Compensating strong coupling with large charge

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based on

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Compensating strong coupling with large charge

Overview

- Motivation for Large Charge Perturbation Theory

- Large-charge vacuum
- Quantum fluctuations
- Anomalous dimensions

- Summary and Outlook

Motivation

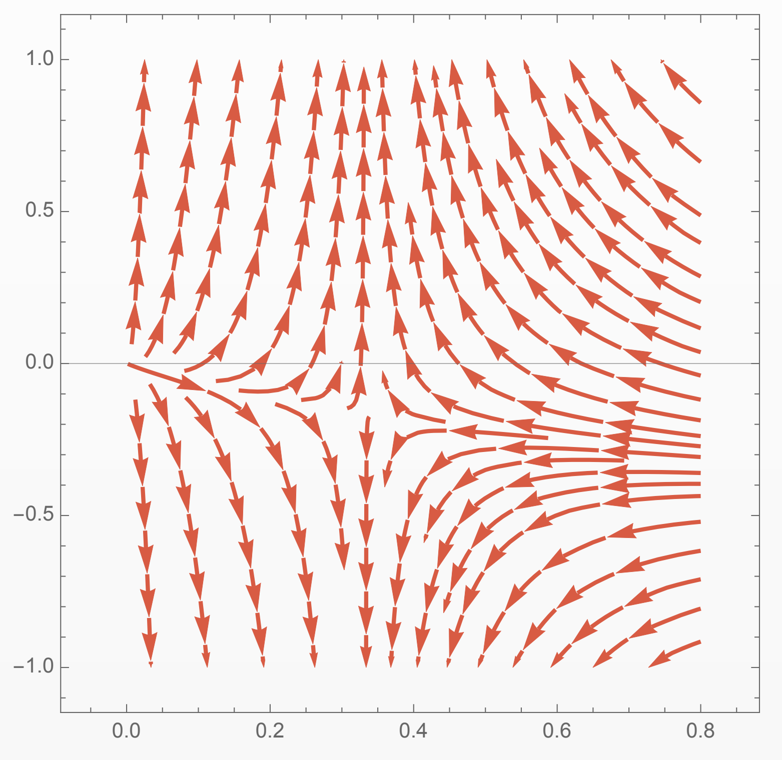
“  ”

$$\left(1 + \alpha + \mathcal{O}(\alpha^2)\right)$$

perform a series of Gaussian integrals

e.g. $\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$

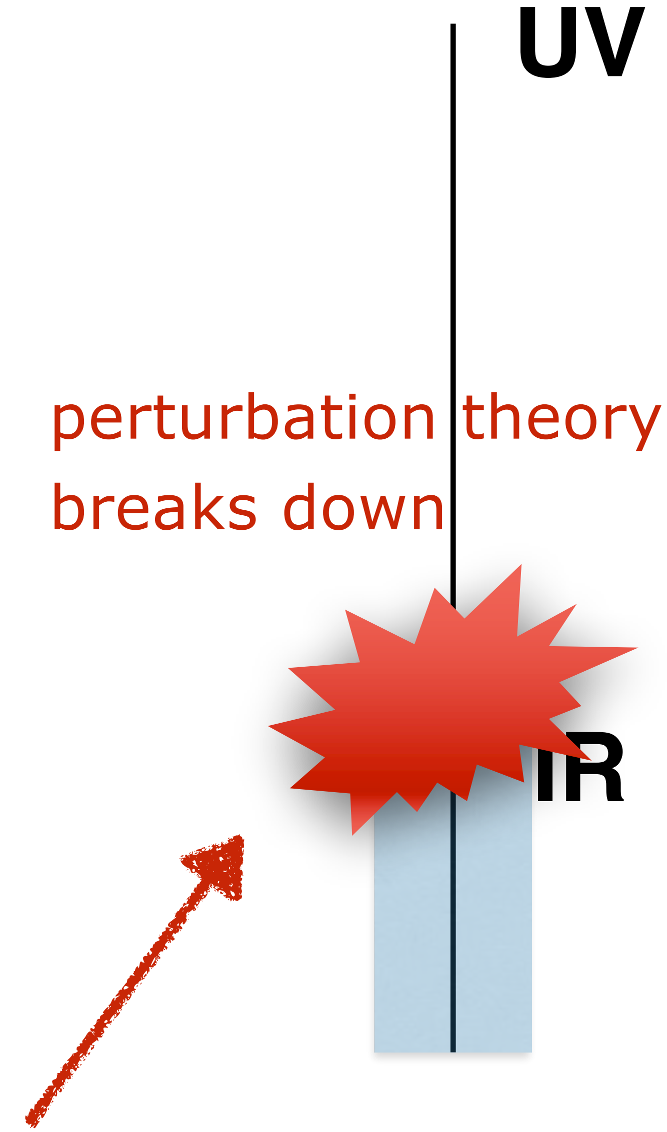
- We want to make perturbation theory useful in the analysis at the fixed points of the RG flow



Introduction: Effective actions

- But, any ***natural*** action does not generically allow perturbative calculations
  - it contains infinite many terms
  - all couplings are ***naturally*** of order one

Wilsonian action
neat,
but mostly useless

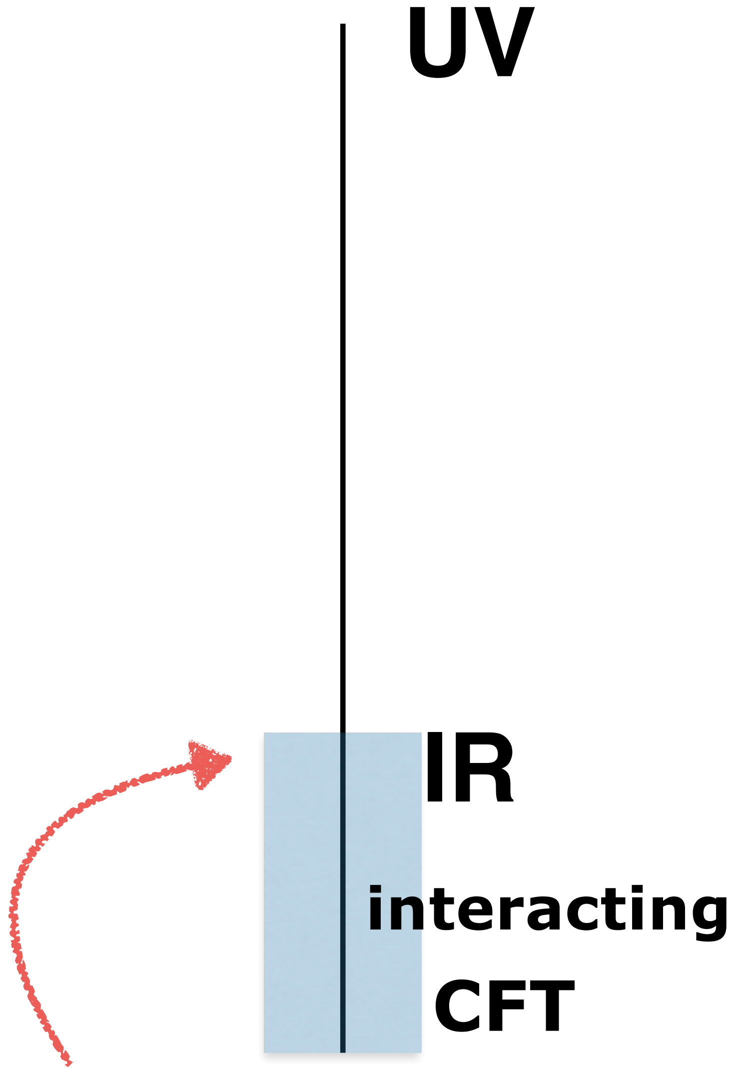


- Insight to a theory with no intrinsically small parameter
  - CFT $(D = 2)$, conformal bootstrap $(D \geq 3)$
  - access via non-perturbative methods, e.g. Lattice QFT
  - other methods...?

Introduction: The main idea

- Assumptions
  - original theory has a *global* symmetry

    —> conserved Noether charge Q
  - IR coupling λ is *finite* (or large)



- Idea
  - fix the charge $Q = Q_0$
  - take $Q_0 \gg 1$

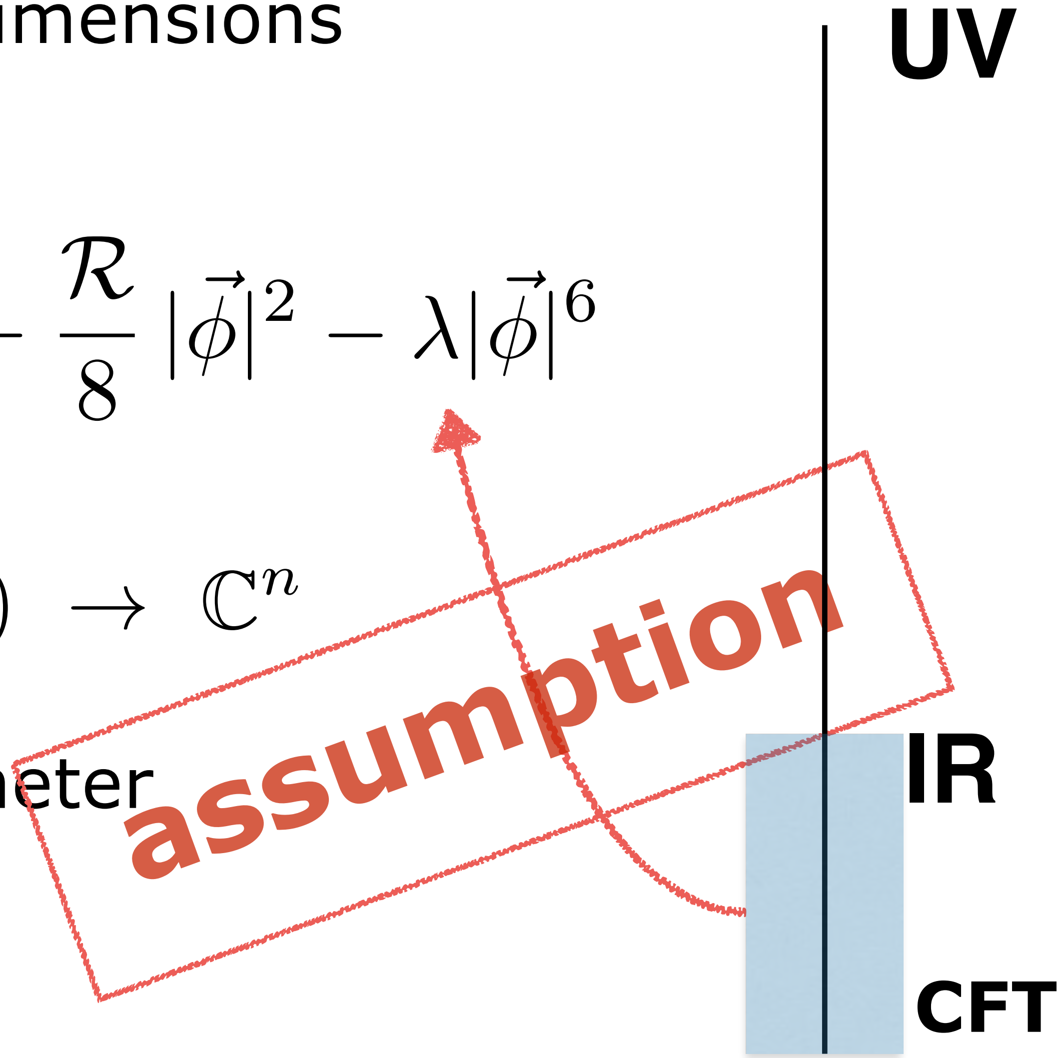
$$\Lambda \ll \sqrt[d]{\frac{Q_0}{V}}$$

vacuum $|Q_0\rangle$ + Goldstone + Q_0 - suppressed corrections

The *O(2n)* vector model

- Concrete class of models in 2+1 dimensions on the two-sphere

$$\mathcal{L} = \partial_\mu \vec{\phi}^\dagger \partial^\mu \vec{\phi} - \frac{\mathcal{R}}{8} |\vec{\phi}|^2 - \lambda |\vec{\phi}|^6$$



  - complex scalar $\vec{\phi} \, : \, \mathbb{R}_t \times S^2(r_0) \; \to \; \mathbb{C}^n$
  - $\lambda \sim \mathcal{O}(1)$ is a Wilsonian parameter
  - $\mathcal{R} = 2/r_0^2$ is the Ricci scalar

  - The action enjoys in real DOFs an $O(2n)$ symmetry

The *O(2n)* vector model

- Fix $k \leq n$ charges Q_i in the Cartan sub-algebra of $O(2n)$

$$O(2n-2k) \times \mathrm{U}(k) \rightarrow O(2n-2k) \times \mathrm{U}(k-1)$$

- True generator of time-translations: effective Hamiltonian

$$H - \mu \frac{Q}{4\pi r_0^2} \quad \text{with} \quad Q \equiv \sum_{i=1}^{k} Q_i \overset{!}{=} Q_0$$

Chemical potential

Only **sum** of charges appears

The *O(2n)* vector model

- Fix $k \leq n$ charges Q_i in the Cartan sub-algebra of $O(2n)$

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- True generator of time-translations: effective Hamiltonian

$$H - \mu \frac{Q}{4\pi r_0^2} \quad \text{with} \quad Q \equiv \sum_{i=1}^{k} Q_i \overset{!}{=} Q_0$$

- Classical ground-state: by $U(n)$ symmetry can be written as

$$\langle \vec{\phi} \rangle = (\underbrace{0, \ldots, 0}_{k-1}, v\, \mathrm{e}^{i\mu t}, \underbrace{0, \ldots, 0}_{n-k})$$

Time-dependent!

with $\mu \sim \mathcal{O}(\sqrt{Q})$

The vacuum of *O(2n)* vector model at fixed charge

$$\left(\frac{Q}{4\pi r_0^2}\right)^2 \frac{1}{2v^2} + \frac{\mathcal{R}}{16}v^2 + \frac{\lambda}{6}v^6$$

centrifugal potential

with $v \sim \mathcal{O}(\sqrt[4]{Q})$

- Classical ground-state:

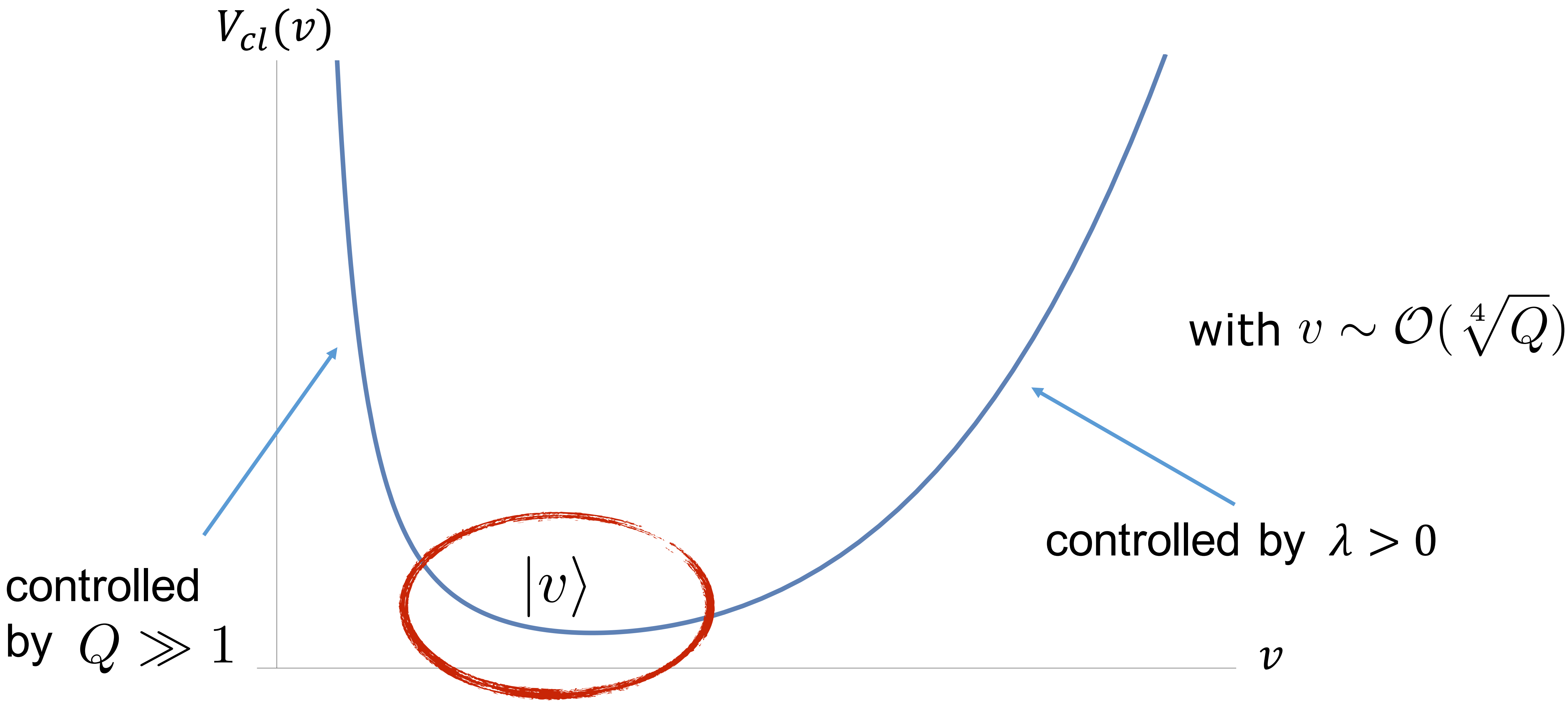
$$\langle\vec{\phi}\rangle = (\underbrace{0,\ldots,0}_{k-1}, v\,\mathrm{e}^{i\mu t}, \underbrace{0,\ldots,0}_{n-k})$$

The vacuum of *O(2n)* vector model at fixed charge

$$0 \stackrel{!}{=} \frac{\partial}{\partial v} \left(\left(\frac{Q}{4\pi r_0^2} \right)^2 \frac{1}{2v^2} + \frac{\mathcal{R}}{16} v^2 + \frac{\lambda}{6} v^6 \right)$$

centrifugal potential

- Minimize centrifugal potential



Fluctuations on top of $|Q_0\rangle$: Low-energy spectrum

- Consider fluctuations $\mathbf{p} \neq 0$ on top of

$$\langle\vec{\phi}\rangle = (\underbrace{0\,,\,\ldots\,,\,0}_{k-1},\, v\,\mathrm{e}^{i\mu t},\, \underbrace{0\,,\,\ldots\,,\,0}_{n-k})$$

non-relativistic Goldstones

$$\omega_{\mathbf{p}} = \frac{|\mathbf{p}|^2}{2\mu}$$

relativistic Goldstone

$$\omega_{\mathbf{p}} = \frac{|\mathbf{p}|}{\sqrt{2}}$$

(all other modes are massive)

up to **quadratic** order in the fluctuating fields

- Consider fluctuations $\mathbf{p} \neq 0$ on top of

$$\langle \vec{\phi} \rangle = (\underbrace{0\,,\,\ldots\,,\,0}_{k-1},\, v\,\mathrm{e}^{i\mu t},\, \underbrace{0\,,\,\ldots\,,\,0}_{n-k})$$

spontaneous breaking of time-translation invariance

superfluid phase

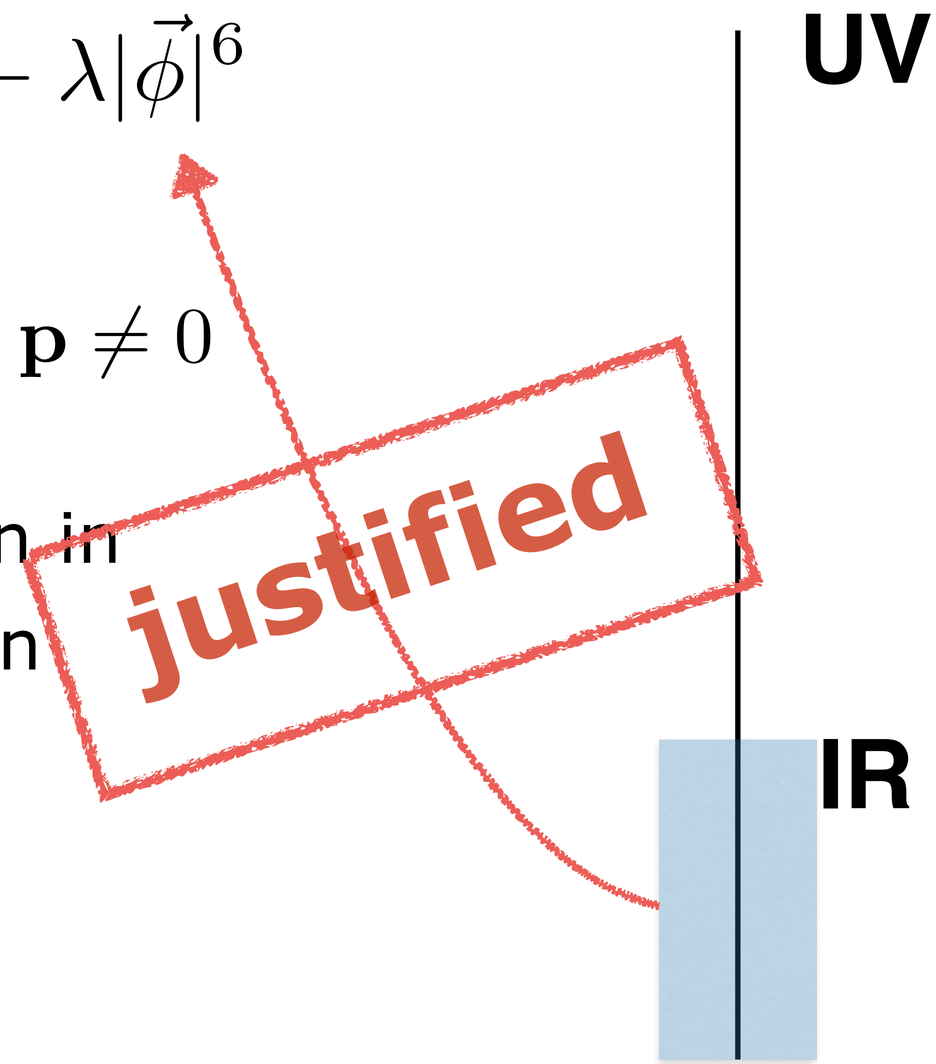
$$\omega_{\mathbf{p}} = \frac{|\mathbf{p}|}{\sqrt{2}}$$

up to **quadratic** order in the fluctuating fields

- Go quantum, in canonical formalism

$$\mathcal{L} = \partial_\mu \vec{\phi}^\dagger \partial^\mu \vec{\phi} - \frac{\mathcal{R}}{8} |\vec{\phi}|^2 - \lambda |\vec{\phi}|^6$$

  - expand in ladder operators for every $\mathbf{p} \neq 0$
  - diagonalize the quadratic Hamiltonian in the oscillators via a Bogoliubov-Valatin transformation
  - all higher orders are **suppressed** by appropriate powers of $\lambda^a / Q^b \ll 1$



$$\Lambda \ll \sqrt[d]{\frac{Q_0}{V}}$$

Computing anomalous dimensions

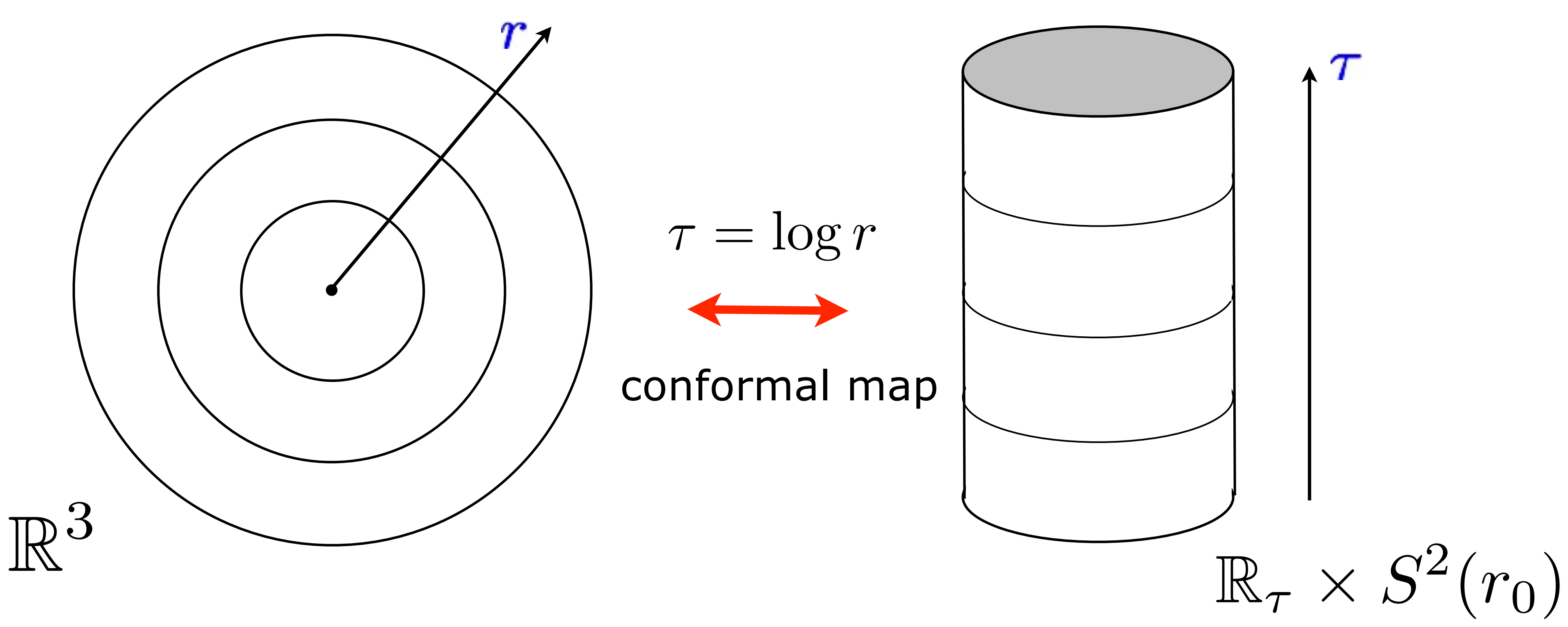
- conformal (anomalous) dimension

$$D = r_0 E$$

of an operator with charge Q on $\mathbb{R}^3$

- energy E on $\mathbb{R}_\tau \times S^2(r_0)$

of a state at fixed charge Q



Computing anomalous dimensions

$$D(Q) = \alpha_{3/2}\, Q^{3/2} + \alpha_{1/2}\, Q^{1/2} - 0.093 + \mathcal{O}(Q^{-1/2})$$

- The energy on $S^2(r_0)$ is dictated by the condensate v and the vacuum energy of the relativistic Goldstone

▸ lesson from large charge Q

$$\langle \vec{\phi} \rangle = (\underbrace{0\,, \ldots\,, 0}_{k-1}, v\, \mathrm{e}^{i\mu t}, \underbrace{0\,, \ldots\,, 0}_{n-k})$$

$$\omega_{\mathbf{p}} = \frac{|\mathbf{p}|}{\sqrt{2}}$$

Computing anomalous dimensions

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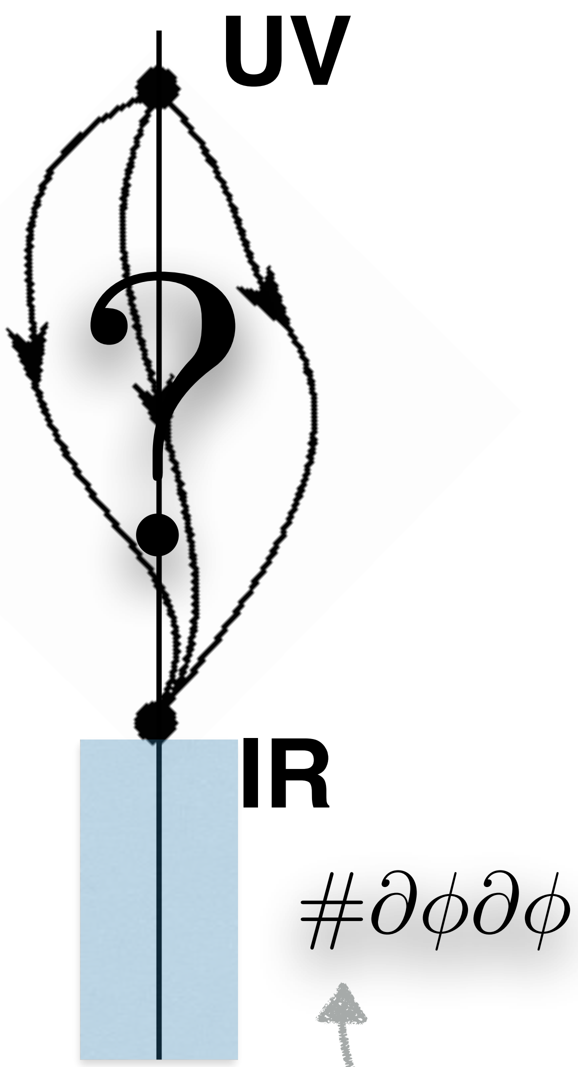
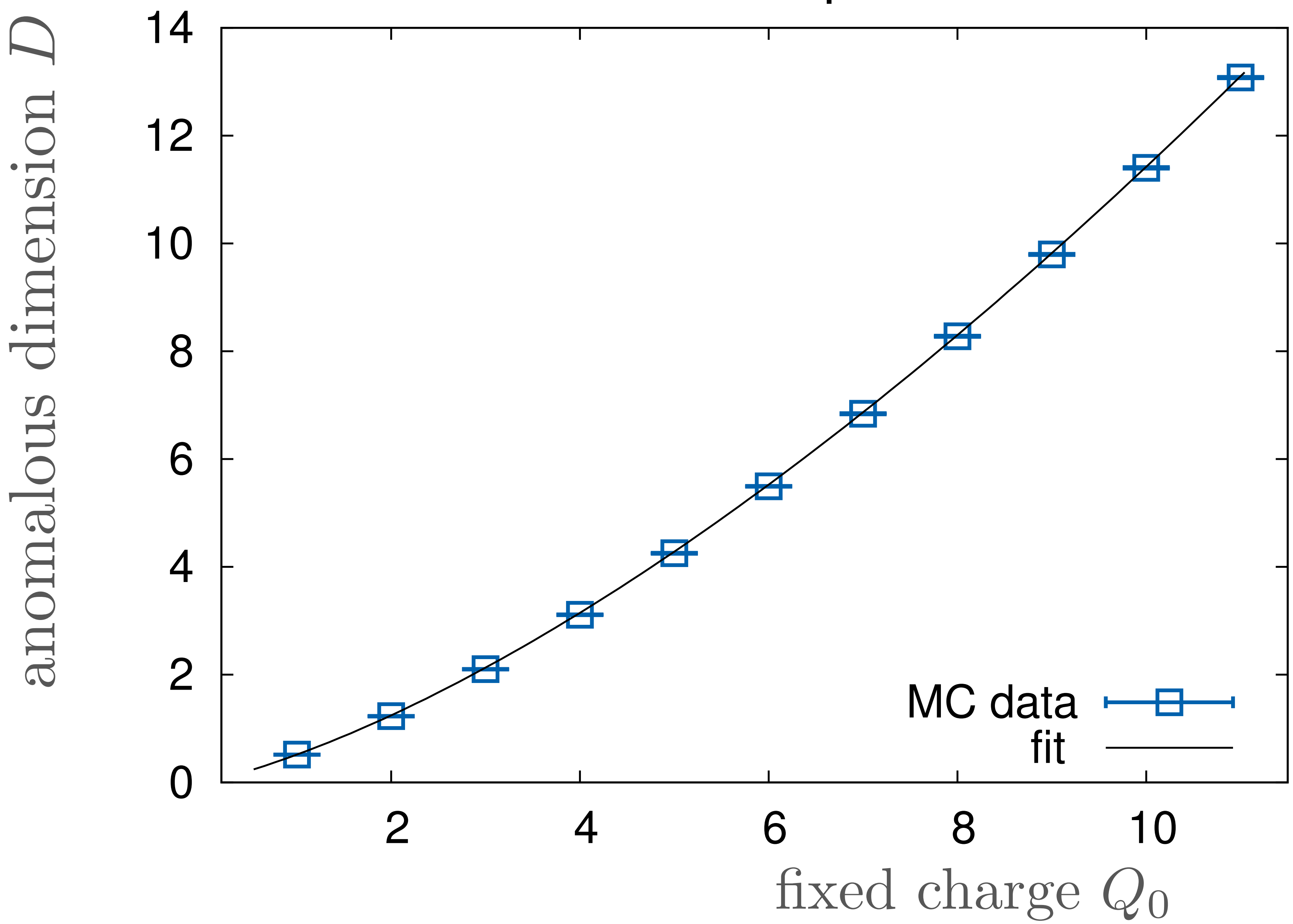


- $\mathcal{O}(1)$ coefficients
- depend on the IR coupling λ and the specifics of the RG flow through

Computing anomalous dimensions

$$D(Q) = \alpha_{3/2}\, Q^{3/2} + \alpha_{1/2}\, Q^{1/2} - 0.093 + \mathcal{O}(Q^{-1/2})$$

- fix them via parameter-fit to lattice data



scalar $O(2)$

at fixed charge

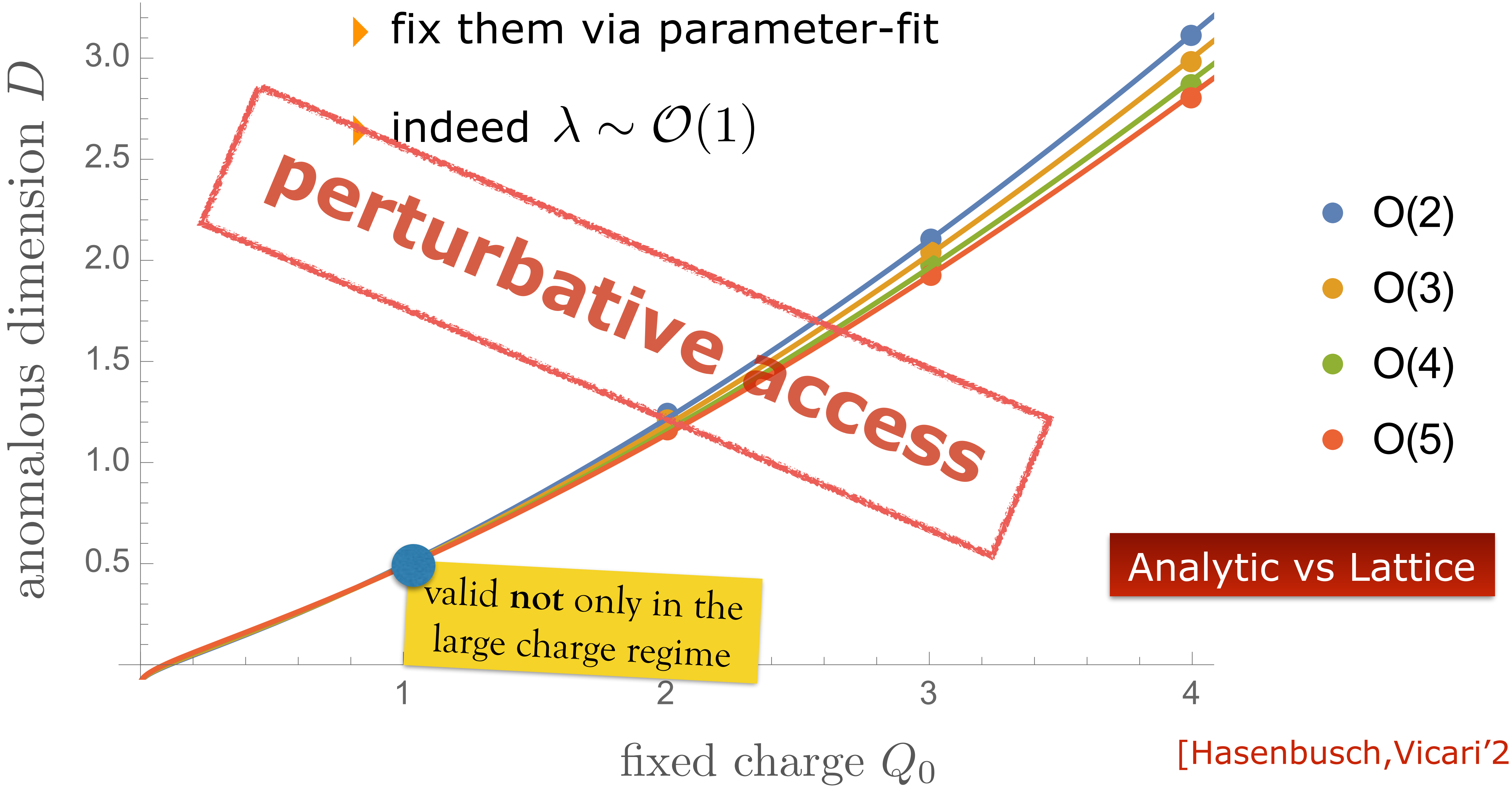
Analytic vs Lattice

[Banerjee,Chandrasekharan, Orlando'2017]

Computing anomalous dimensions

$$D(Q) = \alpha_{3/2}\, Q^{3/2} + \alpha_{1/2}\, Q^{1/2} - 0.093 + \mathcal{O}(Q^{-1/2})$$

- fix them via parameter-fit
- indeed $\lambda \sim \mathcal{O}(1)$



[Hasenbusch,Vicari'2011]

Summary

UV

- Consider CFT at the IR
  - e.g. $O(n)$ models

Wilsonian action becomes useful

- Fix global $U(1)$ charge(s)
- Theory is effectively at weak coupling

IR

setup perturbation series in "$1/Q_0$"

$$\Lambda \ll \sqrt[d]{\frac{Q_0}{V}}$$

vacuum $|Q_0\rangle$ + Goldstone + Q_0 - suppressed corrections

Summary & Outlook

- We can also compute the fusion coefficients of 3pt functions
[Monin,Pirtskhalava,Rattazzi,Seibold'2016] [O.L.,Orlando,Reffert'2017]

- Investigate even broader class of models
  - e.g. matrix models $\supset \mathbb{CP}^2$
  [O.L.,Orlando,Reffert'2017]

- Relation to large spin theories [work in progress]

- Supersymmetric models
[Hellerman,Orlando,Reffert,Watanabe'2015] [Hellerman,Maeda,Watanabe'2017]

- Large charge perturbation techniques for fermions

Thank you
for your attention

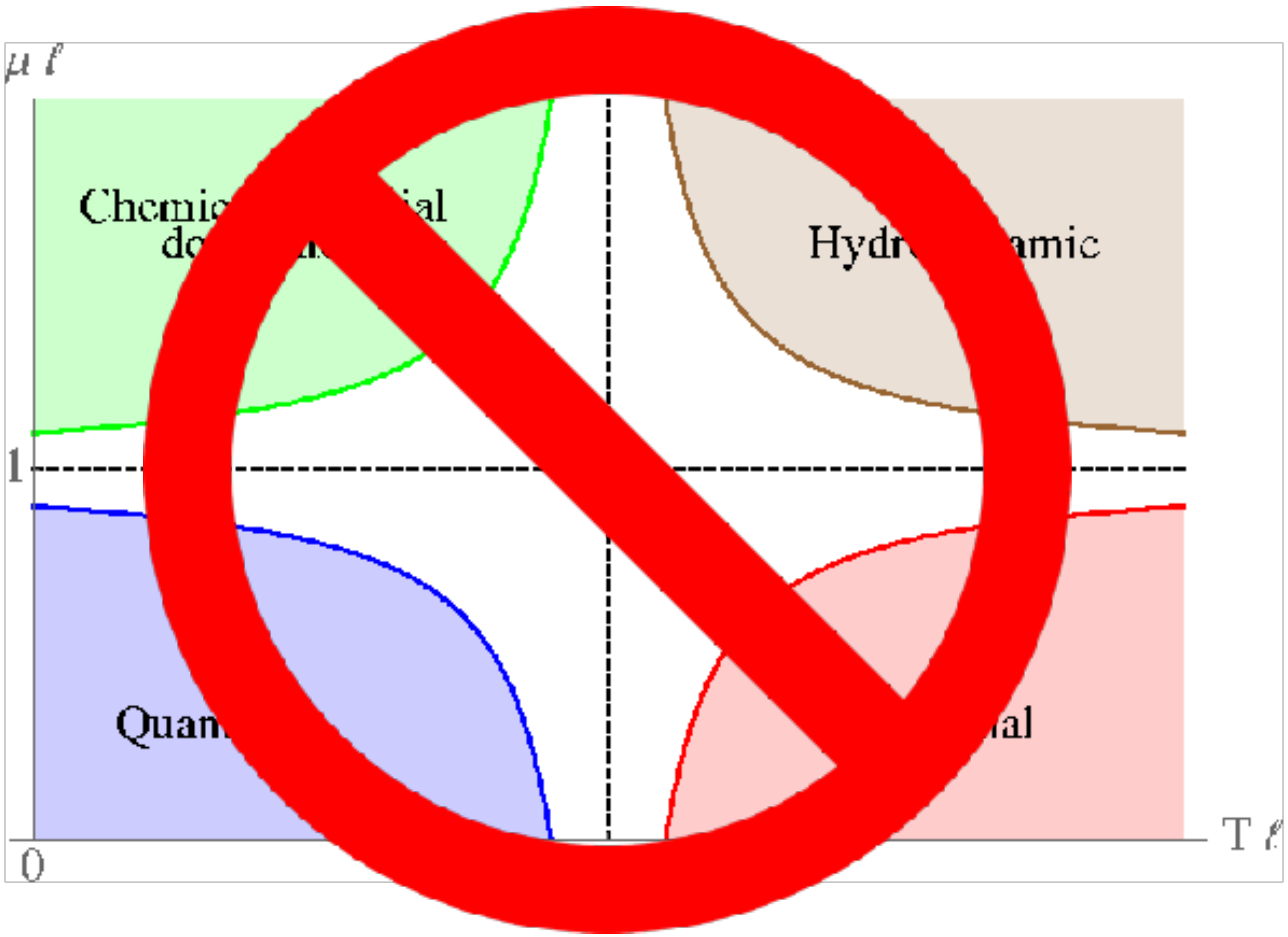
Disclaimer!

- No bootstrap here:
  - our approach is completely orthogonal to bootstrap
  - we can access sectors that are exponentially

    difficult to reach with bootstrap

Disclaimer!

- This is **not** simply a theory at finite chemical potential



Fixed charged theories

- Generically consider partition function or thermal sum at $T = \frac{1}{\beta}$

$$Z(\beta) \; = \; \mathrm{Tr}\, \mathrm{e}^{-\beta \hat{H}}$$

- Insert delta function constraint to fix the charge

$$\delta\left(\hat{Q} - Q_0\right) = \frac{1}{2\pi} \int \mathrm{d}\theta \, \mathrm{e}^{i\theta\left(\hat{Q} - Q_0\right)}$$

- Fixed charge partition sum

Fourier transform

chemical potential

$$Z_{Q_0}(\beta) \equiv \mathrm{Tr} \left\{ \delta\left(\hat{Q} - Q_0\right) \, \mathrm{e}^{-\beta \hat{H}} \right\} = \int \frac{\mathrm{d}\theta}{2\pi} \mathrm{e}^{-i\theta Q_0} \, \mathrm{Tr} \left\{ \mathrm{e}^{i\theta \hat{Q}} \, \mathrm{e}^{-\beta \hat{H}} \right\}$$

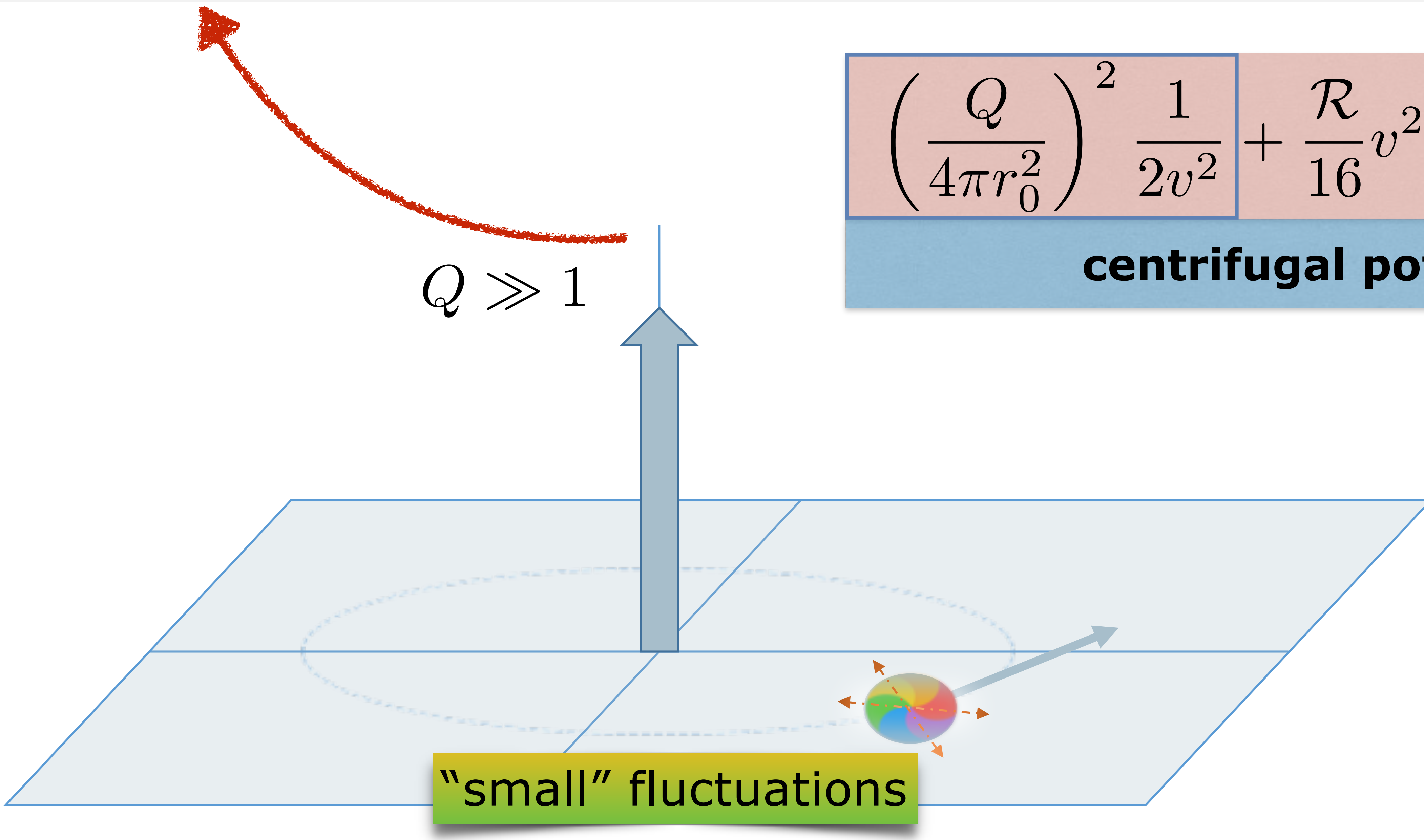
canonical ensemble

Grand-canonical ensemble

The vacuum of *O(2n)* vector model at fixed charge

$$\left(\frac{Q}{4\pi r_0^2}\right)^2 \frac{1}{2v^2} + \frac{\mathcal{R}}{16}v^2 + \frac{\lambda}{6}v^6$$

centrifugal potential



- The classical picture