Quantum aspects of exceptional field theory

Axel Kleinschmidt (Albert Einstein Institute, Potsdam)

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Joint work with Guillaume Bossard

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`[and upcoming preprint]`

Motivation and goal

Supergravity is the (ultra-)low energy effective action of string or M-theory. Certainly not the full story since theory contains many more states: Winding, wrapping, . . .

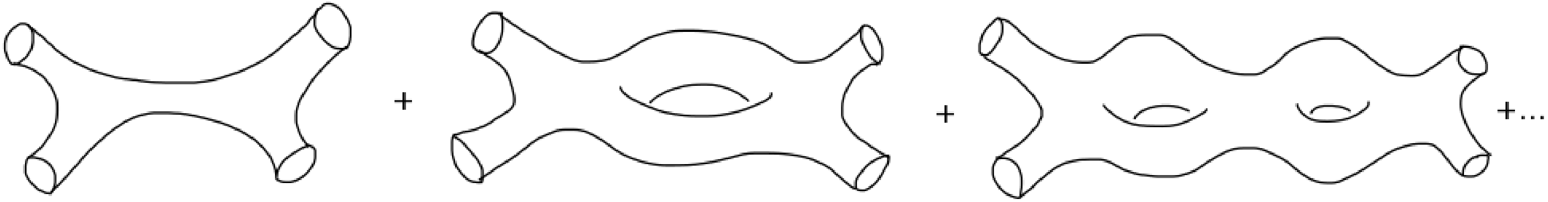
Aim: Study M-theory effective action beyond supergravity, in particular higher derivative corrections in $D = 11 - d$ dimensions with T^d

Tools

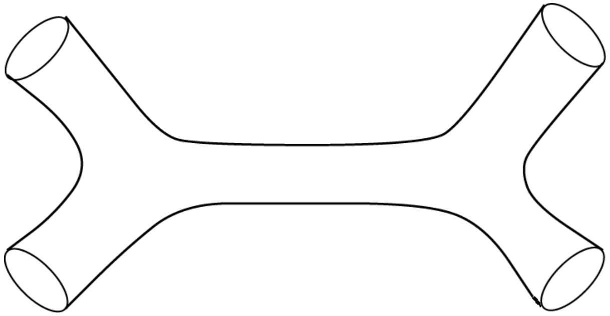
- Hidden symmetries $E_d(\mathbb{R})$ and U-duality $E_d(\mathbb{Z})$
- Exceptional field theory structures
- Relation between field theory loops and BPS-protected string corrections
- Automorphic forms

String theory scattering amplitudes

Low-energy limit of perturbative amplitudes



E.g. four gravitons (in $D = 10$ type II) at tree level



string scale

$\alpha' = \ell_s^2$

$$\mathcal{A}(s,t,u) = g_s^{-2}\frac{4}{stu}\frac{\Gamma(1-\alpha's)\Gamma(1-\alpha't)\Gamma(1-\alpha'u)}{\Gamma(1+\alpha's)\Gamma(1+\alpha't)\Gamma(1+\alpha'u)}\mathcal{R}^4$$

$$= 4g_s^{-2}\mathcal{R}^4\left[\frac{1}{stu} + (\alpha')^3\cdot 2\zeta(3) + (\alpha')^5(s^2+t^2+u^2)\cdot\zeta(5) + \ldots\right]$$

dimensionful → massive string states

Low energy effective action

Higher order α' contributions to $\mathcal{A}$ $\Longleftrightarrow$ higher derivative terms in low energy effective action

Type IIB

$$\Phi = \chi + ie^{-\phi}, \quad e^{\phi} = g_{\mathrm{s}}$$

$$\mathcal{E}^{10B}_{(0,0)}(\Phi) = 2\zeta(3)e^{-3\phi/2} + \ldots$$

$$\begin{aligned} e^{-1}\mathcal{L} = \ell^{2-D}&\left[R - \frac{1}{2}G_{IJ}(\Phi)\partial\Phi^I\partial\Phi^J + \ldots\right] \\ &+ \ell^{8-D}\left[\mathcal{E}^D_{(0,0)}(\Phi)R^4 + \ldots\right] + \ell^{12-D}\left[\mathcal{E}^D_{(1,0)}(\Phi)\nabla^4R^4 + \ldots\right] \\ &+ \ell^{14-D}\left[\mathcal{E}^D_{(0,1)}(\Phi)\nabla^6R^4 + \ldots\right] + \ldots \end{aligned}$$

Scalar moduli fields Φ belong to quantum moduli space

$$E_d(\mathbb{Z})\backslash E_d(\mathbb{R})/K(E_d) \qquad (d = 11 - D)$$

$K(E_d)$: max. compact subgroup of CJ symmetry $E_d(\mathbb{R})$ [Cremmer, Julia]

$E_d(\mathbb{Z})$: Discrete U-duality [Hull, Townsend]

Higher derivative corrections

Coefficient functions $\mathcal{E}^D_{(p,q)}(\Phi)(s^2+t^2+u^2)^p(s^3+t^3+u^3)^q$

- satisfy $\mathcal{E}^D_{(p,q)}(\gamma\Phi k) = \mathcal{E}^D_{(p,q)}(\Phi)$ for $\gamma \in E_d(\mathbb{Z})$, $k \in K(E_d)$
- A lot known for lowest $\mathcal{E}^D_{(p,q)}$ from supersymmetry and internal consistency [Green, Gutperle, Kiritsis, Miller, Obers, Pioline, Russo, Sethi, Vanhove,...]

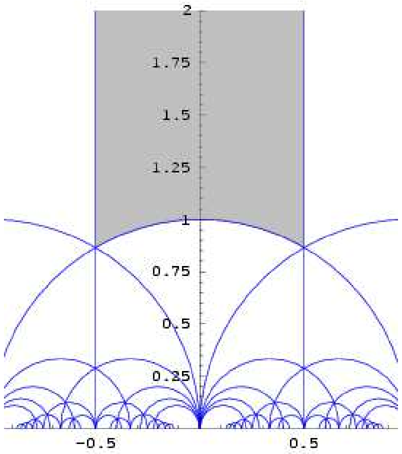
$\mathcal{E}^D_{(0,0)}$	R^4 correction	$\left(\Delta - \lambda^D_{(0,0)}\right)\mathcal{E}^D_{(0,0)} = 0$
$\mathcal{E}^D_{(1,0)}$	$\nabla^4 R^4$ correction	$\left(\Delta - \lambda^D_{(1,0)}\right)\mathcal{E}^D_{(1,0)} = 0$
$\mathcal{E}^D_{(0,1)}$	$\nabla^6 R^4$ correction	$\left(\Delta - \lambda^D_{(0,1)}\right)\mathcal{E}^D_{(0,1)} = -\left(\mathcal{E}^D_{(0,0)}\right)^2$

- Contain perturbative and non-perturbative information

Example: Type IIB

Hidden symmetry $SL(2,\mathbb{R})$; U-duality $SL(2,\mathbb{Z})$. Scalars $\Phi \equiv \tau \equiv \tau_1 + i\tau_2 = \chi + ie^{-\phi}$. Define

$$E_{[s]}(\tau) = \sum_{\substack{c,d\in\mathbb{Z}\\(c,d)\neq(0,0)}} \frac{\tau_2^s}{|c\tau+d|^{2s}} = 2\zeta(2s) \sum_{\gamma\in B(\mathbb{Z})\backslash SL(2,\mathbb{Z})} [\mathrm{Im}(\gamma\cdot\tau)]^s$$



Note: $\mathrm{Im}\,\tau = \tau_2 = e^{-\phi} = g_s^{-1}$. Rewriting is sum over U-duality orbits.

$E_{[s]}(\tau)$ is a non-holomorphic Eisenstein series.

$\mathcal{E}^{10B}_{(0,0)} = E_{[3/2]}$ R^4 correction [Green, Gutperle]

$\mathcal{E}^{10B}_{(1,0)} = \frac{1}{2}E_{[5/2]}$ $\nabla^4 R^4$ correction [Green, Vanhove]

$\mathcal{E}^{10B}_{(0,1)}$ $\nabla^6 R^4$ correction. Not Eisenstein, explicit form by [Green, Miller, Vanhove]

Relation to field theory loops

Four-graviton process is very special. Low order corrections R^4, $\nabla^4 R^4$ and $\nabla^6 R^4$ enjoy (some) SUSY protection.

$\Longrightarrow$ Only BPS states contribute; no other M-theory states visible at low energies

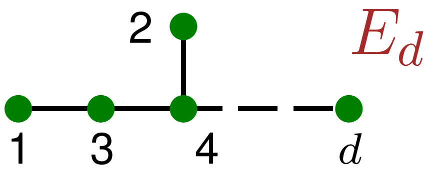
Used by [Green, Vanhove; de Wit, Lüst] to perform supergravity loop calculations including BPS momentum (and winding) states to find $\mathcal{E}^{10}_{(0,0)}$ and $\mathcal{E}^{10}_{(1,0)}$ for type IIA/IIB.

Aim: Investigate $\mathcal{E}^{D}_{(p,q)}$ for $D < 10$ by similar methods in manifestly U-duality covariant formalism

$\Longrightarrow$ Exceptional field theory loops

Exceptional field theory

[de Wit, Nicolai; Hull; Waldram et al.; Hohm, Samtleben; West; ...]



Formalism to make hidden $E_d(\mathbb{R})$ (continuous!) manifest. Combine diffeomorphisms with gauge transformations.

Consider extended space-time

$$\mathcal{M}^D \times \mathcal{M}^{d(\alpha_d)}$$

Coordinates x^μ, y^M with $\mu = 0, ..., D-1$ and $M = 1, ..., d(\alpha_d)$.

$d(\alpha_d) = \dim \mathbf{R}_{\alpha_d}$: hst. weight rep. on node α_d

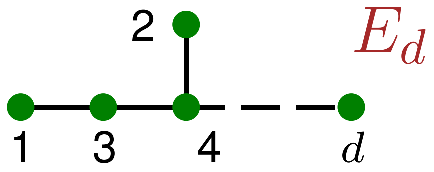
$\mathbf{R}_{\alpha_d}$ decomposes under ‘gravity line’ $GL(d, \mathbb{R}) \subset E_d(\mathbb{R})$

$$y^M = (y^m, y_{[mn]}, y_{[m_1 ... m_5]}, \dots) \qquad (m, n, ... = 1, ..., d)$$

KK momenta M2 wrappings

Generalised coordinates $y^M \in \mathbf{R}_{\alpha_d}$

E_d	$\mathbf{R}_{\alpha_d}$	$\mathbf{R}_{\alpha_1}$
$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$
E_6	$\mathbf{27}$	$\overline{\mathbf{27}}$
E_7	$\mathbf{56}$	$\mathbf{133}$
E_8	$\mathbf{248}$	$\mathbf{3875} \oplus \mathbf{1}$



Generalised coordinates y^M have to obey section constraint

$$\left.\frac{\partial A}{\partial y^M}\,\frac{\partial B}{\partial y^N}\right|_{\mathbf{R}_{\alpha_1}} = 0$$

for any two fields $A(x^\mu, y^M)$, $B(x^\mu, y^M)$. LHS belongs to

$$\mathbf{R}_{\alpha_d} \otimes \mathbf{R}_{\alpha_d} = \mathbf{R}_{\alpha_1} \oplus \dots$$

Section constraint

$$\left.\frac{\partial A}{\partial y^M}\frac{\partial B}{\partial y^N}\right|_{\mathbf{R}_{\alpha_1}} = 0$$

Possible solution: ‘M-theory’: $y^M = (y^m, \cancel{y_{mn}}, \cancel{y_{m_1\ldots m_5}}, \ldots)$

Alternative: Type IIB `[Blair, Malek, Park]`. These are the only two vector space solutions `[BK]`

Here: ‘Toroidal’ extended space for y^M. Conjugate momenta are quantised charges

$$\Gamma_M = (n_m, n^{m_1 m_2}, n^{n_1\ldots n_5}, \ldots) \in \mathbb{Z}^{d(\alpha_d)}$$

Section constraint becomes $\frac{1}{2}$-BPS constraint on charges

$$\left.\Gamma \times \tilde{\Gamma}\right|_{\mathbf{R}_{\alpha_1}} = 0 \qquad \Rightarrow \qquad \text{write } \Gamma \times \tilde{\Gamma} = 0 \text{ for brevity}$$

Amplitudes in ExFT (I)

Exceptional field theory is mainly a classical theory. QFT treatment complicated due to section constraint.

Consider kinetic term in ExFT $\partial\phi\,\partial\phi$

$$\int_{\mathbb{R}^{11-d}} dx \int_{T^{d(\alpha_d)}/\text{section}} dy\, \nabla\phi(x,y)\cdot\nabla\phi(x,y)$$

y-Fourier expand $\phi(x,y) = \sum_{\Gamma\in\mathbb{Z}^{d(\alpha_d)}} \phi_\Gamma(x)e^{i\ell^{-1}\Gamma\cdot y}$

$$\sum_{\substack{\Gamma\in\mathbb{Z}^{d(\alpha_d)}\\ \Gamma\times\Gamma=0}} \int_{\mathbb{R}^{11-d}} dx \left[\partial_\mu\phi_\Gamma\partial^\mu\phi_{-\Gamma} \underbrace{- \ell^{-2}\left\langle Z(\Gamma)|Z(\Gamma)\right\rangle}_{\text{charge dependent mass}} \phi_\Gamma\phi_{-\Gamma}\right]$$

Section constraint on y^M turned into constraint on charges

Amplitudes in ExFT (II)

$\langle Z(\Gamma)|Z(\Gamma)\rangle$ is BPS-mass and depends on scalar moduli Φ

Momenta in propagators are effectively shifted by Kaluza–Klein mass

$$p^2 \quad \longrightarrow \quad p^2 + \ell^{-2}|Z(\Gamma)|^2$$

and section constraint $\Gamma_i \times \Gamma_j = 0$ at every vertex.

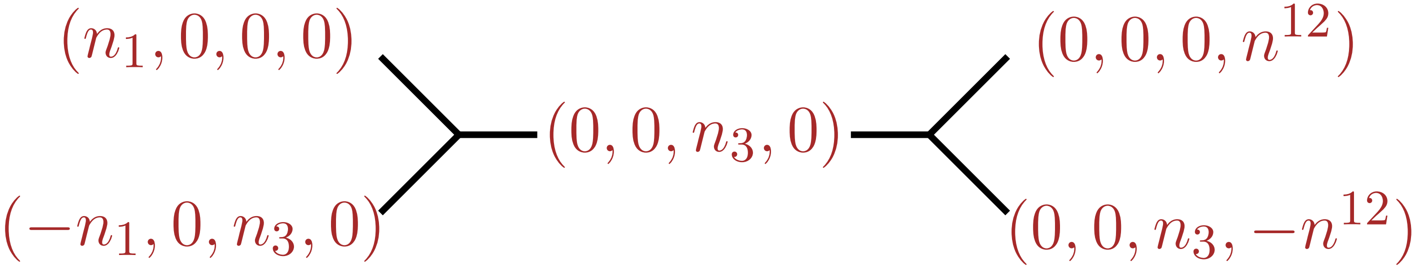
$\Rightarrow$ Use this to compute exceptional field theory amplitudes.

Amplitudes in ExFT (III)

Loop charge sum $\sum_{\substack{\Gamma_l\in\mathbb{Z}^{d(\alpha_d)}\\ \Gamma_{\langle i}\times\Gamma_{j\rangle}=0}}$ affects only adjacent charges.

Can violate section constraint globally!

E.g. $\Gamma=(n_1,n_2,n_3,n^{12})$ on T^3



$(n_1,0,0,0)$

$(-n_1,0,n_3,0)$

$(0,0,n_3,0)$

$(0,0,0,n^{12})$

$(0,0,n_3,-n^{12})$

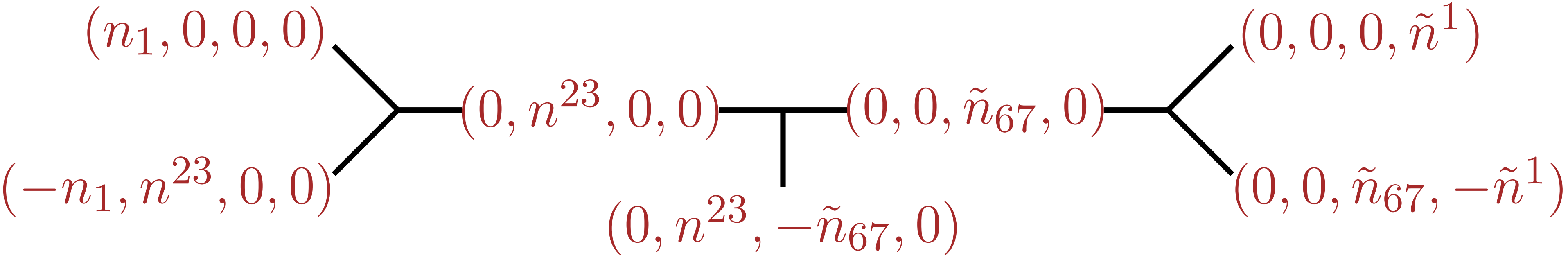
Scattering of two $D=11$ KK-states into two IIB KK-states.

$\Rightarrow$ T-fold transition

Makes sense in ExFT but not in a fixed duality frame (solution to section constraint)

Amplitudes in ExFT (IV)

Other example: $\Gamma = (n_1, n^{23}, n^{12345}, n^{1,1234567})$ on T^7



$\Rightarrow$ S-fold transition

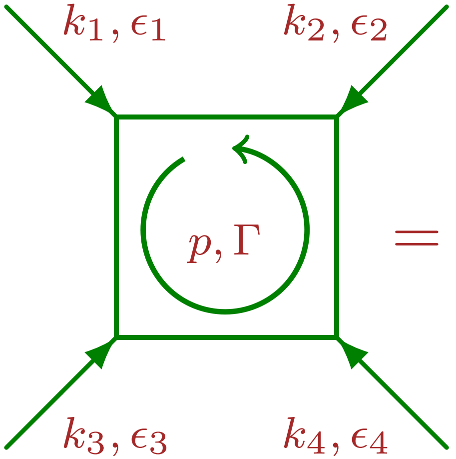
Again makes sense in ExFT but not in a fixed duality frame

Can show that up to two loops: No such complications

Next: Calculate $L = 1$ and $L = 2$ assuming reduction to scalar diagrams as in [Bern et al.; Green, Vanhove]

One-loop in ExFT (I)

Four-graviton amplitude reduces to scalar box



$$= \underbrace{\left[\frac{i\kappa^2}{2} t_8 t_8 \prod_{A=1}^{4} k_A R(k_A, e_A)\right]}_{\mathcal{R}^4} A^{\text{1-loop}}(k_1, k_2, k_3, k_4)$$

Pull out kinematic part

$$A^{\text{1-loop}}(k_1, k_2, k_3, k_4) = \kappa^2 \int \frac{d^{11-d}p}{(2\pi)^{11-d}} \sum_{\substack{\Gamma \in \mathbb{Z}^{d(\alpha_d)} \\ \Gamma \times \Gamma = 0}} \frac{1}{((p-k_1)^2 + \ell^{-2}|Z|^2)}$$

$$\times \frac{1}{(p^2 + \ell^{-2}|Z|^2)((p-k_1-k_2)^2 + \ell^{-2}|Z|^2)((p+k_4)^2 + \ell^{-2}|Z|^2)}$$

$$+ \text{perms.}$$

One-loop in ExFT (II)

$\Gamma = 0$ term corresponds to SUGRA in $D = 11 - d$; usual log threshold contribution $\Rightarrow$ remove for analytic eff. action

Treat loop integral over $d^{11-d}p$ with usual Schwinger and Feynman techniques:

$$A^{\text{1-loop}}(k_1, k_2, k_3, k_4) = 4\pi\ell^{9-d} \sum_{\substack{\Gamma \in \mathbb{Z}_*^{d(\alpha_d)} \\ \Gamma\times\Gamma=0}} \int_0^\infty \frac{dv}{v^{\frac{d-1}{2}}} \int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3$$

$$\times \exp\left[\frac{\pi}{v}\left((1-x_1)(x_2-x_3)s + x_3(x_1-x_2)t - \ell^{-2}|Z|^2\right)\right] + \text{perms.}$$

Low energy from expanding in Mandelstam variables

$$s = -(k_1+k_2)^2, \quad t = -(k_1+k_4)^2, \quad u = -(k_1+k_3)^2.$$

Low energy correction terms

For lowest two orders

R^4 correction

$$A^{\text{1-loop}}(s,t,u) = \pi\ell^6\left(\xi(d-3)E_{\alpha_d,\frac{d-3}{2}} + \frac{\pi^2\ell^4(s^2+t^2+u^2)}{720}\xi(d+1)E_{\alpha_d,\frac{d+1}{2}} + \dots\right)$$

$\nabla^4 R^4$ correction

Notation

- $\xi(s) = \pi^{-s/2}\Gamma(s/2)\zeta(s)$ [completed Riemann zeta]
- $E_{\alpha_d,s} = \frac{1}{2\zeta(2s)}\sum_{\substack{\Gamma\neq 0\\ \Gamma\times\Gamma=0}} |Z(\Gamma)|^{-2s}$ [Eisenstein series]

  Restricted lattice sum rewritable as single U-duality orbit!

Interpretation

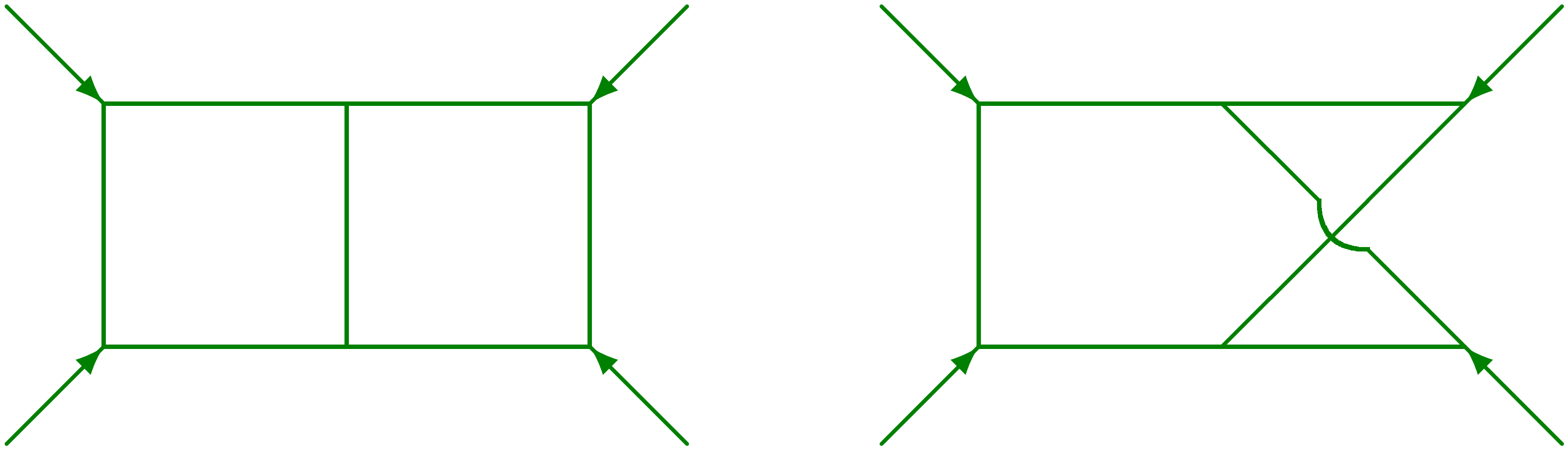
Expressions converge for $\nabla^{2k}R^4$ term on T^d when $k > \frac{3-d}{2}$

- For $k = 0$ (R^4) and $d > 3$ ($D < 8$) find after using Langlands' functional relation the correct correction function $\mathcal{E}^D_{(0,0)}$ (including numerical coefficient).
  For $d = 3$ one has to regularise; related to known one-loop R^4 divergence in SUGRA.
- For $k = 2$ ($\nabla^4 R^4$) expressions converge. For $d \leq 5$ one obtains only one supersymmetric invariant of [Bossard, Verschinin]; for $7 \leq d < 5$ full (unique) invariant with correct coefficient. Should be renormalised. For $d = 8$ ancestor of 3-loop divergence.

Expressions also ok for $d > 8$; Kac–Moody case [Fleig, AK]

Two loops in ExFT (I)

[Bern et al.]: combination of planar and non-planar scalar diagram at $L = 2$



After a few pages of calculation

$$A^{\text{2-loop}}(s,t,u) \sim \ell^6 \sum_{\substack{\Gamma_1,\Gamma_2 \\ \Gamma_i \times \Gamma_j = 0}} \int_0^\infty \frac{d^3\Omega}{(\det \Omega)^{\frac{7-d}{2}}} e^{-\Omega^{ij}\langle Z(\Gamma_i)|Z(\Gamma_j)\rangle}$$

$\nabla^4 R^4$ correction

$\nabla^6 R^4$

$$\times \left[\frac{\ell^4(s^2+t^2+u^2)}{6} + \frac{\ell^6(s^3+t^3+u^3)}{72}\Phi_{(0,1)}(\Omega) + \dots \right]$$

Two loops in ExFT (II)

After a few pages of calculation (⟶ details)

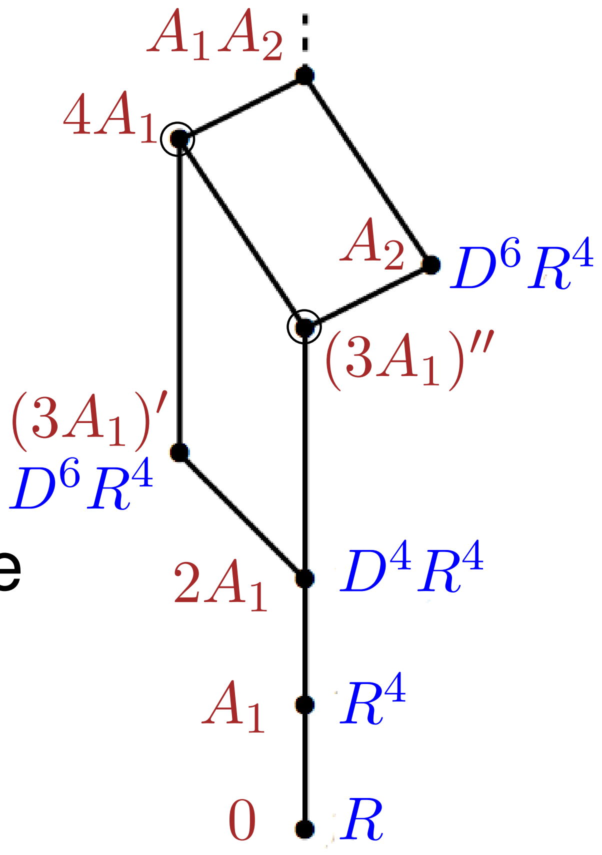
$$A^{\text{2-loop},\nabla^4 R^4}(s,t,u) = 8\pi\ell^{10}\xi(d-4)\xi(d-5)E_{\alpha_{d-1},\frac{d-4}{2}}$$

- This gives the correct function and coefficient for $3 \leq d \leq 8$ with the right coefficient. Case $d = 5$ ($D = 6$) trickier due to IR divergences
- Depends on non-trivial functional identities for Eisenstein series
- Certain doubling of contributions from one loop and two loops. Correct if one-loop result renormalised
- Can be extended to ⟶ three loops

Summary and outlook

- Explicitly evaluated loop amplitudes in ExFT
- Reproduced known $\mathcal{E}_{(p,q)}$ in manifestly U-duality covariant form
- Useful tools for dealing with section constraint
- Analysis of differential equation for higher order corrections and their wavefront sets, relation to nilpotent orbits and non-perturbative instanton effects

Thank you for your attention!



Hasse diagram for $E_{7(7)}$