Strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$

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Based on work with Matthias Gaberdiel, Rajesh Gopakumar and Wei Li
[arXiv:1701.03552], [arXiv:1707.?????].

Summary of the results

- The background supports the large $\mathcal{N} = 4$ superconformal algebra and is thus very interesting, but still mysterious.
- We have analyzed the BPS spectrum of $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ both in string theory and supergravity.
- The sugra calculation shows a discrepancy with the old result of [de Boer, Pasquinucci, Skenderis '99].
- We have made an explicit proposal for the dual CFT.

The large $\mathcal{N} = 4$ algebra A_γ

- R-symmetry [Sevrin, Troost, van Proeyen, Schoutens, Spindel, Theodoridis, Goddard, Schwimmer 88'-90']:
  - $\mathfrak{su}(2)_{k^+} \oplus \mathfrak{su}(2)_{k^-} \oplus \mathfrak{u}(1)$-current algebra
- Central charge:

$$c = \frac{6k^+k^-}{k^+ + k^-}$$

- Global algebra (wedge-algebra): $\mathrm{D}(2,1|\alpha)$.

BPS bound

- Representations are labelled by

$$(h \; ; \; j^+, \; j^-, \; u)$$

conformal weight

spin w.r.t. $A^{+,i}$

spin w.r.t. $A^{-,i}$

$\mathfrak{u}(1)$-charge

- BPS bound [Gunaydin, Petersen, Taormina, van Proeyen '89, Petersen, Taormina '90]:

$$h_{\text{BPS}} = \frac{k^+ j^- + k^- j^+}{k^+ + k^-} + \underbrace{\frac{(j^+ - j^-)^2 + u^2}{k^+ + k^-}}_{\text{invisible in supergravity } k^\pm \to \infty} \; .$$

The global algebra $\mathrm{D}(2,1|\alpha)$

- $\mathfrak{u}(1)$-current decouples: Only global $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$-symmetry remains
- Representations are labelled by

$$(h\ ;\ \ j^+,\ \ j^-)$$

conformal weight spin w.r.t. $A^{+,i}$ spin w.r.t. $A^{-,i}$

- BPS bound [de Boer, Pasquinucci, Skenderis '99]:

$$h_{\mathrm{BPS}} = \frac{k^+ j^- + k^- j^+}{k^+ + k^-}\ .$$

BPS bounds

- The A_γ (stringy) BPS-bound is stronger than the $\mathrm{D}(2,1|\alpha)$ (sugra) BPS-bound, equality only for $j^+ = j^-$ and $u = 0$.
- Strange consequence: Sugra BPS states with $j^+ \neq j^-$ have to acquire non-trivial quantum corrections to even satisfy the stringy BPS bound [de Boer, Pasquinucci, Skenderis '99; Gukov, Martinec, Moore, Strominger '04].
- According to the analysis of [de Boer, Pasquinucci, Skenderis '99], sugra contains such BPS states.

WZW model

- For pure NS-NS background, the worldsheet theory of the string can be described by a supersymmetric WZW model based on [Elitzur, Feinerman, Giveon, Tsabar '99]:

$$\mathfrak{sl}(2,\mathbb{R})_k^{(1)} \oplus \mathfrak{su}(2)_{k^+}^{(1)} \oplus \mathfrak{su}(2)_{k^-}^{(1)} \oplus \mathfrak{u}(1)^{(1)} \ .$$

- Criticality of the string theory requires the total central charge to be 15:

$$k = \frac{k^+ k^-}{k^+ + k^-} \ .$$

- The $\mathfrak{sl}(2,\mathbb{R})$-spin j is identified with the conformal weight of the state in the dual CFT [Elitzur, Feinerman, Giveon, Tsabar '99].

  $\Rightarrow$ Can study BPS spectrum

Worldsheet BPS spectrum: unflowed sectors

- In NS-sector: Use the one fermion to lower the $\mathfrak{sl}(2,\mathbb{R})$-spin.

$$j = -\frac{1}{2} + \sqrt{\frac{1}{4} + k\left(\frac{j^+(j^++1)}{k^+} + \frac{j^-(j^-+1)}{k^-}\right)}\ .$$

- Compare with the BPS bound

$$j \geq \frac{k^+j^- + k^-j^+}{k^+ + k^-} + \frac{(j^+ - j^-)^2}{k^+ + k^-}\ .$$

- The BPS bound is saturated only for $j^+ = j^-$.

Complete worldsheet BPS spectrum

- Spectrally flowed (long string) sectors contribute more BPS states.
- Complete BPS spectrum: [LE, Gaberdiel, Gopakumar, Li '17]:

$$\bigoplus_{j\in\frac{1}{2}\mathbb{Z}\backslash\left(\frac{1}{2}\lfloor k\mathbb{Z}\rfloor\backslash\frac{1}{2}\mathrm{lcm}(k^+,k^-)\mathbb{Z}\right)}^{\frac{c}{12}} [j,j,u=0]_S \otimes \overline{[j,j,u=0]_S}\ .$$

- Taking into account the missing chiral primaries:

$$\bigoplus_{j\in\frac{1}{2}\mathbb{Z}}^{\frac{c}{12}} [j,j,u=0]_S \otimes \overline{[j,j,u=0]_S}\ .$$

- This should be matched with supergravity, which corresponds to the regime $k \to \infty$.

Sugra BPS spectrum

- Structure of the result:

$$h = +\frac{1}{2} + \sqrt{\frac{1}{4} + k\left(\frac{j^+(j^++1)}{k^+} + \frac{j^-(j^-+1)}{k^-}\right)}$$

wrong sign!

$$\Big\updownarrow\, m^2 = h(h-1)$$

$$m^2 = \underbrace{\frac{j^+(j^++1)}{k^+} + \frac{j^-(j^-+1)}{k^-}}_{\text{Laplacian on } \mathrm{S}^3 \times \mathrm{S}^3}$$

$\Rightarrow$ Looks like a KK-reduction, suggests that same conclusion should also hold true in supergravity.

Sugra BPS spectrum

- To fix the sign and to confirm this, we performed an explicit KK-reduction of 9d supergravity on $\mathrm{S}^3 \times \mathrm{S}^3$.
- Result:
  - The spectrum arranges itself into $\mathrm{D}(2,1|\alpha)$-multiplets.
  - Confirms the string theory result [LE, Gaberdiel, Gopakumar, Li '17]:

The only BPS states have $j^+ = j^-$.

Comparison with de Boer et al.

- Gives an elegant resolution of the previous puzzle: Sugra has no BPS states for $j^+ \neq j^-$
  $\Rightarrow$ There is no need for miraculous quantum corrections in string theory.
- Previously excluded candidates as dual CFTs are again "back in the game".

Matching of the BPS spectrum with the dual CFT

- At generic points in the moduli space and in the large spin limit, the BPS spectrum was independently analysed by [Baggio, Ohlsson Sax, Sfondrini, Stefanski, Torielli '17] using integrability techniques.
  - $\Rightarrow$ This suggests that the full BPS spectrum is the same everywhere in moduli space, i.e., also the dual CFT should just have BPS states with $j^+ = j^-$.

Brane construction

- Wrap a special Lagrangian S^3:

	0	1	2	3	4	5	6	7	8	9
Q_5^+ D5 branes	$\times$					$\times$	$\times$	$\times$	$\times$	$\times$
Q_1 D1 branes	$\times$					$\times$	$\sim$	$\sim$	$\sim$	$\sim$
Q_5^- D5 fluxes							$\circ$	$\circ$	$\circ$	

- This configuration gives the near-horizon geometry $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$.

Worldvolume theory

- The low-energy theory on the 6-dimensional D5-brane worldvolume is a 3-dimensional $\mathrm{U}(Q_5^+)$ Chern-Simons theory living in 059.
- Near-horizon limit: overall $\mathrm{U}(1)$ decouples.
  $\Rightarrow$ Subtle issue: End up with $\mathrm{SU}(Q_5^+)$ or $\mathrm{SU}(Q_5^+)/\mathbb{Z}_{Q_5^+}$.
- [Witten '99]: The latter is anomalous unless $\mathrm{Q}_5^+ \,|\, \mathrm{Q}_5^-$.
  $\Rightarrow$ Brane picture is not consistent unless $\mathrm{Q}_5^+ \,|\, \mathrm{Q}_5^-$.

Instanton moduli space

- The dual CFT should be identified with the low-energy theory living on the D1-D5 brane intersection.
- D1-branes can be viewed as instantons in the D5-branes, living on the transverse direction of the D1-branes: $\mathrm{S}^3 \times \mathrm{S}^1$.
- The dual CFT is the supersymmetric σ-model on the moduli space $\mathcal{M}_{Q_1,Q_5^+,Q_5^-}$ of Q_1 instantons of $\mathrm{SU}(Q_5^+)$ on $\mathrm{S}^3_{Q_5^- - Q_5^+} \times \mathrm{S}^1$.

Instanton moduli space

- For $Q_5^+=1$, the moduli space is easy to determine:

$$\mathcal{M}_{Q_1,1,Q_5^-} \cong \mathrm{Sym}^{Q_1}(\mathrm{S}^3_{Q_5^- -1} \times \mathrm{S}^1)\ .$$

- In general hard, but when $Q_5^+ \,|\, Q_5^-$ there is a natural guess:

$$\mathcal{M}_{Q_1,Q_5^+,Q_5^-} \cong \mathrm{Sym}^{Q_1 Q_5^+}(\mathrm{S}^3_{Q_5^-/Q_5^+ -1} \times \mathrm{S}^1)\ .$$

- Supersymmetric σ-models on these spaces support the large $\mathcal{N}=4$ algebra with the correct levels.

The theory $\mathcal{S}_\kappa$

- $\mathcal{S}_\kappa$ is the $\mathcal{N}=1$ WZW model on $\mathrm{S}^3\times\mathrm{S}^1\cong\mathrm{SU}(2)\times\mathrm{U}(1)$ [Sevrin, Troost, van Proeyen '88].
- κ is the level of the bosonic $\mathfrak{su}(2)$-algebra.
- Fermions generate the current algebra $\mathfrak{su}(2)_1\oplus\mathfrak{su}(2)_1$.
- Theory supports the A_γ algebra with levels $k^+=1$, $k^-=\kappa+1$.

The symmetric orbifold of $\mathcal{S}_\kappa$

- Moduli spaces were of the form $\mathrm{Sym}^N(\mathcal{S}_\kappa)$.
- This supports the large $\mathcal{N}=4$ algebra with levels $(N, N(\kappa+1))$.
- The same theories were considered before in [Elitzur, Feinerman, Giveon, Tsabar '98; Gukov, Martinec, Moore, Strominger '04], but discarded because of the wrong BPS spectrum.

The BPS spectrum of the symmetric orbifold of $\mathcal{S}_\kappa$ and comparison

- Complete low-lying BPS spectrum:
  [Gukov, Martinec, Moore, Strominger '04; LE, Gaberdiel, Li '17]

$$\bigoplus_{j=0}^{\frac{c}{12}} [j,j,u=0]_S \otimes \overline{[j,j,u=0]_S} \ .$$

- Perfect agreement with the string theory prediction!

Conclusions

- We have shown that the BPS spectrum of string theory and sugra on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ agrees and contains only states with $j^+ = j^-$.
- We analyzed the BPS spectrum of the symmetric product of the theory $\mathcal{S}_\kappa$ and found precisely the same BPS spectrum.
- Very convincing evidence in favour of the duality!

Thank you!