Logarithmic Corrections to Black Hole Entropy

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Corrections to Black Hole Entropy

- Agenda: ***study quantum corrections*** to black hole entropy ***far from the BPS limit*** .

- Methods:

  - ***Explicit computation***.

  - Study ***symmetries in effective quantum field theory.***

- Conventional wisdom: quantum corrections far from the BPS limit are large and complicated.

- Results:

  - Generally the corrections are complicated, as expected.

  - However, they greatly simplify in some ***environments.***

Environmental Dependence

- Example: consider a ***Kerr-Newman black hole*** as a ***solution to the Einstein-Maxwell theory***

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- Now add a field that appears only quadratically in the action (such as a fermion ψ.)

- We can study the "same" solution (unchanged ***geometry and gauge field***): assume the additional field vanishes $\psi = 0$.

- Environmental dependence: ***quantum corrections depend on such additional fields*** (for example, they run in loops).

- The upshot: ***Kerr-Newman black holes simplify in an environment with $\mathcal{N} \geq 2$ supersymmetry***.

This Talk

- ***Embedding*** of Kerr-Newman black holes into theories with $\mathcal{N} \geq 2$ SUSY.

- ***Quantum corrections*** to black hole entropy: explicit computation.

- A ***non-renormalization theorem.***

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N=2 SUGRA

- The baseline: $\mathcal{N}=2$ ***minimal*** SUGRA has bosonic part identical to Einstein-Maxwell theory.

- Any bosonic solution remains a solution after the two gravitini are added because fermions can be consistently set to zero.

- More challenging: ***couple to n_V N=2 vector multiplets***:

$$\mathcal{L} = \frac{1}{2\kappa^2} R - g_{\alpha\bar{\beta}} \nabla^\mu z^\alpha \nabla_\mu z^{\bar{\beta}} + \frac{1}{2} \mathrm{Im} \left[\mathcal{N}_{IJ} F^{+I}_{\mu\nu} F^{+\mu\nu J} \right]$$

- Comments:
  - Complex scalar fields in vector multiplets: z^α, $\alpha = 1, \ldots, n_V$.
  - Vector fields A^I_μ include the graviphoton so $I = 0, \ldots, n_V$ (one more value).
  - Kähler metric $g_{\alpha\bar{\beta}}$ and ***vector couplings $\mathcal{N}_{IJ}$ depend on scalars*** as specified by special geometry.

Adding Scalars to Kerr-Newman

- Kerr-Newman does not have scalars so to maintain the "same" solution we take the $\mathcal{N} = 2$ ***scalars constant***.

- An obstacle: ***generally the vector fields source the scalars*** so they cannot be constant.

- Solution: first specify the projective coordinates X^I for the scalars, then specify the $\mathcal{N} = 2$ vectors ($F^I_{\mu\nu}$) in terms of the Einstein-Maxwell field strength ($F_{\mu\nu}$) and the scalars as:

$$F^{+I}_{\mu\nu} = X^I F^+_{\mu\nu} \,.$$

- Then the sources on the scalars cancel so ***it is consistent to have constant scalars***.

General Embedding: the Upshot

- We consider ***all theories with*** $\mathcal{N} \geq 2$ ***SUSY***.
- Summarize the matter content in terms of $\mathcal{N} = 2$ fields:
  - one SUGRA multiplet
  - $\mathcal{N} - 2$ (massive) gravitini
  - n_V vector multiplets
  - n_H hyper multiplets
- This ***decomposition is useful for both BPS and non-BPS***.
- Our embedding takes the geometry unchanged, matter fields "minimal", and guarantees that all equations of motion of $\mathcal{N} \geq 2$ SUGRA are satisfied.

Quantum Corrections: Generalities

- The entropy of a large black hole allows the expansion:

$$S = \frac{A}{4G} + \frac{1}{2} D_0 \log A + \dots \ .$$

- ***Take all parameters with dimension length large***: so area $A \sim (2MG)^2$ (up to dimensionless ratios J/M^2 and Q/M).

- The logarithmic correction: $\log A \sim \log 2MG$ up to the coefficient D_0, a nontrivial function of dimensionless ratios J/M^2 and Q/M.

- The area A and the coefficient D_0 can both be ***computed from the low energy theory***: only massless fields contribute.

- They each offer an ***infrared window into the ultraviolet theory***.

Quantum Fluctuations: Strategy

- All contributions from ***quadratic fluctuations*** around the classical geometry take the schematic form

$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \, .$$

- The quantum corrections are encoded in the heat kernel

$$D(s) = \mathrm{Tr} \, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} \, .$$

- The effective action becomes

$$W = -\int_{\epsilon^2}^{\infty} \frac{ds}{2s} D(s) = -\int_{\epsilon^2}^{\infty} \frac{ds}{2s} \int d^D x K(s) \, .$$

- The leading corrections are encoded in the ***the*** s***-independent term in*** $D(s)$ (denoted D_0).

- Disclosure: zero modes are suppressed in this talk.

Aside: the Trace Anomaly

- The coefficient D_0 is closely related to the ***integrated*** trace anomaly.

- Gravity is far from conformal so there is ***no trace anomaly***.

- Classically the effective action satisfies the scaling relation

$$\left(M \frac{\partial}{\partial M} - 2 \right) \Gamma = 0$$

  so the classical on-shell action scales as $\Gamma \sim M^2 \sim A$

- We study the ***quantum effective action*** that satisfies the anomalous scaling relation

$$\left(M \frac{\partial}{\partial M} - 2 + D_0 \right) \Gamma =$$

Interactions

- ***In principle***: computations are straightforward applications of techniques from the 70's.

- But: our embedding into SUGRA gives ***nonminimal couplings***.

- For example, for fermions in $\mathcal{N} = 2$ hypermultiplets the background EM-field enters through ***Pauli couplings***

$$\mathcal{L}_{\text{hyper}} = -2\overline{\zeta}_A \gamma^\mu D_\mu \zeta^A - \frac{1}{2}\left(\overline{\zeta}^A F_{\mu\nu}\Gamma^{\mu\nu}\zeta^B \epsilon_{AB} + \text{h.c.}\right) \ .$$

- Such nonminimal couplings force us to compute some new heat kernels.

- Schematic scaling in gravity: $[D + F, D + F] \sim R + F^2$. Like Einstein's equation but unlike typical QFTs (minimal couplings).

Example

- Lagrangian for hyperfermion in the $\mathcal{N} = 2$ SUGRA multiplet:

$$\mathcal{L}_{\text{hyper}} = -2\bar{\zeta}_A \gamma^\mu D_\mu \zeta^A - \frac{1}{2}\left(\bar{\zeta}^A F_{\mu\nu}\Gamma^{\mu\nu}\zeta^B \epsilon_{AB} + \text{h.c.}\right) \ .$$

  Recall: there is Pauli coupling involving field strength $F_{\mu\nu}$.

- Heat kernel coefficient (computed using standard technology):

$$(4\pi)^2 a_4^{\text{hyper}}(x) = -\frac{1}{360}\left(-7R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 232R_{\mu\nu}R^{\mu\nu} - \frac{45}{4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{45}{4}(F^{\mu\nu}\tilde{F}_{\mu\nu})^2\right) \ .$$

- Consistent with scaling $R \sim F^2$,

- Generally there are terms of schematic form R^2, RF^2, F^4 (allowed by scaling symmetry).

Simplifications

- General form of 2nd Seeley-deWitt coefficient:

$$a_4(x) = \alpha_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \ldots$$

- After simplifications using Einstein equation, Bianchi identities, ...

$$a_4(x) = \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} E_4 \ ,$$

  Euler density

$$E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \ .$$

- Note: ***all dependence on field strength can be traded for curvature terms***.

- Final results can be expressed in terms of c, a only!

Duality: Einstein-Maxwell Theory

- ***Why*** can all explicit dependence on field strength be eliminated?
- Electromagnetic duality requires that ***four derivative terms are duality invariant*** (even though two derivative terms are not).
- A unique duality invariant tensor: $\mathcal{I}_{\mu\nu\rho\sigma} = F^{+}_{\mu\nu} F^{-}_{\rho\sigma}$
- All Lorentz invariants (eg. $\mathcal{I}_{\mu\nu\rho\sigma}\mathcal{I}^{\mu\nu\rho\sigma}$) can be recast in terms of:

$$\mathcal{I}_{(\mu\ \nu)\rho}^{\ \ \rho} = F^{+\rho}_{\mu} F^{-}_{\rho\nu} = R_{\mu\nu}$$

- Upshot: duality precludes explicit dependence on $F_{\mu\nu}$ so ***anomaly coefficients expressible in terms of geometry alone.***

Duality: $\mathcal{N}=2$ Supergravity

- Duality in $\mathcal{N}=2$ supergravity: symplectic invariance

- Embedding shows that the ***Maxwell field*** $F^{+}_{\mu\nu}$ ***is duality invariant***

$$F^{+I}_{\mu\nu} = X^I F^{+}_{\mu\nu} \ .$$

- $U(1)_R$ symmetry: $F^{+I}_{\mu\nu}$ neutral, X^I charged, so $F^{+}_{\mu\nu}$ ***is (negatively) charged***.

- Electromagnetic ***duality symmetry of Einstein-Maxwell descends from*** $U(1)_R$ ***symmetry of*** $\mathcal{N}=2$ ***supergravity***.

- Upshot: $U(1)_R$ symmetry precludes explicit dependence on $F_{\mu\nu}$, ***anomaly coefficients expressible in terms of geometry alone.***

Results: Logarithms from Bosons

- Contributions from bosons in $\mathcal{N} \geq 2$ theory:

$$c^{\text{boson}} = \frac{1}{60}(137 + 12(\mathcal{N} - 2) - 3n_V + 2n_H)$$
$$a^{\text{boson}} = \frac{1}{90}(106 + 31(\mathcal{N} - 2) + n_V + n_H)$$

- The bosons in the n_H hyper multiplets and $\mathcal{N} - 2$ gravitino multiplets are minimally coupled so these values for a, c are standard.

- The bosons in the n_V vector multiplets and the supergravity multiplet couple to the field strength so, after eliminating F^2 in favor of R, these values of a, c are nonstandard.

- For fermions the situation is reversed.

Integrals

- General form of quantum corrections to the entropy:

$$\delta S = \frac{1}{2} D_0\left(\frac{Q}{M}, \frac{J}{M^2}\right) \log A_H$$

- The ***a-term gives a universal contribution*** to D_0 (independent of black hole parameters) because

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{-g}\, E_4 = 2 \, .$$

- The ***c-term gives a complicated contribution*** to D_0:

$$\int d^4x \sqrt{-g}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} = 64\pi^2 + \frac{\pi\beta Q^4}{b^5 r_H^4 (b^2 + r_H^2)} \Big[4b^5 r_H + 2b^3 r_H^3 + 3(b^2 - r_H^2)(b^2 + r_H^2)^2 \tan^{-1}\left(\frac{b}{r_H}\right) + 3 b r_H^5 \Big] \, .$$

$b = J/M$, $r_H = M + \sqrt{M^2 - b^2}$, $\beta = 1/T$.

Logarithms from Fermions

- Contributions from bosons in $\mathcal{N} \geq 2$ theory:

$$c^{\text{boson}} = \frac{1}{60}(137 + 12(\mathcal{N} - 2) - 3n_V + 2n_H)$$
$$a^{\text{boson}} = \frac{1}{90}(106 + 31(\mathcal{N} - 2) + n_V + n_H)$$

- Contributions from fermions in $\mathcal{N} \geq 2$ theory:

$$c^{\text{fermion}} = \frac{1}{60}(-137 - 12(\mathcal{N} - 2) + 3n_V - 2n_H)$$
$$a^{\text{fermion}} = \frac{1}{360}(-589 + 41(\mathcal{N} - 2) + 11n_V - 19n_H)$$

- The c ***coefficent vanishes in*** $\mathcal{N} \geq 2$ ***theory!***

- A ***huge simplification***: Weyl^2 terms are complicated in general backgrounds.

- It is ***a surprise***: SUSY of the background $\Rightarrow$ AdS$_2 \times S^2 \Rightarrow$ $\mathrm{Weyl}^2 = 0 \Rightarrow$ vanishing ***coefficient*** of Weyl^2 not noticed.

Higher Derivative Corrections

- ***Why*** is the anomaly coefficient $c = 0$?

- Background is generally not supersymmetric so ***fluctuations are not organized in supermultiplets***.

- Background field formalism realizes symmetry explicitly: dependence on background fields respect $\mathcal{N} = 2$ supersymmetry.

- Schematic form of effective action

  $$\mathcal{L}_4 = g_W(\mathrm{Weyl}^2 + \mathrm{SUSY\ partners}) - g_E(\mathrm{Euler} + \mathrm{SUSY\ partners})$$

  Coefficients g_W, g_E are ***running couplings*** with β-function related to c, a.

Higher Derivatives and $\mathcal{N} = 2$ SUSY

- Details of the action: off-shell formalism from reduction of superconformal supersymmetry, a lot of auxiliary fields,...... (details involve hard work).

- SUSY completion of Weyl^2 has been known for a long time.

- Schematic of on-shell structure:

$$\mathrm{Weyl}^2 + \mathrm{SUSY\ partners} = E_4$$

Cartoon: there is an elaborate cancellation between gravitational terms (Weyl^2), their matter partners (F^4), and cross-terms (RF^2).

Higher Derivatives and $\mathcal{N}=2$ SUSY

- SUSY completion of E_4 was identified only in the last few years.

- Schematic of on-shell structure:

$$E_4 + \text{SUSY partners} = E_4$$

  Cartoon: the matter terms vanish on-shell.

- So: all matter terms can be eliminated in favor of geometry alone.

- And: both four-derivative $\mathcal{N}=2$ invariants reduce to the Euler invariant E_4.

- ***The anomaly $c=0$ because W^2 is inconsistent with $\mathcal{N}=2$ SUSY.***

Summary

- Logarithmic corrections to black hole entropy in $\mathcal{N} \geq 2$ SUGRA simplify greatly even when BHs preserve no SUSY.

- The coefficient is universal: it ***depends only on the theory*** (not on parameters of the black hole)

$$\delta S = \frac{1}{12}\left(23 - 11(\mathcal{N} - 2) - n_V + n_H\right) \log A_H \ .$$

- These corrections can be reproduced from microscopic theory in some BPS cases.

- The IR theory is a window into the UV theory: apparently the ***deformation (far!) off extremality is independent of coupling!***

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