Berry Phases of Boundary Gravitons

Blagoje Oblak

(ETH Zurich)

July 2017

Based on arXiv 1703.06142 + work in progress

MOTIVATION

Gravity has rich **asymptotic symmetries**

- Virasoro for AdS_3 [Brown-Henneaux 1986]
- BMS for Minkowski [Bondi *et al.* 1962]

Aspt symmetries act on space-time metric

- **Boundary gravitons**
- Observable quantities ?

Berry phases for loops in space of $\begin{cases} \text{metrics} \\ \text{reference frames} \end{cases}$

- Generalize Thomas precession [Thomas 1926]

Plan of the talk

1. Berry phases in group reps
2. Virasoro group
3. Virasoro Berry phases

1. Berry phases in group representations

Berry phases & groups

System with **parameters** $p_1, \ldots, p_n$

- Coordinates on manifold $\mathcal{M}$
- Hamiltonian $H(p)$ with $p \in \mathcal{M}$
- Eigenvalue $E(p)$ & eigenvector $|\phi(p)\rangle$

Adiabatic variation of parameters

- Path $\gamma(t)$ in $\mathcal{M}$
- Time-dependent Hamiltonian $H(\gamma(t))$
- Solve Schrödinger with $|\psi(0)\rangle = |\phi(\gamma(0))\rangle$

Berry phases & groups

$$|\psi(t)\rangle = \exp\left[-i\int_0^t d\tau E(\gamma(\tau)) + i\int_\gamma A\right]|\phi(\gamma(t))\rangle$$

- $A = i\langle\phi(\cdot)|d|\phi(\cdot)\rangle$

- Closed paths : $\gamma(T) = \gamma(0)$
- **Berry phase** : $B_\phi[\gamma] = \oint_\gamma A$ [Berry 1984]

Berry phases & groups

Group G, algebra $\mathfrak{g}$

- **Time translations** $= e^{tX}$ for some $X \in \mathfrak{g}$

Let $\mathcal{U}$ = unitary rep of G

- Evolution operator $\mathcal{U}[e^{tX}] = e^{t\,\mathfrak{u}[X]}$
- Hamiltonian $H = i\,\mathfrak{u}[X]$

This relies on a **choice of frame** !

- Let $f \in G$ be a change of frame
- Hamiltonian $\mathcal{U}[f]H\mathcal{U}[f]^{-1}$
- $G \sim$ space of parameters
- Berry phases ?

BERRY PHASES & GROUPS

Let $|\phi\rangle$ = eigenstate of H
Let $f(t)$ = closed path in G

- Berry phase : $B_\phi[f(t)] = i\oint_0^T dt\,\langle\phi|\mathfrak{u}[f^{-1}\cdot\dot{f}]|\phi\rangle$

$\downarrow$ **Maurer-Cartan form**

Note : Berry phase vanishes for $f(t)$ in stabilizer of $|\phi\rangle$

- Parameter space is G/G_ϕ

2. Maurer-Cartan form of Virasoro

Maurer-Cartan in Virasoro

Diff S^1 = group of **circle diffeos**

- Half of 2D conformal group
- Elements are fcts $f(\varphi)$ with

$$f(\varphi+2\pi)=f(\varphi)+2\pi \quad \text{and} \quad f'(\varphi)>0$$

- Group operation is composition : $f\cdot g=f\circ g$
- Lie algebra = Vect S^1 = Witt algebra $\ell_m \propto e^{im\varphi}\partial_\varphi$

Path $f(t,\varphi)$

- Maurer-Cartan : $f^{-1}\cdot\dot{f}=\dfrac{\dot{f}(t,\varphi)}{f'(t,\varphi)}\,\partial_\varphi$

Maurer-Cartan in Virasoro

Virasoro group = Central extension of $\text{Diff}\, S^1 = \text{Diff}\, S^1 \times \mathbb{R}$

- Elements = pairs (f, α)

Maurer-Cartan form ?

- Let $f(t, \varphi)$ = path in $\text{Diff}\, S^1$
- $$(f^{-1}, 0) \cdot (\dot{f}, 0) = \left(\frac{\dot{f}}{f'} \partial_\varphi \,,\, \frac{1}{48\pi} \int_0^{2\pi} d\varphi \, \frac{\dot{f}}{f'} \left(\frac{f''}{f'} \right)' \right)$$

[Alekseev-Shatashvili 1989]

3. Virasoro Berry phases

VIRASORO BERRY PHASES

Highest weight rep of Virasoro, central charge c

- Highest weight state $|h\rangle$:

$$\mathfrak{u}[\ell_0]|h\rangle = h|h\rangle \qquad \mathfrak{u}[\ell_n]|h\rangle = 0 \quad \text{if } n > 0$$

- Stabilizer $= \mathrm{U}(1)$ if $h > 0$
- Parameter space $= \text{Diff}\, S^1/S^1$

Path $f(t,\varphi)$ with closed projection on $\text{Diff}\, S^1/S^1$

- Berry phase :

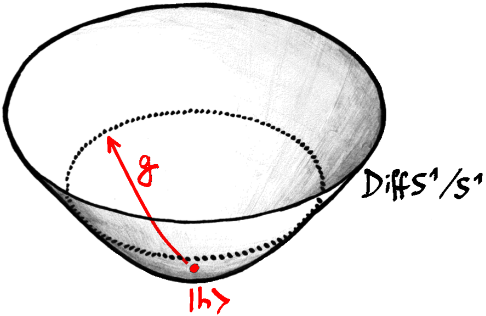
$$B_{c,h}[f] = -\frac{1}{2\pi}\oint dt \int d\varphi \, \frac{\dot{f}}{f'}\left[h - \frac{c}{24} + \frac{c}{24}\left(\frac{f''}{f'}\right)'\right] + \left(h - \frac{c}{24}\right) f^{-1}(0, f(T,0))$$

Virasoro Berry phases

Circular paths : $f(t,\varphi) = g(\varphi) + \omega t$

- Closes at $t = 2\pi/\omega$
- Berry phase :

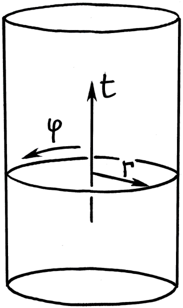
$$B_{c,h}[f] = -\int \frac{d\varphi}{g'}\left[h - \frac{c}{24} + \frac{c}{24}\left(\frac{g''}{g'}\right)'\right] + 2\pi\left(h - \frac{c}{24}\right)$$



Virasoro Berry phases

Interpretation in **AdS$_3$** ?

- Infinity = time-like cylinder
- Light-cone coordinates $x^{\pm} = \frac{t}{\ell} \pm \varphi$



Include gravity

- **Asymptotic symmetries** : Diff $S^1 \times$ Diff S^1
  [Brown-Henneaux 1986]
- $(x^+, x^-) \mapsto \big(f(x^+), \bar{f}(x^-)\big)$
- UIRREPs = **particles dressed with gravitons**

Berry phases appear
when particles undergo cyclic changes of frames

Virasoro Berry phases

Circular path : $\begin{cases} f(t,x^+) = g(x^+) + \omega t \\ \bar{f}(t,x^-) = \bar{g}(x^-) - \omega t \end{cases}$

- Take $g = \bar{g} =$ **boost** :

$$e^{i g(\varphi)} = \frac{e^{i\varphi}\cosh(\lambda/2) + \sinh(\lambda/2)}{e^{i\varphi}\sinh(\lambda/2) + \cosh(\lambda/2)}$$

- Berry phase :

$$B = \; -2\pi \, s \, (\cosh\lambda - 1)$$

- **Thomas precession** ! [Thomas 1926]

VIRASORO BERRY PHASES

Circular path : $\begin{cases} f(t,x^+) = g(x^+) + \omega t \\ \bar{f}(t,x^-) = \bar{g}(x^-) - \omega t \end{cases}$

- Take g, $\bar{g}$ = **generalized boosts** :

$$e^{in\,g(\varphi)} = \frac{e^{in\varphi}\cosh(\lambda/2) + \sinh(\lambda/2)}{e^{in\varphi}\sinh(\lambda/2) + \cosh(\lambda/2)}$$

- Berry phase :

$$B = -2\pi\left(h + \frac{c}{24}(n^2 - 1)\right)(\cosh\lambda - 1) + \text{same with bars}$$

- **Thomas precession** for **dressed particles** [Oblak 2017]

Thank you for listening !