RG Flows Across Dimensions and Holography

Nikolay Bobev

Instituut voor Theoretische Fysica
KU Leuven

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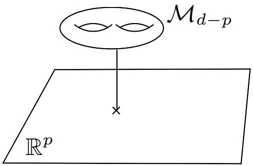
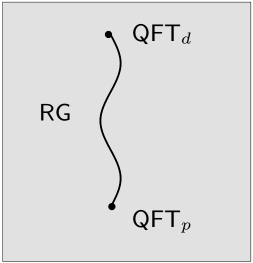
Francesco Benini, P. Marcos Crichigno
Francesco Azzurli, Vincent Min, Krzysztof Pilch, Orestis Vasilakis, Alberto Zaffaroni

RG flows across dimensions

Consider a QFT on $M_d = \mathbb{R}^p \times \mathcal{M}_{d-p}$ and flow to the IR:





Questions:

- What are the properties of such RG flows?
- What is the p-dimensional QFT at low energies?
- Quantitative tools to study these systems?

Employ supersymmetry to simplify the problem.

Motivation

- Construct and explore large classes of interacting p-dimensional superconformal field theories (SCFTs) obtained from a d-dimensional theory "compactified" on the manifold $\mathcal{M}_{d-p}$. [Vafa-Witten], [Witten], [Bershadsky-Johansen-Sadov-Vafa], [Klemm-Lerche-Mayr-Vafa-Warner], [Maldacena-Núñez], [Kapustin], [Gaiotto], [Gaiotto-Moore-Neitzke], [Dimofte-Gukov-Gaiotto], [Cecotti-Córdova-Vafa], [Gadde-Gukov-Putrov], [Benini-NB], [Bah-Beem-NB-Wecht], ...
- This setup leads to interesting "dualities" between the p-dimensional SCFT and a ("topological") theory on $\mathcal{M}_{d-p}$. [Alday-Gaiotto-Tachikawa], [Gadde-Pomoni-Rastelli-Razamat], [Dimofte-Gaiotto-Gukov], [Cecotti-Córdova-Vafa], ...
- Obtain insight into the original d-dimensional theory and RG flows across dimensions.
- The p-dimensional SCFTs typically admit a "large N" limit and have holographic duals which can be explicitly constructed. New examples of AdS/CFT. [Maldacena-Núñez], ...

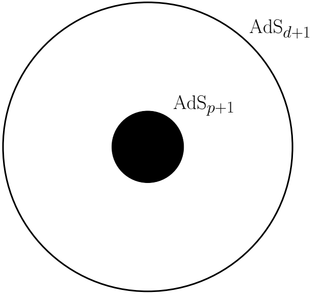
Tools

- Anomaly matching ['t Hooft]
- Anomaly polynomials [Alvarez-Gaumé-Ginsparg], [Witten], [Harvey-Minasian-Moore], ...
- Topological twists [Witten], ...
- a-maximization [Intriligator-Wecht], F-maximization [Jafferis], [Closset-Dumitrescu-Fesctuccia-Komargodski-Seiberg], c-extremization [Benini-NB]
- Supersymmetric lozalization [Witten], [Pestun],...
- Unitarity bounds [Hofman-Maldacena]
- Wrapped branes [Bershadsky-Sadov-Vafa], [Maldacena-Núñez], [Gauntlett-Waldram et al.], ...
- Holography [Maldacena], [GKP], [Witten], ...

Disclaimer: Here I always take $\mathcal{M}_{d-p}$ to be compact and (in holography) with an Einstein metric. Generalizations are possible and very interesting!
[Anderson-Beem-NB-Rastelli]; [Gaiotto-Maldacena], [Bah]

Holography

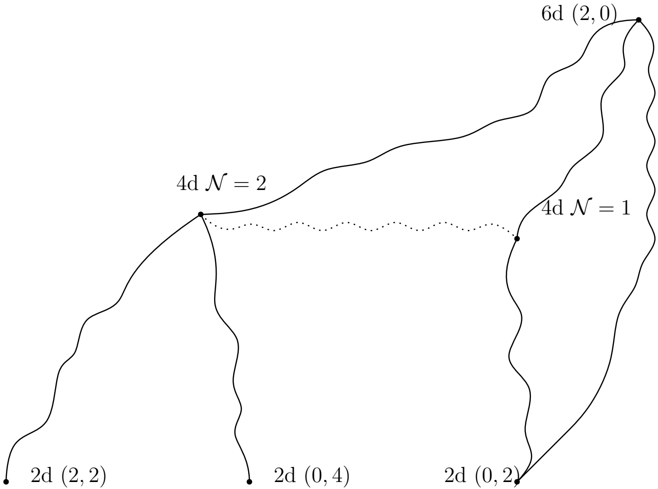
Holography is an important tool to deduce the existence of these RG flows and study their properties.



The holographic dual description is a domain wall (or black brane) interpolating between (asymptotically locally) AdS_{d+1} and AdS_{p+1}. [Maldacena-Núñez], ...

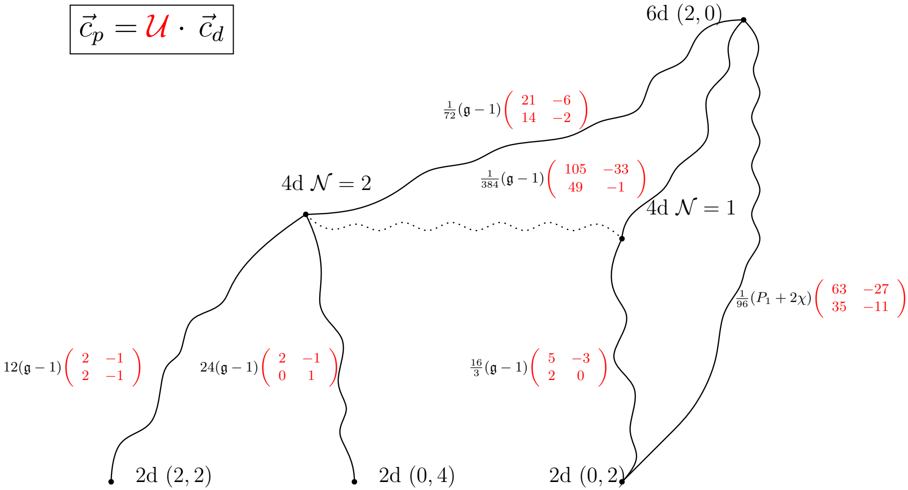
Numerous holographic RG flows of this type can be explicitly constructed and embedded in string theory by using branes wrapped on $\mathcal{M}_{d-p}$.

Universal flows across even dimensions



Universal flows across even dimensions

$$\vec{c}_p = \mathcal{U} \cdot \vec{c}_d$$



Universal flows across odd dimensions

- No 't Hooft anomalies $\Rightarrow$ technically harder.
- Make progress using supersymmetric localization. [Kapustin-Willett-Yaakov], [Benini-Zaffaroni], ...

3d $\mathcal{N} = 2$ on Σ_g

At large N one finds $F_{S^1 \times \Sigma_\mathfrak{g}} = (\mathfrak{g} - 1) F_{S^3}$

1d SUSY QM

- In holography this maps to a (magnetically charged) black hole in AdS_4. Microscopic count of the BH entropy!

4d $\mathcal{N} = 1$ SCFTs on $\Sigma_{\mathfrak{g}}$

4d $\mathcal{N}=1$ SCFTs on $\Sigma_{\mathfrak{g}}$

Consider a general 4d $\mathcal{N}=1$ SCFTs on $\mathbb{R}^2 \times \Sigma_{\mathfrak{g}}$ and perform a (partial) "topological twist", i.e. use the R-symmetry to cancel the space-time curvature [Witten]

$$\mathcal{A}_\mu^R = -\frac{1}{4}\omega_\mu \ , \qquad \rightarrow \qquad \widetilde{\nabla}_\mu \epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu + \mathcal{A}_\mu^R\right)\epsilon = \partial_\mu \epsilon = 0 \ .$$

Generally the global symmetry group is $U(1)_R \times G_F$.

Turn a background field only for $\mathcal{A}_\mu^R$ to preserve 2d $\mathcal{N}=(0,2)$ supersymmetry (ensure proper R-charge quantization on $\Sigma_{\mathfrak{g}}$)

A "universal twist" for all 4d $\mathcal{N}=1$ SCFTs.

General background fluxes in the Cartan of G_F do not break additional supersymmetry. The construction depends on the details of the 4d $\mathcal{N}=1$ theory, i.e. non-universal $\rightarrow$ Rich families of 2d $\mathcal{N}=(0,2)$ SCFTs.

[Almuhairi-Polchinski], [Kutasov-Lin], [Franco-Lee-Vafa et al.], [Schäfer-Nameki-Weigand], [Amariti-Cassia-Penati]...

4d $\mathcal{N}=1$ SCFTs on $\Sigma_{\mathfrak{g}}$

The anomaly polynomials in 4d and 2d are

$$\mathcal{I}_6 = \frac{k_{RRR}}{6} c_1(\mathcal{F}_R^{(4)})^3 - \frac{k_R}{24} c_1(\mathcal{F}_R^{(4)}) p_1(\mathcal{T}_4) + I_6^F \ ,$$
$$\mathcal{I}_4 = \frac{k_{RR}}{2} c_1(\mathcal{F}_R^{(2)})^2 - \frac{k}{24} p_1(\mathcal{T}_2) + I_4^F \ .$$

Superconformal Ward identities relate conformal and 't Hooft anomalies

[Anselmi-Freedman-Grisaru-Johansen]

$$a = \frac{9}{32} k_{RRR} - \frac{3}{32} k_R \ , \quad c = \frac{9}{32} k_{RRR} - \frac{5}{32} k_R \ , \quad c_r = 3 k_{RR} \ , \quad c_r - c_l = k \ .$$

Use ($\kappa = 1$ for S^2, $\kappa = 0$ for T^2, and $\kappa = -1$ for H^2)

$$\mathcal{F}_R^{(4)} \to \mathcal{F}_R^{(2)} - \frac{\kappa}{2} t_{\mathfrak{g}} \ , \qquad \text{and} \qquad \mathcal{I}_4 = \int_{\Sigma_{\mathfrak{g}}} \mathcal{I}_6 \ ,$$

to extract the 2d conformal anomalies

$$\boxed{\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{16}{3}(\mathfrak{g}-1) \begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix}}$$

4d $\mathcal{N}=1$ SCFTs on $\Sigma_{\mathfrak{g}}$

Comments:

- The R-charges have to be properly quantized (i.e. rational) in order to be able to perform the universal twist.
- Notice that 't Hooft anomaly matching amounts to

$$k_{RR} = (\mathfrak{g}-1)k_{RRR}\,, \qquad\qquad k = (\mathfrak{g}-1)k_R\,.$$

- Unitarity implies that $\mathfrak{g} > 1$. In addition we should have

$$\frac{3}{5} < \frac{a}{c}\,, \quad \text{compatible with} \quad \frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2} \quad \text{[Hofman-Maldacena]}\,.$$

  Are there any interacting 4d $\mathcal{N}=1$ SCFTs with $\frac{1}{2} < \frac{a}{c} < \frac{3}{5}$?

- For $a=c$ we have $c_r = c_l = \frac{32}{3}(\mathfrak{g}-1)a$. To be tested holographically!

The holographic dual

The generality of the field theory construction suggests an universal treatment in supergravity.

Use 5d minimal $\mathcal{N} = 2$ gauged supergravity.

$$(g_{\mu\nu}, A_\mu) \,, \qquad \text{dual to} \qquad (T_{\mu\nu}, j_\mu^R) \,.$$

There is a BPS black string solution of the 5d theory [Klemm-Sabra], [Benini-NB]

$$ds^2 = e^{2f(r)}(-dt^2 + dz^2 + dr^2) + e^{2h(r)} \frac{dx^2 + dy^2}{y^2} \,, \qquad A = \frac{dx}{y}$$

- Analytic solution for $f(r)$ and $h(r)$. Here $\Sigma_{\mathfrak{g}} = \mathbb{H}^2/\Gamma$.
- Asymptotically locally AdS_5 background at $r \to \infty$.
- $AdS_3 \times \Sigma_{\mathfrak{g}}$ background at $r \to 0$.

The holographic dual

Using standard holographic technology one finds

$$c_r = c_l = \frac{3L_{AdS_3}}{2G_N^{(3)}} = \frac{32}{3}(\mathfrak{g}-1)\frac{\pi L^3_{AdS_5}}{8G_N^{(5)}} = \frac{32}{3}(\mathfrak{g}-1)a.$$

Uplift of this simple solution to many distinct string and M-theory backgrounds by using supergravity uplift formulas. [Gauntlett-Varela]

Particular examples:

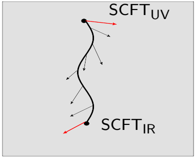
- IIB compactifications on SE_5 manifolds, i.e. 4d $\mathcal{N}=1$ quiver gauge theories. [Klebanov-Witten], [Morrison-Plesser]
- M-theory compactifications of the Maldacena-Núñez type, i.e. 4d $\mathcal{N}=1$ "class $\mathcal{S}$" SCFTs. [Bah-Beem-NB-Wecht].

4d $\mathcal{N}=1$ on $\Sigma_{\mathfrak{g}}$

More general twists. Turn on fluxes for $G_F \to$ non-universal flows

$$\mathcal{A}_{\text{back}} = \mathcal{A}^R + b_i \mathcal{A}^i \,, \qquad \to \qquad R_{IR} = R_{UV} + \epsilon_i(b) F^i$$

Mixing between the flavor and R-symmetries along the RG flow!



Consider a trial R-current Ω_μ^{tr}

$$\Omega_\mu^{\text{tr}}(t) = \Omega_\mu + \sum_{i \neq R} t_i J_\mu^i \,.$$

Construct a trial $c_r^{\text{tr}}(t)$ from the anomaly of the trial R-symmetry. For unitary SCFTs one finds c-extremization [Benini-NB]

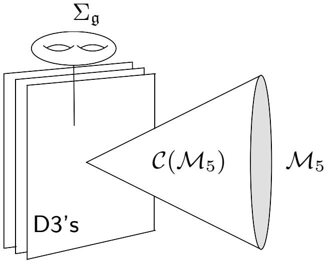
$$\frac{\partial c_r^{\text{tr}}(t^*)}{\partial t^i} = 0 \,, \quad \forall\, i \neq R \,, \quad \to \quad c_r^{\text{tr}}(t^*) = c_r \,.$$

Similar to a-maximization in 4d and F-maximization in 3d. [Intriligator-Wecht], [Jafferis], [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg]

4d $\mathcal{N} = 1$ on $\Sigma_{\mathfrak{g}}$

Explicit examples from string theory: 4d $\mathcal{N} = 1$ SCFTs from D3-branes at conical singularities [Klebanov-Witten], [Morrison-Plesser],...

- Transverse space $\mathbb{R}_+ \times \mathcal{M}_5 = \mathcal{C}(\mathcal{M}_5)$.
- $\mathcal{M}_5$ is Sasaki-Einstein $\Rightarrow \mathcal{C}(\mathcal{M}_5)$ is Calabi-Yau.
- $\mathcal{M}_5 = S^5 \Rightarrow \mathcal{N} = 4$ SYM.
- $\mathcal{M}_5 = Y^{1,0} \Rightarrow \mathcal{N} = 1$ KW.
- More examples based on $Y^{p,q}$ [Gauntlett-Martelli-Sparks-Waldram] and dP_n surfaces.



One can analyze in the same way the Leigh-Strassler $\mathcal{N} = 1$ mass deformation of $\mathcal{N} = 4$ SYM. [Gubser-Freedman-Pilch-Warner], [NB-Pilch-Vasilakis]

A new feature: "Baryonic" and R-symmetries can mix along such RG flows.

Holographic description

Example: $Y^{p,0}$ on $\Sigma_{\mathfrak{g}>1}$ with baryonic flux b ($\mathbb{Z}_p$ orbifold of the KW theory).

Field theory: Apply anomaly matching and c-extremization to find

$$c_r = \frac{32}{3}(\mathfrak{g}-1)a(Y^{p,0}) + 24(\mathfrak{g}-1)p^3 b^2 N^2 - 2p(\mathfrak{g}-1)$$

For $b=0 \rightarrow$ universal twist.

IIB supergravity: Solve the IIB supergravity supersymmetry variations

$$ds^2_{10} = ds^2_{\mathsf{AdS}_3} + \frac{v^2+v+1}{4v} ds^2_{\Sigma_\mathfrak{g}} + \frac{v^2+v+1}{4(v+1)}\Big[d\theta^2 + \sin^2\theta\, d\phi^2$$

$$+\frac{1}{v}\left(dw^2+\sin^2 w\, d\nu^2\right)\Big] + \frac{1}{4}\left(d\psi - \cos\theta\, d\phi - \cos w\, d\nu - \frac{dx_1}{x_2}\right)^2 ,$$

$$G_{(5)} = \mathsf{vol}_{\mathsf{AdS}_3} \wedge \left(\frac{(v+1)^2}{2v}\mathsf{vol}_{\Sigma_\mathfrak{g}} + \frac{1}{2(v+1)}\left(v^2\,\mathsf{vol}_{S^2_{\theta\phi}} + \frac{1}{v}\mathsf{vol}_{S^2_{w\nu}}\right)\right) ,$$

where $v = \frac{1+4pb}{1-4pb}$.

The holographic central charge is

$$c_{\text{sugra}} = \frac{32}{3}(\mathfrak{g}-1)a(Y^{p,0}) + 24(\mathfrak{g}-1)p^3 b^2 N^2 .$$

4d $\mathcal{N} = 2$ SCFTs on $\Sigma_{\mathfrak{g}}$

4d $\mathcal{N} = 2$ on $\Sigma_{\mathfrak{g}}$

All 4d $\mathcal{N} = 2$ SCFTs have $SU(2)_R \times U(1)_r$ R-symmetry. There are two "universal" twists. [Kapustin]

- α-twist with $\mathcal{N} = (2,2)$. Background flux for $U(1)_R$. The central charges are
$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = 24(\mathfrak{g}-1)\begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} a \\ c \end{pmatrix} .$$

- β-twist with $\mathcal{N} = (0,4)$. Background flux for $U(1)_r$. The central charges are
$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = 24(\mathfrak{g}-1)\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ c \end{pmatrix} .$$

Notice that $\frac{1}{3}\alpha + \frac{4}{3}\beta$ is the "universal" twist with 2d $\mathcal{N} = (0,2)$ supersymmetry.

4d $\mathcal{N} = 2$ on $\Sigma_\mathfrak{g}$

Comments

- Unitarity implies that $\mathfrak{g} > 1$ and

$$\frac{1}{2} < \frac{a}{c} \,, \quad \text{compatible with the HM bound} \quad \frac{1}{2} \leq \frac{a}{c} \leq \frac{5}{4} \,.$$

- A curious relation between the α-twist and the Shapere-Tachikawa formula

$$c_l = c_r = 12(\mathfrak{g}-1)(2a-c) = 3(\mathfrak{g}-1)\sum_i \Delta_{\mathcal{C}}^{(i)} = 3(\mathfrak{g}-1)d_G \,.$$

- Holographic dual of the α-twist: a black string solution in minimal 5d $\mathcal{N} = 4$ gauged supergravity [Romans] with an $AdS_3 \times \Sigma_\mathfrak{g}$ near horizon region.

- Uplift of this solution to IIB and 11d supergravity. [Gauntlett-Varela] Reproduce the CFT central charges holographically.

- No AdS_3 solution for the β-twist. Maybe no normalizable vacuum? Similarities with the $\mathcal{N} = (4,4)$ twist of 4d $\mathcal{N} = 4$ SYM where one finds a 2d σ-model onto the Hitchin moduli space [Bershadsky-Johansen-Sadov-Vafa]?

3d $\mathcal{N} = 2$ SCFTs on $\Sigma_{\mathfrak{g}}$

3d $\mathcal{N}=2$ on $\Sigma_{\mathfrak{g}}$

Consider 3d $\mathcal{N}=2$ SCFTs on $S^1 \times \Sigma_{\mathfrak{g}}$. The global symmetry group is $U(1)_R \times G_F$.

Turn a background field only for $\mathcal{A}^R_\mu$ to preserve 2 supercharges in 1d.

To study this theory use the "topologically twisted index" [Benini-Zaffaroni], [Benini-Hristov-Zaffaroni]. Computed by localization in the same spirit as the S^3 partition function. [Kapustin-Willett-Yaakov], [Drukker-Marino-Putrov], [Jafferis-Klebanov-Pufu-Safdi]

At large N a simple relation between the two partition functions [Hosseini-Zaffaroni]

$$F_{S^1\times\Sigma_{\mathfrak{g}}}(\Delta_I,\mathfrak{n}_I) = (\mathfrak{g}-1)F_{S^3}\left(\Delta_I/\pi\right) + \sum_I \left(\frac{\mathfrak{n}_I}{1-\mathfrak{g}} - \frac{\Delta_I}{\pi}\right)\frac{\pi}{2}\partial_{\Delta_I}F_{S^3}\left(\Delta_I/\pi\right)$$

Where: $\Delta_I \to$ chemical potentials for global symmetries; $\mathfrak{n}_I \to$ magnetic charges.

The universal twist amounts to: $\mathfrak{n}_I = (1-\mathfrak{g})\Delta_I/\pi$

$$\boxed{F_{S^1\times\Sigma_{\mathfrak{g}}} = (\mathfrak{g}-1)F_{S^3}}$$

Black holes in AdS_4

A simple supersymmetric BH solution of 4d minimal gauged supergravity
[Romans], [Caldarelli-Klemm]

$$ds_4^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2_{\Sigma_\mathfrak{g}} \ , \qquad F = \frac{1}{2\sqrt{2}}\mathrm{vol}_{\Sigma_\mathfrak{g}}$$

Asymptotic to AdS_4 for $\rho \to \infty$ and to $AdS_2 \times \Sigma_\mathfrak{g}$ for $\rho \to \frac{1}{\sqrt{2}}$.

Uplift to M-theory: [Gauntlett-Kim-Waldram], [Gauntlett-Varela]

$$ds^2_{11} = L^2 \left(ds_4^2 + 16\, ds^2_{\mathsf{SE}_7}\right) \ , \qquad G_{(4)} \neq 0 \, ,$$

Dual to M2-branes at conical singularities $\mathcal{C}(\mathcal{M}_7)$.

In massive IIA: a new solution! Deformation of a recently constructed AdS_4 vacuum in massive IIA [Guarino-Jafferis-Varela], [Fluder-Sparks]

$$ds^2_{10} = e^{2\lambda} L^2 \left(ds_4^2 + ds_6^2\right)$$

with

$$ds_6^2 = \omega_0^2 \left[e^{\varphi - 2\phi} X^{-1} d\alpha^2 + \sin^2\alpha (\Delta_1^{-1} ds^2_{\mathsf{KE}_4} + X^{-1}\Delta_2^{-1}\eta^2)\right]$$
$$e^{2\lambda} \equiv (\cos(2\alpha)+3)^{1/2}(\cos(2\alpha)+5)^{1/8} \, ,$$

and $L, \omega_0, \varphi, \phi$ - constants. Nontrivial F_2, H_3, and F_4 fluxes in massive IIA.

Large class of new massive IIA solutions with CFT duals.

3d $\mathcal{N} = 2$ on $\Sigma_\mathfrak{g}$

For the black hole entropy one finds

$$S_{\text{BH}} = \frac{\pi}{2G_N^{(4)}}(\mathfrak{g} - 1) = (\mathfrak{g} - 1)F_{S^3}$$

Thus $F_{S^1 \times \Sigma_\mathfrak{g}}$ computes the entropy of these BHs!

The same universal results holds for $\mathcal{N} = 2$ mass deformations of the ABJM theory. [Corrado-Pilch-Warner], [Jafferis-Klebanov-Pufu-Safdi], [NB-Min-Pilch]

Comments:

- At large N one has $F_{S^1 \times \Sigma_\mathfrak{g}} \sim N^{3/2}$ in M-theory and $F_{S^1 \times \Sigma_\mathfrak{g}} \sim N^{5/3}$ in massive IIA.
- To generalize this setup turn on background flux for the G_F global symmetry of the CFT. Find the correct R-symmetry in the IR by extremizing the "twisted index". [Benini-Hristov-Zaffaroni]
- The holographic dual construction is realized by M2-branes at the tip of a conical singularity wrapping $\Sigma_\mathfrak{g}$. A large landscape of AdS_2 vacua in M-theory. [in progress]
- One can also add electric charges to these 4d black holes and modify correspondingly the "twisted index" [Benini-Hristov-Zaffaroni], [in progress]

Other flows

- This general setup is applicable to SCFTs in other dimensions and on other manifolds. A prominent example is the 6d $(2,0)$ theory on 4-manifolds. [Benini-NB], [Gadde-Gukov-Putrov], [Ganor]...
- The field theory tools are less developed for 5d $\mathcal{N}=1$ SCFTs on $\mathbb{R}^3 \times \Sigma_{\mathfrak{g}}$ or $\mathbb{R}^2 \times \mathcal{M}_3$ (absence of anomalies or localization results).
- A holographic analysis in "minimal" gauged supergravity always yields an AdS vacuum. Suggestive results and "predictions" from holography.
- Interpretation in terms of wrapped branes in string/M-theory.

Summary

- Evidence for many new SCFTs arising from RG flows across dimensions.
- Understanding of some of their properties through nonperturbative QFT tools.
- Explicit "top-down" holographic constructions that are dual to the SCFTs at hand.
- Useful spin-off: microscopic entropy counting for AdS black holes.

Outlook

Many things to understand

- Field theory understanding of the large "zoo" of 2d SCFTs. Is there a "T_N type" building block? A "baby version" of AGT? The (2,2) twist seems particularly curious.

- $1/N$ corrections to the leading order holographic results. Possible for AdS_3 and AdS_5 vacua. [Baggio-Halmagyi-Mayerson-Robbins-Wecht] Important to understand this for the AdS_4 black holes.

- 5d SCFTs on $\mathbb{R}^3 \times \Sigma_{\mathfrak{g}}$. Holography suggests a universal relation between F_{S^5} and $F_{S^3 \times \Sigma_{\mathfrak{g}}}$. Derivation in QFT from supersymmetric localization?

- Flows between even and odd dimensions? Various universal flows suggested by holography [NB-Crichigno]. Insights from QFT?

- Holographic duals of QFTs with a full topological twist? [Benini-NB-Gautason-Hristov]

- Holographic/geometric dual of c-extremization. Is there some "generalized volume" minimization? [Martelli-Sparks-Yau]

GRAZIE!

DANKE!

MERCI!

GRAZIA!

6d $\mathcal{N} = (2, 0)$ SCFTs on 4-manifolds

6d $\mathcal{N} = (2,0)$ SCFTs on 4-manifolds

In 6d the conformal anomaly is

$$\langle T^\mu_\mu \rangle \sim a\, E_d + \sum_{i=1}^{3} \hat{c}_i\, W_i \,,$$

and the anomaly polynomial is

$$\mathcal{I}_8 = \frac{1}{4!}\left(\alpha\, c_2^2(\mathcal{F}_R) - \beta\, c_2(\mathcal{F}_R) p_1(\mathcal{T}_6) + \gamma\, p_1^2(\mathcal{T}_6) + \delta\, p_2(\mathcal{T}_6)\right) + \mathcal{I}_8^{\text{flavor}}$$

$\mathcal{N} = (2,0)$ SCFTs: $\mathcal{I}_8^{\text{flavor}} = 0$ and there are only two independent conformal anomalies. [Córdova-Dumitrescu-Intriligator], [Beccaria-Tseytlin]

$$a_{6d} = \frac{16}{7}(\alpha - \frac{9}{8}\beta)\,, \qquad c_{6d} = 4(\alpha - \frac{3}{2}\beta)\,.$$

Note: $c_{6d} = \frac{7}{4} a_{6d}$ in the large N limit.

The twist: Take a Kähler manifold $\mathcal{M}_4$. The structure group is $SU(2) \times U(1)$. Turn on a background $U(1) \in SU(2)_{\text{diag}} \in SO(4) \in SO(5)_R$. This preserves 2d $\mathcal{N} = (0,2)$ supersymmetry.

6d $\mathcal{N} = (2,0)$ SCFTs on 4-manifolds

Use $\mathcal{I}_4 = \int_{\mathcal{M}_4} \mathcal{I}_8$ to find

$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{1}{96}(P_1 + 2\chi)\begin{pmatrix} 63 & -27 \\ 35 & -11 \end{pmatrix}\begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

There is an $AdS_3 \times \mathcal{M}_4$ BPS solution of maximal 7d sugra for negatively curved $\mathcal{M}_4$ with a Kähler-Einstein metric.

This can be uplifted to 11d supergravity to find the holographic central charge

$$c_r = \frac{21}{128}(P_1 + 2\chi)a_{6d} = \frac{21}{256\pi^2}\text{Vol}(\mathcal{M}_4)a_{6d}$$

Comments:

- This setup corresponds to M5-branes wrapped on a Kähler 4-cycle in CY_4. Different from the MSW $\mathcal{N} = (0,4)$ CFT. [Maldacena-Strominger-Witten]
- Supergravity (once again) fixes the geometry of the 4d manifold. Why?

There are many other possible twists for the $(2,0)$ theory on various $\mathcal{M}_4$. Generically one has to rely on c-extremization to compute the correct central charges. [Benini-NB], [Gadde-Gukov-Putrov]