2}{\tilde{\mathbf{k}}^2}-\frac{k(1-\tilde{\mathbf{v}}^2)}{2\tilde{\mathbf{v}}^2}\frac{{}_2\tilde{F}_1\left(\frac{1}{2},\frac{k+2}{2};\frac{\tilde{n}+4}{2};-\frac{\tilde{\mathbf{k}}^2}{\tilde{\mathbf{v}}^2}\right)}{{}_2\tilde{F}_1\left(-\frac{1}{2},\frac{k}{2};\frac{\tilde{n}+2}{2};-\frac{\tilde{\mathbf{k}}^2}{\tilde{\mathbf{v}}^2}\right)}=0$$

$$1-\frac{n\Omega_2^2R_2^2}{\tilde{\mathbf{k}}^2}-\frac{k\Omega_2^2R_2^2}{2\tilde{\mathbf{v}}^2}\frac{{}_2\tilde{F}_1\left(\frac{1}{2},\frac{k+2}{2};\frac{\tilde{n}+4}{2};-\frac{\tilde{\mathbf{k}}^2}{\tilde{\mathbf{v}}^2}\right)}{{}_2\tilde{F}_1\left(-\frac{1}{2},\frac{k}{2};\frac{\tilde{n}+2}{2};-\frac{\tilde{\mathbf{k}}^2}{\tilde{\mathbf{v}}^2}\right)}=0$$

$$R_\phi\Omega\sim\frac{9}{25}\quad,\quad R_2\Omega_2=\frac{1}{2}$$

arXiv:1504.01393 by JA & M. Blau

Supergravity

PART V

arXiv:1606.09644 by JA, J.Gath, N.Obers, V. Niarchos, A.V. Perdersen

Supergravity

SUPERGRAVITY BACKGROUND

$g_{\mu\nu}(L)$

(1) Same asymptotics $\frac{r_0}{R} \ll 1$

(2) Different asymptotics $\frac{r_0}{L} \ll 1$

Supergravity

Consider the democratic formulation:

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}_{10}}\left[\star R - \frac{1}{2}d\phi\wedge\star d\phi - \frac{1}{2}e^{-\phi}H_{(3)}\wedge\star H_{(3)} - \frac{1}{4}\sum_q e^{a_q\phi}\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}\right]$$

$$G_{\mu\nu} = 8\pi G T_{M\,\mu\nu}$$

$$\tilde{F}_{q+2} = (-1)^{[q/2]+1}e^{-a_q\phi}\star\tilde{F}_{D-q-2}$$

$$16\pi G\mathbf{T}^{\mu\nu}_{(F)} = \frac{e^{a_q\phi}}{(q+1)!}\left(F_{q+2}^{\mu\mu_1\ldots\mu_{q+1}}F_{q+2}{}^{\nu}{}_{\mu_1\ldots\mu_{q+1}} - \frac{1}{2(q+2)}g^{\mu\nu}F^2_{q+2}\right)$$

$$16\pi G\mathbf{T}^{\mu\nu}_{(\phi)} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi \quad .$$

Supergravity

SUPERGRAVITY BACKGROUND

$$r \gg r_0$$

$$(G_{\mu\nu} - 8\pi G\mathbf{T}_{\mathbf{M}\mu\nu})\,|_{\mathbf{linear}} = 8\pi G\mathbf{T}_{\mu\nu}$$

Supergravity

Coupling to the democratic formulation:

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}_{10}}\left[\star R - \frac{1}{2}d\phi\wedge\star d\phi - \frac{1}{2}e^{-\phi}H_{(3)}\wedge\star H_{(3)} - \frac{1}{4}\sum_q e^{a_q\phi}\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}\right]$$

$$G_{\mu\nu} = 8\pi G\left(\mathbf{T}_{\mathbf{M}\mu\nu} + \mathbf{T}_{\mu\nu}\right) \longrightarrow \nabla_\mu \mathbf{T}^{\mu\nu} = -\nabla_\mu \mathbf{T}^{\mu\nu}_{\mathbf{M}}$$

$$d\left(\star(e^{-\phi}H_{(3)}) - \sum_q \frac{1}{2!}e^{a_q\phi}\left[\star\tilde{F}_{(q)}\wedge C_{(q-3)}\right]\right) = -16\pi G\star j_{(2)} \quad ,$$

$$d\star(e^{a_q}\tilde{F}_{(q)}) + (-1)^q e^{a_q+2\phi}\left[\star\tilde{F}_{(q+2)}\wedge H_{(3)}\right] = (-1)^q 16\pi G\star J_{(q-1)} \quad ,$$

$$\Box\phi + \frac{1}{2}e^{-\phi}\star(H_{(3)}\wedge\star H_{(3)}) - \sum_q \frac{a_q}{4}e^{a_q\phi}\star(\tilde{F}_{(q)}\wedge\star\tilde{F}_{(q)}) = -16\pi G j_\phi$$

Supergravity

Effective equations:

$$G_{\mu\nu} = 8\pi G\left(\mathbf{T}_{\mathbf{M}\mu\nu} + \mathbf{T}_{\mu\nu}\right) \longrightarrow \nabla_\mu \mathbf{T}^{\mu\nu} = -\nabla_\mu \mathbf{T}^{\mu\nu}_{\mathbf{M}}$$

$$d\left(\star(e^{-\phi}H_{(3)}) - \sum_q \frac{1}{2!}e^{a_q\phi}\left[\star\tilde{F}_{(q)} \wedge C_{(q-3)}\right]\right) = -16\pi G \star j_{(2)} \quad ,$$

$$d\star(e^{a_q}\tilde{F}_{(q)}) + (-1)^q e^{a_q+2\phi}\left[\star\tilde{F}_{(q+2)} \wedge H_{(3)}\right] = (-1)^q 16\pi G \star J_{(q-1)} \quad ,$$

$$\Box\phi + \frac{1}{2}e^{-\phi}\star(H_{(3)} \wedge \star H_{(3)}) - \sum_q \frac{a_q}{4}e^{a_q\phi}\star(\tilde{F}_{(q)} \wedge \star\tilde{F}_{(q)}) = -16\pi G j_\phi$$

$$dH_3 = 16\pi G \star \mathbf{j}_6 \quad ,$$

$$d\tilde{F}_{q+2} - H_3 \wedge \tilde{F}_q = 16\pi G\left(\star\mathcal{J}_{D-q-3} - (\star\mathbf{j}_6 \wedge C_{q-1})\right)$$

Supergravity

Effective equations:

$$
\begin{aligned}
\nabla_\mu \mathbf{T}^{\mu\nu} = {} & \frac{1}{2!} H_3^{\nu\mu_1\mu_2} \mathbf{j}_{2\mu_1\mu_2} + \frac{e^{-\phi}}{6!} H_7^{\nu\mu_1\ldots\mu_6} \mathbf{j}_{6\mu_1\ldots\mu_6} + \mathbf{j}_\phi \partial^\nu \phi \\
& + \sum_q \frac{1}{(q+1)!} \left(\tilde{F}_{q+2}^{\nu\mu_1\ldots\mu_{q+1}} + (-1)^{q+1} \frac{q(q+1)}{2!} H_3^{\nu\mu_1\mu_2} C_{q-1}^{\mu_3\ldots\mu_{q+1}} \right) \mathbf{J}_{q+1\mu_1\ldots\mu_{q+1}} \\
& + \sum_q \frac{e^{a_q\phi}}{(\tilde{q}+1)!} \left(\tilde{F}_{\tilde{q}+2}^{\nu\mu_1\ldots\mu_{\tilde{q}+1}} + (-1)^{\tilde{q}+1} \frac{\tilde{q}(\tilde{q}-1)}{2!} H_3^{\nu\mu_1\mu_2} C_{\tilde{q}-1}^{\mu_3\ldots\mu_{q+1}} \right) \mathcal{J}_{\tilde{q}+1_{\mu_1\ldots\mu_{\tilde{q}+1}}} \\
& + \frac{1}{4!} \left(\tilde{F}_5^{\nu\mu_1\ldots\mu_4} + 3 H_3^{\nu\mu_1\mu_2} C_2^{\mu_3\ldots\mu_4} \right) \mathbf{J}_{4\mu_1\ldots\mu_4} \\
& - \sum_q \frac{e^{a_q\phi}}{(q+2)!} \tilde{F}_{q+2}^{\mu_1\ldots\mu_{q+2}} [\star \mathbf{j}_6 \wedge C_{q-1}]^\nu{}_{\mu_1\ldots\mu_{q+2}} ,
\end{aligned}
$$

Supergravity

Current equations:

$$d\star \mathbf{J}_{q+1} + (-1)^{q+1} \star \mathbf{J}_{q+3} \wedge H_3 + (-1)^{q+1} e^{a_q + 2\phi} \star \tilde{F}_{q+4} \wedge \star \mathbf{j}_6 = 0 \quad , \quad q = 0, \ldots, 3$$

$$d\star \mathcal{J}_{D-q-3} = 0 \quad , \quad q = -1, 0 \quad , \quad d\star \mathcal{J}_{D-q-3} = H_3 \wedge \star \mathcal{J}_{D-q-1} \quad , \quad q = 1, 2 \quad ,$$

$$d\star \mathbf{j}_2 = 0 \quad , \quad d\star \mathbf{j}_6 = 0 \quad .$$

Supergravity

The Dp-F1 black brane:

$$\mathrm{d}s^2 = D^{\frac{1-p}{8}} H^{\frac{p-7}{8}} \left(-f u_a u_b + v_a v_b\right) \mathrm{d}\sigma^a \mathrm{d}\sigma^b + D^{\frac{9-p}{8}} H^{\frac{p-7}{8}} \left(\Delta_{ab} - v_a v_b\right) \mathrm{d}\sigma^a \mathrm{d}\sigma^b + D^{\frac{1-p}{8}} H^{\frac{p+1}{8}} \left(f^{-1}\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_{8-p}^2\right) \ ,$$

$$B = e^{-a_{\mathrm{F1}}\varphi_0/2} \sin\xi \left(H^{-1} - 1\right) \coth\alpha \ u \wedge v \ ,$$

$$A_{p-1} = e^{-a_{p-2}\varphi_0/2} \tan\xi \left(DH^{-1} - 1\right) \ \star_{(p+1)} (u \wedge v) \ ,$$

$$A_{p+1} = e^{-a_p\varphi_0/2} \cos\xi \left(H^{-1} - 1\right) \coth\alpha \ \star_{(p+1)} \mathbf{1} \ ,$$

Supergravity

The Dp-F1 black brane:

$$\varepsilon = \frac{\Omega_{n+1}}{16\pi G} r_0^n (n + 1 + n \sinh^2 \alpha) \ ,$$

$$P = -\frac{\Omega_{n+1}}{16\pi G}(1 + n \sinh^2 \alpha) \ , \quad P_\perp = -\frac{\Omega_{n+1}}{16\pi G}(1 + n \sinh^2 \alpha \cos^2 \xi)$$

$$T_{ab} = \varepsilon u_a u_b + P v_a v_b + P_\perp \perp_{ab}$$

$$j_2 = \mathcal{Q}_{\mathrm{F1}} u \wedge v \, , \quad J_{p+1} = \mathcal{Q}_p *_{p+1} \mathbf{1}$$

$$\mathcal{Q}_{p-2} = \Phi_p \mathcal{Q}_{\mathrm{F1}} = \Phi_{\mathrm{F1}} \mathcal{Q}_p \, , \quad J_{p-1} = \mathcal{Q}_{p-2} *_{p+1} (u \wedge v)$$

Supergravity

Effective equations:

$$\nabla_a T^{ab} = \frac{1}{(p-1)!}\left[\mathcal{F}_p^{ba_1\ldots a_{p-1}} J_{p-1 a_1\ldots a_{p-1}} + (-1)^{p+1}\frac{1}{2}\mathcal{H}_3^{ba_1a_2}\mathcal{C}_{p-1}^{a_3\ldots a_{p+1}} J_{p+1 a_1\ldots a_{p+1}}\right] + \frac{1}{2}\mathcal{H}_3^{ba_1a_2} j_{2a_1a_2} + j_\phi \partial^b \varphi\,,$$

$$\nabla_a j_2^{ab} = 0\,,\quad \nabla_a J_{p-1}^{ab_1\ldots b_{p-2}} = \frac{1}{3!}\mathcal{H}_{3abc} J_{p+1}^{abcb_1\ldots b_{p-2}}\,,\quad \nabla_a J_{p+1}^{ab_1\ldots b_p} = 0\,,$$

$$j_\phi = \frac{1}{2}\left(a_{\rm F1}\mathcal{Q}_{\rm F1}\Phi_{\rm F1} + a_p Q_p \Phi_p\right)$$

Supergravity

Effective equations:

$$\mathcal{F}^{\mu}_{H} = \frac{1}{2}\mathcal{H}_3^{\mu\mu_1\mu_2}\left(j_{2\mu_1\mu_2} + (-1)^{p+1}\frac{1}{(p-1)!}\mathcal{C}_{p-1}^{\mu_3\dots\mu_{p+1}}J_{p+1\mu_1\dots\mu_{p+1}}\right)$$

$$\mathcal{F}^{\mu}_{F_p} = \frac{1}{(p-1)!}\mathcal{F}_p^{\mu\mu_1\dots\mu_{p-1}}J_{p-1\mu_1\dots\mu_{p-1}}\,,$$

$$\mathcal{F}^{\mu}_{\tilde{F}_{p+2}} = \frac{1}{(p+1)!}\tilde{\mathcal{F}}_{p+2}^{\mu\mu_1\dots\mu_{p+1}}J_{p+1\mu_1\dots\mu_{p+1}}\,, \quad \mathcal{F}^{\mu}_{\phi} = j_{\phi}\partial^{\mu}\varphi\,.$$

$$T^{ab}K_{ab}{}^{i} = n^{i}{}_{\mu}\left(\mathcal{F}^{\mu}_{H} + \mathcal{F}^{\mu}_{F_p} + \mathcal{F}^{\mu}_{F_{p+2}} + \mathcal{F}^{\mu}_{\phi}\right)$$

Summary and future work

- The method allows to scan for black hole horizons and topologies in higher dimensions
- The method can be applied to supergravity in flux backgrounds
- It would be interesting to find solutions to these equations in several setups
- Fluids with q-brane charge in the context of AdS/CFT can lead to interesting results

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THANK YOU