PUZZLES IN 3D CHERN-SIMONS-MATTER THEORIES

Silvia Penati, University of Milano-Bicocca and INFN

Ascona, July 3 2017

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SUSY WILSON LOOPS

Why BPS Wilson Loops?

BPS Wilson Loops in supersymmetric gauge theories: gauge invariant non–local operators that preserve some supercharges

The prototype example in N=4 SYM

1/2 BPS WL

$$WL = \text{Tr} P e^{-i\int_\Gamma d\tau(\dot{x}^\mu A_\mu + i|\dot{x}|\theta_I \Phi^I)}$$

It includes couplings to the six scalars (Maldacena, PRL80 (1998) 4859)
(Drukker, Gross, Ooguri, PRD60 (1999) 125006; Zarembo, NPB 643)

- They are in general non–protected operators and their expectation value can be computed exactly by using **localization techniques**.
- **Dual description** in terms of fundamental strings or M2–branes. The expectation value at strong coupling is given by the exponential of a minimal area surface ending on the WL contour. Matching with localization results provides a crucial test of the AdS/CFT correspondence.

Why BPS WL in 3D SCSM theories?

We will focus on

- $\mathcal{N} = 6$ ABJ(M) Aharony, Bergman, Jafferis, Maldacena, 0806.1218
  Aharony, Bergman, Jafferis, 0807.4924
- $\mathcal{N} = 4$ orbifold ABJM and more general SCSM with $\Pi_{l=1}^{r} U(N_{2l-1}) \times U(N_{2l})$ and alternating levels Gaiotto, Witten, 0804.2907
  Hosomichi, Lee, Lee, Lee, Park, 0805.3662

BPS WL in 3D SCSM theories exhibit a richer spectrum of interesting properties compared to the 4D case. Among them:

- Due to dimensional reasons also **fermions** together with scalars can enter the definition of BPS WL. In general they increase the number of susy charges preserved by WL. Therefore, we have a richer spectrum of BPS WL.
- **Framing factors** appear as overall complex phases in localization results for $\langle WL \rangle$.

Plan of the talk

- Systematic construction of classical WL via Higgsing
- How to compute $\langle WL \rangle$ at $\star$weak coupling, $\star$strong coupling or $\star$exactly
- Solved and unsolved puzzles
  - Framing factor in ABJ(M) ✓
  - Fermionic WLs in ABJ(M) ✓
  - Degeneracy of WLs in $\mathcal{N}=4$ orbifold ABJM ✓
  - Comparison with localization result for orbifold ABJM ✓
  - Comparison with localization result for $\mathcal{N}=4$ SCSM theories Alert!
- Conclusions and Perspectives

M.S. Bianchi, L. Griguolo, J-j. Zhang, M. Leoni, A. Mauri, D. Seminara
PLB753, JHEP 1606, JHEP 1609, arXiv:1705.02322 + in progress

Construction via Higgsing – Field theory

Time–like WL $\to$ phase of a very heavy quark moving in the gauge background.

In SCFTs there are no massive particles $\Rightarrow$ Higgsing procedure

$SU(N+1)$ $\mathcal{N}=4$ SYM

$$\langle \Phi_I \rangle = v \to \infty \qquad\qquad SU(N+1) \to SU(N)\times U(1)$$

$\mathcal{N}=4 \to \mathcal{N}=2$ with an infinitely massive multipet $m=v$.

Particle modes

$$W_\mu = \frac{1}{\sqrt{2v}} w_\mu\, e^{-imt}$$
$$R_i = \frac{1}{\sqrt{2v}} r_i\, e^{-imt}$$

Anti–particle modes

$$\bar W_\mu = \frac{1}{\sqrt{2v}} \bar w_\mu\, e^{imt}$$
$$\bar R_i = \frac{1}{\sqrt{2v}} \bar r_i\, e^{imt}$$

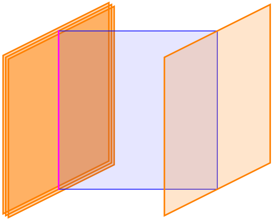
Non–relativistic lagrangian

$$\mathcal{L} = i\bar w_a \mathcal{D}_0 w_a + i\bar r_i \mathcal{D}_0 r_i \qquad \mathcal{D}_0 = \partial_0 \ + \ \underbrace{i(A_0 \pm \Phi_I)}_{(\mathcal{A}_I)_0}$$

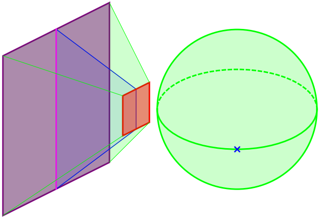
1/2 BPS $\qquad WL_I = \mathcal{P}\exp\Big(-i\int d\tau \mathcal{A}_I(\tau)\Big) \qquad I=5,\dots,9$

They preserve complementary sets of supercharges

Construction via Higgsing – String dual



$(N+1) \to (N) + 1$ D3 branes



$\mathrm{AdS}_5 \times \mathrm{S}^5$ limit

In the $\mathrm{AdS}_5 \times \mathrm{S}^5$ limit the fundamental string is localized in the internal space. The position of the point is related to $\langle \Phi_I \rangle$.

We can excite fundamental strings or anti–strings

Procedure generalizable to 3D SCSM theories with string or M–theory duals

Prototype examples of WLs in ABJ(M)

(K-M. Lee, S. Lee, JHEP09 (2009) 030)

$\mathcal{N} = 6$ **susy ABJ(M) model** for $U(N_1)_k \times U(N_2)_{-k}$ CS-gauge vectors A_μ, $\hat{A}_\mu$ minimally coupled to

$SU(4)$ complex scalars C_I, $\bar{C}^I$ and fermions ψ_I, $\bar{\psi}^I$

in the (anti)bifundamental representation of the gauge group with non-trivial potential.

Bosonic 1/6 BPS WLs $\quad W_{1/6} = \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau (A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^{\ I} C_I \bar{C}^J)\right]$

Fermionic 1/2 BPS WL $\quad W_{1/2} = \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\tau)\right]$

$$\mathcal{L}(\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^{\ I} C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}} |\dot{x}| \eta_I \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}} |\dot{x}| \psi_I \bar{\eta}^I & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| \hat{M}_J^{\ I} \bar{C}^J C_I \end{pmatrix}$$

Cohomological equivalence

$$W_{1/2} = \frac{N_1 W_{1/6} + N_2 \hat{W}_{1/6}}{N_1 + N_2} + QV$$

How to compute $\langle WL \rangle$ in SCSM theories

$$\langle WL \rangle \sim \int D[A, \hat{A}, C, \bar{C}, \psi, \bar{\psi}]\, e^{-S}\, \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\tau)\right]$$

- Weak coupling $N_1/k, N_2/k \ll 1$ Perturbative evaluation
- Strong coupling $N_1/k, N_2/k \gg 1$ Holographic evaluation
- $N_1/k, N_2/k \sim 1$ Localization techniques reduce $\langle WL \rangle$ to a Matrix Model

For ABJ(M) → non–gaussian MM (Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089)

$$\langle W_{1/6} \rangle = \int \prod_{a=1}^{N_1} d\lambda_a \; e^{i\pi k \lambda_a^2} \prod_{b=1}^{N_2} d\hat{\lambda}_b \; e^{-i\pi k \hat{\lambda}_b^2} \times \left(\frac{1}{N_1} \sum_{a=1}^{N_1} e^{2\pi\lambda_a} \right) \frac{\prod_{a<b}^{N_1} \sinh^2(\pi(\lambda_a - \lambda_b)) \prod_{a<b}^{N_2} \sinh^2(\pi(\hat{\lambda}_a - \hat{\lambda}_b))}{\prod_{a=1}^{N_1} \prod_{b=1}^{N_2} \cosh^2(\pi(\lambda_a - \hat{\lambda}_b))}$$

(Drukker, Marino, Putrov, CMP306 (2011) 511)

Puzzles

Puzzles typically arise in 3D SCSM theories when we try to match perturbative results with localization predictions

- Perturbative results are at framing 0 while localization predictions are at framing 1. **We need identify the correct framing factor**

- Localization predictions are based on **cohomological equivalence** of WLs that is valid at classical level.

$$W_{ferm} = W_{bos} + Q\,V \qquad \Longrightarrow \qquad \langle W_{ferm} \rangle = \langle W_{bos} \rangle$$

Puzzle 1: Framing factor in ABJ(M)

For the $U(N)_k$ pure Chern–Simons theory (topological theory)

$$S_{CS} = -i\frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho + \frac{2}{3}iA_\mu A_\nu A_\rho\right)$$

On a closed path Γ and in fundamental representation

$$\begin{aligned}\langle \mathcal{W}_{\mathrm{CS}}\rangle &= \langle \mathrm{Tr}\, P\, e^{-i\int_\Gamma dx^\mu A_\mu(x)}\rangle \\ &= \sum_{n=0}^{+\infty} \mathrm{Tr}P\int dx_1^{\mu_1}\cdots dx_n^{\mu_n}\,\langle A_{\mu_1}(x_1)\cdots A_{\mu_n}(x_n)\rangle\end{aligned}$$

1) either by using semiclassical methods in the large k limit

(Witten, CMP121 (1989) 351)

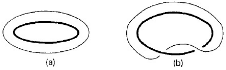
2) or perturbatively (n-pt correlation functions)

(Guadagnini, Martellini, Mintchev, NPB330 (1990) 575)

Regularize singularities in $\langle A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n) \rangle$ at coincident points.

Using point-splitting regularization

$$\Gamma_f : \quad y^\mu(\tau) \to y^\mu(\tau) + \epsilon\, n^\mu(\tau)$$



$$\lim_{\epsilon \to 0} \oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \, \langle A_\mu(x) A_\nu(y) \rangle = -i\pi\lambda\, \chi(\Gamma, \Gamma_f) \qquad \lambda = \frac{N}{k}$$

Gauss linking number

$$\chi(\Gamma, \Gamma_f) = \frac{1}{4\pi} \oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \; \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$

Higher-order contributions exponentiate the one–loop result

$$\langle \mathcal{W}_{\rm CS} \rangle = \underbrace{e^{-i\pi\lambda\chi(\Gamma,\Gamma_f)}}_{\textbf{framing factor}} \rho(\Gamma)$$

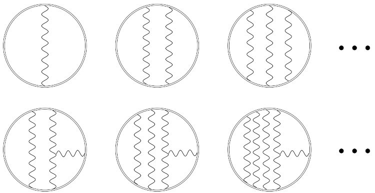
Exponentiation of one–loop framing term relies on the following distinguishing properties

(Alvarez, Labastida, NPB395 (1993) 198)

1. The gauge propagator is one–loop exact. In Landau gauge

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \delta^{ab} \frac{i}{2k} \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$

2. Only diagrams with collapsible propagators contribute to framing



3. Factorization theorem

$\mathcal{N} = 2$ susy CS theory

We are primarily interested in supersymmetric theories for which localization can be used.

$$\langle W_{\rm SCS} \rangle = \langle {\rm Tr}\, P\, e^{-i \int_\Gamma d\tau (\dot{x}^\mu A_\mu(x) - i|\dot{x}|\sigma)} \rangle$$

(Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089)

Localization always provides the result at framing $\chi(\Gamma, \Gamma_f) = -1$. This follows from requiring consistency between point–splitting regularization and supersymmetry used to localize: The only point-splitting compatible with susy is the one where the contour and its frame wrap two different Hopf fibers of S^3

Localization is sensible to framing!

Framing identified as imaginary contributions

$$\langle W_{\rm SCS} \rangle = e^{i\pi\lambda} \rho(\Gamma)$$

Adding matter → ABJ(M) case

1/6-BPS Wilson loop (Drukker, Plefka, Young, JHEP 0811 (2008) 019
Chen, Wu, NPB 825 (2010) 38, Rey, Suyama, Yamaguchi, JHEP 0903 (2009))

$$\langle W_{1/6} \rangle = \langle \mathrm{Tr} P \exp \left[-i \int_\Gamma d\tau (A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J{}^I C_I \bar{C}^J) \right] \rangle$$

$$M_I{}^J = \mathrm{diag}(+1, +1, -1, -1)$$

- **Localization result**. $\langle WL \rangle$ → non-gaussian MM computed exactly
  (Drukker, Marino, Putrov, CMP 306 (2011); Klemm, Marino, Schiereck, Soroush, Naturforsch. A68 (2013))

Weak coupling expansion and **planar limit** $(\lambda_1 = N_1/k, \lambda_2 = N_2/k \ll 1)$

$$\langle W_{1/6} \rangle = \underbrace{e^{i\pi\lambda_1}} \left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) \underbrace{-i\frac{\pi^3}{2}\lambda_1\lambda_2^2} + \mathcal{O}(\lambda^4) \right)$$

$\Downarrow$ pure CS framing (-1) factor

$\Downarrow$ extra imaginary term ???

- **Perturbation theory** (framing $= 0$) $\rightarrow$ no contributions at odd orders

(Rey, Suyama, Yamaguchi, JHEP 0903 (2009)

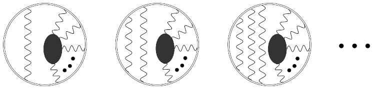
Conjecture: Matter contributes to framing

PROOF: **perturbative 3-loop calculation at framing (-1)**

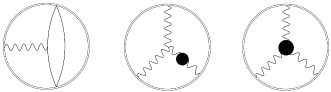
Matter contributes to framing in two different ways:

1) Matter gives non-trivial corrections to the the gauge propagator (FINITE at two loops). Collapsible propagators

$$\langle A_\mu(x) A_\nu(y) \rangle \to \frac{i}{2k} \left[1 - \frac{\pi^2}{2} \left(\underbrace{\lambda_2^2} + \underbrace{\lambda_1 \lambda_2 \left(\frac{1}{4} + \frac{2}{\pi^2} \right)} \right) \right] \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$



2) Matter vertex-like diagrams cancel lower-transcendentality terms



Exponentiation still works, so we can write

$$\langle W_{1/6}\rangle_1 = \underbrace{e^{i\pi\left(\lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^5)\right)}}\left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) + \mathcal{O}(\lambda^4)\right)$$

$$\Downarrow$$

perturbative framing function $f(\lambda_1, \lambda_2) = \lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^5)$

$$\langle W_{1/6}\rangle_0 = \left|\langle W_{1/6}\rangle_1\right|$$

Puzzle solved ✓

Puzzle 2: Fermionic WLs in ABJ(M)

Fermionic 1/2 BPS WLs

- $W_{1/2} = \left(\frac{N_1 W_{1/6} + N_2 \hat{W}_{1/6}}{N_1 + N_2} \right) + QV \equiv \tilde{W}_{1/6} + QV$
- Therefore, using Q to localize the path integral (**framing one**) we have

$$\langle W_{1/2} \rangle_1 = \langle \tilde{W}_{1/6} \rangle_1$$

- $\langle \tilde{W}_{1/6} \rangle_1$ can be computed exactly. Weak coupling expansion

$$\langle W_{1/2} \rangle_1 = 1 + i\pi(\lambda_1 - \lambda_2) + \frac{\pi^2}{6} \left(-4\lambda_1^2 - 4\lambda_2^2 + 10\lambda_1\lambda_2 + 1 \right) + \mathcal{O}(\frac{1}{k^3})$$

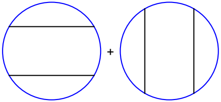
- Perturbative calculation (**framing zero**) up to two loops (Bianchi, Giribet, Leoni, SP, JHEP 10 (2013); Griguolo, Martelloni, Poggi, Seminara, JHEP 09 (2013))
- Matching the two results (Drukker, Trancanelli, JHEP 02 (2010) 058)

Framing exponentiates

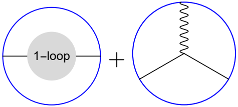
$$\langle W_{1/2} \rangle_1 = e^{i\pi(\lambda_1 - \lambda_2)} \langle W_{1/2} \rangle_0$$

Fermions contribute to framing

Comparing the perturbative calculation with the framing–one result we can make an **educated guess** (M.S. Bianchi, JHEP 1609 (2016) 047)



$$\Big|_{f=0}^{f=1} = -\frac{3\pi^2}{2}\lambda_1\lambda_2$$



$$\Big|_{f=0}^{f=1} = 2\pi^2\lambda_1\lambda_2$$

Although we do not have a direct check yet, they should contribute to exponentiate the framing factor.

Fermionic 1/6 BPS WLs

(Ouyang, Wu, Zhang, NP B 910 (2016) 496)

$$W^{(F)}_{1/6}(\alpha,\beta) = \frac{1}{N_1+N_2}\mathrm{Tr}\mathcal{P}e^{-i\oint d\tau L_{1/6}(\alpha,\beta)}$$

$$L_{1/6}(\alpha,\beta) = \begin{pmatrix} A_\mu \dot{x}^\mu + \frac{2\pi i}{k} U^I{}_J C_I \bar{C}^J |\dot{x}| & \sqrt{\frac{4\pi}{k}}\bar{\alpha}_I u_+ \psi^I |\dot{x}| \\ \sqrt{\frac{4\pi}{k}}\bar{\psi}_I u_- \beta^I |\dot{x}| & B_\mu \dot{x}^\mu + \frac{2\pi i}{k} U^I{}_J \bar{C}^J C_I |\dot{x}| \end{pmatrix}$$

$$\bar{\alpha}_I = (\bar{\alpha}_1, \bar{\alpha}_2, 0, 0), \quad \beta^I = (\beta^1, \beta^2, 0, 0)$$

$$U^I{}_J = \begin{pmatrix} 1-2\beta^2\bar{\alpha}_2 & 2\beta^1\bar{\alpha}_2 & & \\ 2\beta^2\bar{\alpha}_1 & 1-2\beta^1\bar{\alpha}_1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

They interpolate between $\tilde{W}_{1/6} = W^{(F)}_{1/6}(0,0)$ and $W_{1/2} = W^{(F)}_{1/6}(\alpha^I, \alpha^I/|\alpha|^2)$

Cohomological equivalence

$$W^{(F)}_{1/6}(\alpha,\beta) = \tilde{W}_{1/6} + QV(\alpha,\beta)$$

- Framing–one result from localization

$$\langle W^{(F)}_{1/6}(\alpha,\beta)\rangle_1 = \langle \tilde{W}_{1/6}\rangle_1$$
$$= 1 + i\pi(\lambda_1-\lambda_2) + \frac{\pi^2}{6}\left(-4\lambda_1^2 - 4\lambda_2^2 + 10\lambda_1\lambda_2 + 1\right) + \mathcal{O}(\frac{1}{k^3})$$

No parameter dependence

- Framing–zero result from perturbation theory

$$\langle W^{(F)}_{1/6}(\alpha,\beta)\rangle_0 = 1 + \frac{\pi^2}{6}\{-\lambda_1^2 - \lambda_2^2 + [9(\bar{\alpha}_I\beta^I)^2 - 12\bar{\alpha}_I\beta^I + 7]\lambda_1\lambda_2 + 1\} + O(\frac{1}{k^3})$$

Comparing the two results

$$\langle W^{(F)}_{1/6}(\alpha,\beta)\rangle_0 = e^{-i\pi(\lambda_1-\lambda_2)}\langle \tilde{W}_{1/6}\rangle_1 + \underbrace{\frac{\pi^2}{6}[9(\bar{\alpha}_I\beta^I)^2 - 12\bar{\alpha}_I\beta^I + 3]\lambda_1\lambda_2}_{\text{Remnant}}$$

Educated guess gives fermionic contributions at framing one that depend on the parameters. They should cancel against the framing–independent terms in order to restore the localization result.

A perturbative calculation at framing one is required in order to confirm framing dependence in vertex–like fermionic diagrams

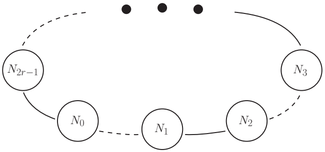
How do we interpret the Remnant? **Puzzle only partially solved ✓**

Puzzle 3: WL degeneracy in $\mathcal{N}=4$ SCSM theories

(Gaiotto, Witten, JHEP 06 (2010) 097; Hosomichi, Lee^3, Park, JHEP 07 (2008) 091)

$\Pi_{l=1}^{r} U(N_{2l-1}) \times U(N_{2l})$ quiver gauge theories with alternating $\pm k$ levels

Matter in (anti)bifundamental representation of adiacent gauge groups and in $(2,1)$ and $(1,2)$ of $SU(2)\times \hat{SU}(2)$ R–symmetry $\quad \phi^I \quad \phi^{\hat{I}}$



Dual to M–theory on $\mathrm{AdS}_4 \times S^7/(Z_r \otimes Z_r)/Z_k$

Orbifold ABJM: $\quad N_0 = N_1 = \cdots = N_{2r-1}$

Dual to M–theory on $\mathrm{AdS}_4 \times S^7/(Z_r \otimes Z_{rk})$

BPS WL defined locally for quiver nodes $(2l-1, 2l) \to W^{(l)}$

or globally $W = \sum_{l=1}^{r} W^{(l)}$

Higgsing procedure allows to construct two classes of 1/2 BPS WLs

Exiting heavy particle dof $\to$ class $\mathcal{C}$

Exiting heavy anti–particle dof $\to$ class $\hat{\mathcal{C}}$

For ABJ(M) models, representatives of different classes preserve different sets of supercharges only partially overlapping.

In $\mathcal{N} = 4$ SCSM theories, for each ψ_1 representative in $\mathcal{C}$ we can find a representative ψ_2 in $\hat{\mathcal{C}}$ that preserves the **same set of supercharges**.
(Crooke, Drukker, Trancanelli, JHEP 10 (2015) 140; Lietti, Mauri, Zhang, SP, 1705.03322) **Puzzle??**

We have proved that (embedding S^7 in $\mathrm{R}^8 \cong \mathrm{C}^4 \to z_{1,2,3,4}$)

$\psi_1 \to$ **M2–brane** wrapped on an internal circle $|z_1| = 1$ and localized at $z_{2,3,4} = 0$
$\psi_2 \to$ **M2–antibrane** wrapped on a different circle $|z_2| = 1$ and at $z_{1,3,4} = 0$

The two brane configurations preserve the **same set of supercharges**.

Puzzle solved ✓

$$\psi_i = \frac{1}{N_1 + N_2} \operatorname{Tr} P \exp\left(-i \int_\Gamma d\tau \mathcal{L}_{\psi_i}(\tau)\right)$$

where

$$\mathcal{L}_{\psi_1} = \begin{pmatrix} \mathcal{A}_{(1)} & \bar{c}_\alpha \psi^\alpha_{(1)\hat{1}} \\ c^\alpha \bar{\psi}^{\hat{1}}_{(1)\alpha} & \mathcal{A}_{(2)} \end{pmatrix}$$

$$\mathcal{A}_{(1)} = \dot{x}^\mu A_{(1)\mu} - \frac{i}{k}\left(q^I_{(1)} \delta_I{}^J \bar{q}_{(1)J} + \bar{q}_{(0)\hat{I}} (\sigma_3)^{\hat{I}}{}_{\hat{J}}\, q^{\hat{J}}_{(0)}\right)|\dot{x}|$$

$$\mathcal{A}_{(2)} = \dot{x}^\mu A_{(2)\mu} - \frac{i}{k}\left(\bar{q}_{(1)I} \delta^I{}_J\, q^J_{(1)} + q^{\hat{I}}_{(2)} (\sigma_3)_{\hat{I}}{}^{\hat{J}}\, \bar{q}_{(2)\,\hat{J}}\right)|\dot{x}|$$

$$\mathcal{L}_{\psi_2} = \begin{pmatrix} \mathcal{B}_{(1)} & \bar{d}_\alpha \psi^\alpha_{(1)\hat{2}} \\ d^\alpha \bar{\psi}^{\hat{2}}_{(1)\alpha} & \mathcal{B}_{(2)} \end{pmatrix}$$

$$\mathcal{B}_{(1)} = \dot{x}^\mu A_{(1)\mu} - \frac{i}{k}\left(-q^I_{(1)} \delta_I{}^J \bar{q}_{(1)J} + \bar{q}_{(0)\hat{I}} (\sigma_3)^{\hat{I}}{}_{\hat{J}}\, q^{\hat{J}}_{(0)}\right)|\dot{x}|$$

$$\mathcal{B}_{(2)} = \dot{x}^\mu A_{(2)\mu} - \frac{i}{k}\left(-\bar{q}_{(1)I} \delta^I{}_J\, q^J_{(1)} + q^{\hat{I}}_{(2)} (\sigma_3)_{\hat{I}}{}^{\hat{J}}\, \bar{q}_{(2)\,\hat{J}}\right)|\dot{x}|$$

Cohomological equivalence

$$\psi_1 = W_{1/4} + QV_1 \qquad \psi_2 = W_{1/4} + QV_2$$

At quantum level?

Cohomological equivalence

$\psi_1 = W_{1/4} + QV_1 \qquad \psi_2 = W_{1/4} + QV_2$

- Localization (framing–one) $\quad \langle \psi_1 \rangle_1 = \langle \psi_2 \rangle_1 = \langle W_{1/4} \rangle_1$

  We expect $\langle \psi_1 \rangle_0 = \langle \psi_2 \rangle_0 = |\langle W_{1/4} \rangle_1|$ (Proved at 3 loops)

- Perturbation theory (framing–zero): For planar contour

$$\langle \psi_1 \rangle_0^{(L)} = (-1)^L \langle \psi_2 \rangle_0^{(L)}$$

(Bianchi, Griguolo, Leoni, Mauri, SP, Seminara, JHEP 1609 (2016) 009)

Consistency requires

$$\langle \psi_1 \rangle_0^{(2L+1)} = \langle \psi_2 \rangle_0^{(2L+1)} = 0$$

Is it true?

- From localization $|\langle W_{1/4}\rangle_1|$ vanishes at odd orders (checked up to three loops).

- From a perturbative calculation: One loop result vanishes. We need a **3–loop** calculation

  - Orbifold ABJM: Too many diagrams to compute. Still open question

  - $\mathcal{N}=4$ SCSM theories: The number of diagrams can be drastically reduced by restricting to the range–3 color sectors $N_{l-1}N_lN_{l+1}$

  For $l=1$ we have found (Bianchi, Griguolo, Leoni, Mauri, SP, Seminara, JHEP 1609 (2016) 009)

$$\langle\psi_1\rangle^{(3L)} = -\langle\psi_2\rangle^{(3L)} = \frac{5}{8\pi}\,\frac{N_0N_1^2N_2+N_1N_2^2N_3}{(N_1+N_2)\,k^3}$$

Alerting puzzle!

Possible explanation?

- It is a matter of fact that $\langle \frac{\psi_1+\psi_2}{2} \rangle^{(odd)} = 0$ and matches the localization result.
- However, neither $\langle \psi_1 \rangle^{(3L)}$ nor $\langle \psi_2 \rangle^{(3L)}$ match the localization result.
- It is hard to believe that two non–BPS operators give rise to a BPS operator when linearly combined.
- If the dual description works as in the orbifold case, it points towards the fact that both ψ_1 and ψ_2 should be BPS at quantum level. But we don't know ...

Only possibility: ψ_1 and ψ_2 are BPS, but the cohomological equivalence is broken by quantum effects

$$\langle \psi_1 \rangle = \langle W_{1/4} \rangle + \mathcal{A} \qquad\qquad \langle \psi_2 \rangle = \langle W_{1/4} \rangle - \mathcal{A}$$

such that $\frac{\psi_1+\psi_2}{2}$ is BPS and Q–equivalent to $W_{1/4}$.

A direct check requires computing $\langle \psi_1 \rangle_1$ and $\langle \psi_2 \rangle_1$ at framing one in perturbation theory. This implies understanding framing contributions from fermions.

Puzzle unsolved X

Conclusions

- We have understood the framing mechanism in CS theories with matter. But
  - Better understand contributions from vertex-like diagrams.
  - Framing from matter in fermionic WLs: understand framing from fermionic diagrams
  - What happens at higher orders? Divergences?
- Cohomological equivalence in $\mathcal{N} = 4$ SCSM theories is still an open problem
- Framing at strong coupling?
- WLs in theories with vanishing CS levels (Imamura, Kimura, JHEP 0810 (2008) 040)