Perturbative structure of bimetric gravity and a cosmological application

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String Theory and Quantum Gravity
Ascona, 04/07/2017

Contents

Motivation

Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

massless & massive

Spin 2: graviton $g_{\mu\nu}$

MASSLESS !

How do we make a spin-2 field massive ?

Massless + Massive Spin-2 Fields

General Relativity

Einstein-Hilbert action: $S_{\rm EH}[g] = M_{\rm P}^2 \int {\rm d}^4x\sqrt{g}\left(R(g) - 2\Lambda\right)$

Einstein's equations: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$

Maximally symmetric solutions: $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$

Linear perturbations of Einstein's equations, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma}\delta g_{\rho\sigma} = 0 \qquad \bar{\mathcal{E}} \sim \nabla\nabla + \Lambda$$

➩ equation for a massless spin-2 field with 2 degrees of freedom,

tensor analogue of $\Box\phi = 0$

Hamiltonian analysis ➩ 2 d.o.f. also at the nonlinear level

General Relativity

=

unique description of

self-interacting massless spin-2 field

Linear Massive Gravity

Fierz & Pauli (1939)

Equation for a massive spin-2 field:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

tensor analogue of $\Box\phi - m^2\phi = 0$

- propagates 5 degrees of freedom for $\mathbf{a} = \mathbf{1}$
- for $\mathbf{a} \neq \mathbf{1}$ there is an additional scalar mode which gives rise to a ghost instability (negative kinetic energy)

⇨ need extra constraint to remove the 

Can we write down
a nonlinear mass term ?

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

Examples: scalar (spin 0) vector (spin 1)

$$-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2 \qquad - g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} - m^2 g^{\mu\nu}A_\mu A_\nu$$

For the spin-2 tensor contracting indices of the metric gives: $g^{\mu\nu}g_{\mu\nu} = 4$

This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu}f_{\mu\nu} = \mathrm{Tr}\left(g^{-1}f\right) \qquad f^{\mu\nu}g_{\mu\nu} = \mathrm{Tr}\left(f^{-1}g\right)$$

⇨ Massive gravity action: $S_{\mathrm{MG}}[g] = S_{\mathrm{EH}}[g] - \int \mathrm{d}^4x\, V(g,f)$

kinetic term mass term

Nonlinear Mass Term

... should [...] derivatives nor loose indices.

Examples: sc[...] 1)

$-g$ [...] $g^{\mu\nu} A_\mu A_\nu$

For the spin-2 [...] $\nu = 4$

This is not a ma[...]

Simplest way ou[...]:

$g^{\mu\nu} f_{\mu\nu}$ [...] $g)$

What determines $f_{\mu\nu}$?

Shouldn't it be dynamical ?

➪ Massive gr[...] action: $S_{\mathrm{MG}}[g] = S_{\mathrm{EH}}[g] - \int \mathrm{d}^4 x\, V(g, f)$

kinetic term mass term

Bimetric Theory

Nonlinear action for two interacting tensors:

$$
\begin{aligned}
S_\mathrm{b}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, \Big(R(g) - 2\Lambda\Big) \\
&+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, \Big(R(f) - 2\tilde{\Lambda}\Big) - \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks good, but what about  ?

The Nonlinear Ghost

Can we extend the Fierz-Pauli mass term by nonlinear interactions ?

$$\frac{m_{\mathrm{FP}}^2}{2}\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) + \mathbf{c_1}\delta g_{\mu}{}^{\rho}\delta g_{\rho\nu} + \mathbf{c_2}\delta g\delta g_{\mu\nu} + \dots$$

Can we choose coefficients $\mathbf{c_i}$ such that the  remains absent ?

Boulware & Deser (1972): Beyond linear order this is impossible!

No consistent nonlinear massive gravity / bimetric theory ?

The Ghost-Free Theory

 - free Bimetric Theory

de Rham, Gabadadze, Tolley (2010);
Hassan, Rosen, ASM, von Strauss (2011)

$$S_{\mathrm{b}}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)$$

$$V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right)$$

- arbitrary spin-2 mass scale m
- 3 interaction parameters β_n
- square-root matrix S defined through $S^2 = g^{-1}f$

 - free Bimetric Theory

de Rham, Gabadadze, Tolley (2010);
Hassan, Rosen, ASM, von Strauss (2011)

$$S_{\rm b}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)$$

$$V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)$$

elementary symmetric polynomials:

$$e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)$$

$$e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)$$

Ghost-free bimetric theory

=

unique description of
massless + massive spin-2

Perturbative Expansion

Proportional solutions

Hassan, ASM, von Strauss (2012)

Ansatz: $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with $c =$ const.

- gives two copies of Einstein's equations $(\alpha \equiv m_f/m_g)$:

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$
$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

- consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines c

⇨ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

Mass spectrum

Hassan, ASM, von Strauss (2012)

Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

Can be diagonalised into mass eigenstates:

$$\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu} \quad \text{massless (2 d.o.f.)}$$

$$\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \quad \text{massive (5 d.o.f.)}$$

Linearised equations:

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta G_{\rho\sigma} = 0$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta M_{\rho\sigma} + \frac{m_{\rm FP}^2}{2} \left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \delta M \right) = 0$$

with Fierz-Pauli mass $m_{\rm FP} = m_{\rm FP}(\alpha, \beta_n, c)$

Structure of Vertices

Hassan, ASM, von Strauss (2012)

(bimetric action expanded in mass eigenstates)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$

what about higher orders?

$$S_{(2)} = \tfrac{1}{2}\int \mathrm{d}^4x\Big[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma} + \delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} - \tfrac{m_{\rm FP}^2}{2}(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2) - \tfrac{1}{m_{\rm Pl}}\Big(\delta G^{\mu\nu} - \alpha\,\delta M^{\mu\nu}\Big)T_{\mu\nu}\Big]$$

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\rm GM} = &- \frac{m^2_{\rm FP}(1+\alpha^2)(\beta_1+\beta_2)}{4\alpha\,\mu^2}\, e_3(\delta M) \\
&- \frac{m^2_{\rm FP}}{24\alpha}\Big[-2[\delta M]^3 + 9[\delta M][\delta M^2] - 7[\delta M^3] \\
&\qquad + \alpha\left(-3[\delta G][\delta M]^2 + 12[\delta M][\delta G\delta M] + 3[\delta G][\delta M^2] - 12[\delta G\delta M^2]\right) \\
&\qquad + \alpha^2\left([\delta M]^3 - 6[\delta M][\delta M^2] + 5[\delta M^3]\right)\Big] \\
&- \frac{\Lambda}{4}\Big[[\delta G][\delta M]^2 - 4[\delta M][\delta G\delta M] - 2[\delta G][\delta M^2] + 8[\delta G\delta M^2]\Big] \\
&+ \frac{1}{4}\Big[\delta G^{\mu\nu}\Big(\nabla_\mu\delta M_{\rho\sigma}\nabla_\nu\delta M^{\rho\sigma} - \nabla_\mu\delta M\nabla_\nu\delta M + 2\nabla_\nu\delta M\nabla_\rho\delta M_\mu{}^\rho + 2\nabla_\nu\delta M_\mu{}^\rho\nabla_\rho\delta M \\
&\quad - 2\nabla_\rho\delta M\nabla^\rho\delta M_{\mu\nu} + 2\nabla_\rho\delta M_{\mu\nu}\nabla_\sigma\delta M^{\rho\sigma} - 4\nabla_\nu\delta M_{\rho\sigma}\nabla^\sigma\delta M_\mu{}^\rho - 2\nabla_\rho\delta M_{\nu\sigma}\nabla^\sigma\delta M_\mu{}^\rho \\
&\quad + 2\nabla_\sigma\delta M_{\nu\rho}\nabla^\sigma\delta M_\mu{}^\rho\Big) \\
&\quad + \frac{1}{2}\delta G\left(\nabla_\rho\delta M\nabla^\rho\delta M - \nabla_\rho\delta M_{\mu\nu}\nabla^\rho\delta M^{\mu\nu} - 2\nabla_\rho\delta M\nabla_\mu\delta M^{\mu\rho} + 2\nabla_\rho\delta M_{\mu\nu}\nabla^\nu\delta M^{\mu\rho}\right)\Big] \\
&+ \frac{1}{2}\Big[\delta M^{\mu\nu}\Big(\nabla_\mu\delta G_{\rho\sigma}\nabla_\nu\delta M^{\rho\sigma} - \nabla_\mu\delta G\nabla_\nu\delta M + \nabla^\rho\delta G_{\rho\mu}\nabla_\nu\delta M + \nabla_\nu\delta G_{\mu\rho}\nabla^\rho\delta M \\
&\quad - \nabla_\rho\delta G_{\mu\nu}\nabla^\rho\delta M + \nabla_\rho\delta G^{\rho\sigma}\nabla_\sigma\delta M_{\mu\nu} - 2\nabla_\mu\delta G^{\rho\sigma}\nabla_\sigma\delta M_{\nu\rho} + \nabla_\mu\delta G\nabla^\rho\delta M_{\rho\nu} \\
&\quad + \nabla^\rho\delta G_{\mu\nu}\nabla^\sigma\delta M_{\rho\sigma} - 2\nabla_\rho\delta G_{\mu\sigma}\nabla_\nu\delta M^{\rho\sigma} - 2\nabla^\rho\delta G_{\mu\sigma}\nabla^\sigma\delta M_{\nu\rho} + 2\nabla^\rho\delta G_{\mu\sigma}\nabla_\rho\delta M_\nu{}^\sigma \\
&\quad + \nabla^\rho\delta G\nabla_\nu\delta M_{\mu\rho} - \nabla^\rho\delta G\nabla_\rho\delta M_{\mu\nu}\Big) \\
&\quad + \frac{1}{2}\delta M\Big(\nabla_\rho\delta G\nabla^\rho\delta M - \nabla_\rho\delta G_{\mu\nu}\nabla^\rho\delta M^{\mu\nu} - \nabla_\rho\delta G\nabla_\sigma\delta M^{\rho\sigma} \\
&\quad - \nabla_\rho\delta G^{\rho\sigma}\nabla_\sigma\delta M + 2\nabla_\rho\delta G_{\mu\nu}\nabla^\nu\delta M^{\mu\rho}\Big)\Big]
\end{aligned}
$$

Higher-Order Vertices

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$

Cubic (suppressed by $m_{\rm Pl}^{-1}$)

δG^3	$\delta G^2 \delta M$	$\delta G \delta M^2$	δM^3
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$	$\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$ $\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$

- self-interactions of massless spin-2 sum up to Einstein-Hilbert action

$$S(g,f)|_{\delta M=0} = m_{\rm Pl}^2 \int \mathrm{d}^4x\,\sqrt{|G|}\,\left(R(G) - 2\Lambda\right) \qquad G_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{m_{\rm Pl}}\delta G_{\mu\nu}$$

- no terms linear in massive fluctuation

Ghost-free bimetric theory

=

Massive spin-2 in background set by massless spin-2

What is the physical metric ?

How much does the theory differ from GR ?

General Relativity Limit

Matter Coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \;-\; \int \mathrm{d}^4x\, V(g,f) \\
&\qquad + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g,\phi)
\end{aligned}
$$

Absence of ghosts: only one metric can couple to matter!

⇨ $g_{\mu\nu}$ is gravitational metric

Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

$$S_{\rm b}[g,f] = m_g^2 \int d^4x \sqrt{g}\, R(g) + m_f^2 \int d^4x \sqrt{f}\, R(f) - \int d^4x\, V(g,f) + \int d^4x \sqrt{g}\, \mathcal{L}_{\rm matter}(g,\phi)$$

(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu} \qquad (\alpha \equiv m_f/m_g)$$

massless massive

The gravitational metric is not massless but a superposition of mass eigenstates.

Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

$$S_{\rm b}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\rm matter}(g,\phi)$$

(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu} \qquad (\alpha \equiv m_f/m_g)$$

massless massive

- for small $\alpha = m_f/m_g$ gravity is dominated by the massless mode
- the massive spin-2 field interacts only weakly with matter

Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

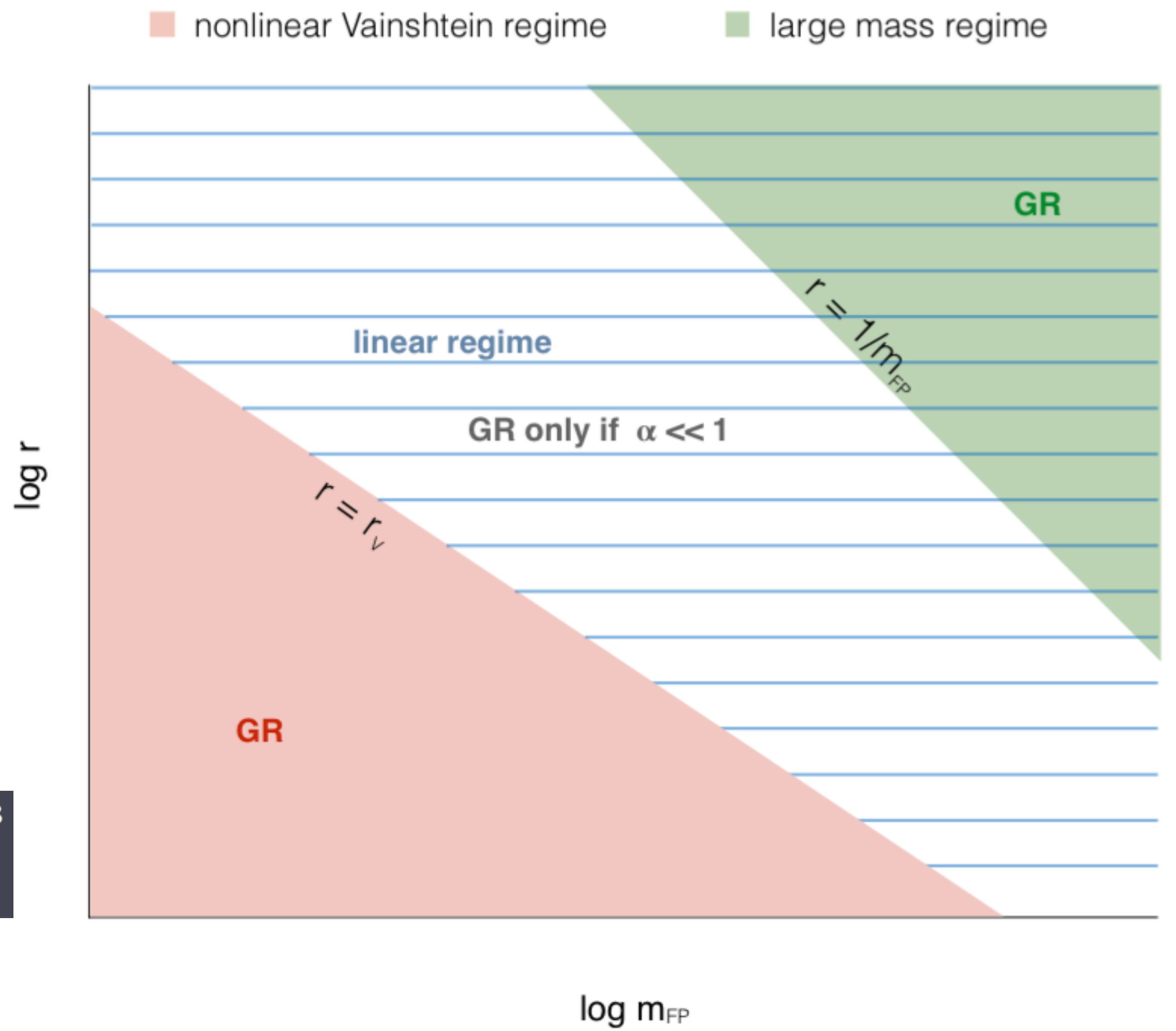
$$
\begin{aligned}
S_{\rm b}[g,f] \quad = \quad & m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
+ \quad & m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \; - \; \int \mathrm{d}^4x \, V(g,f) \\
& + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\rm matter}(g,\phi)
\end{aligned}
$$

$$\alpha = m_f / m_g \to 0$$

is the General Relativity limit of bimetric theory

Recovery of GR

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)



$$r_V = \left(\frac{r_S}{m_{FP}^2}\right)^{1/3}$$

Ghost-free bimetric theory

=

General Relativity +
additional tensor field

Cosmology

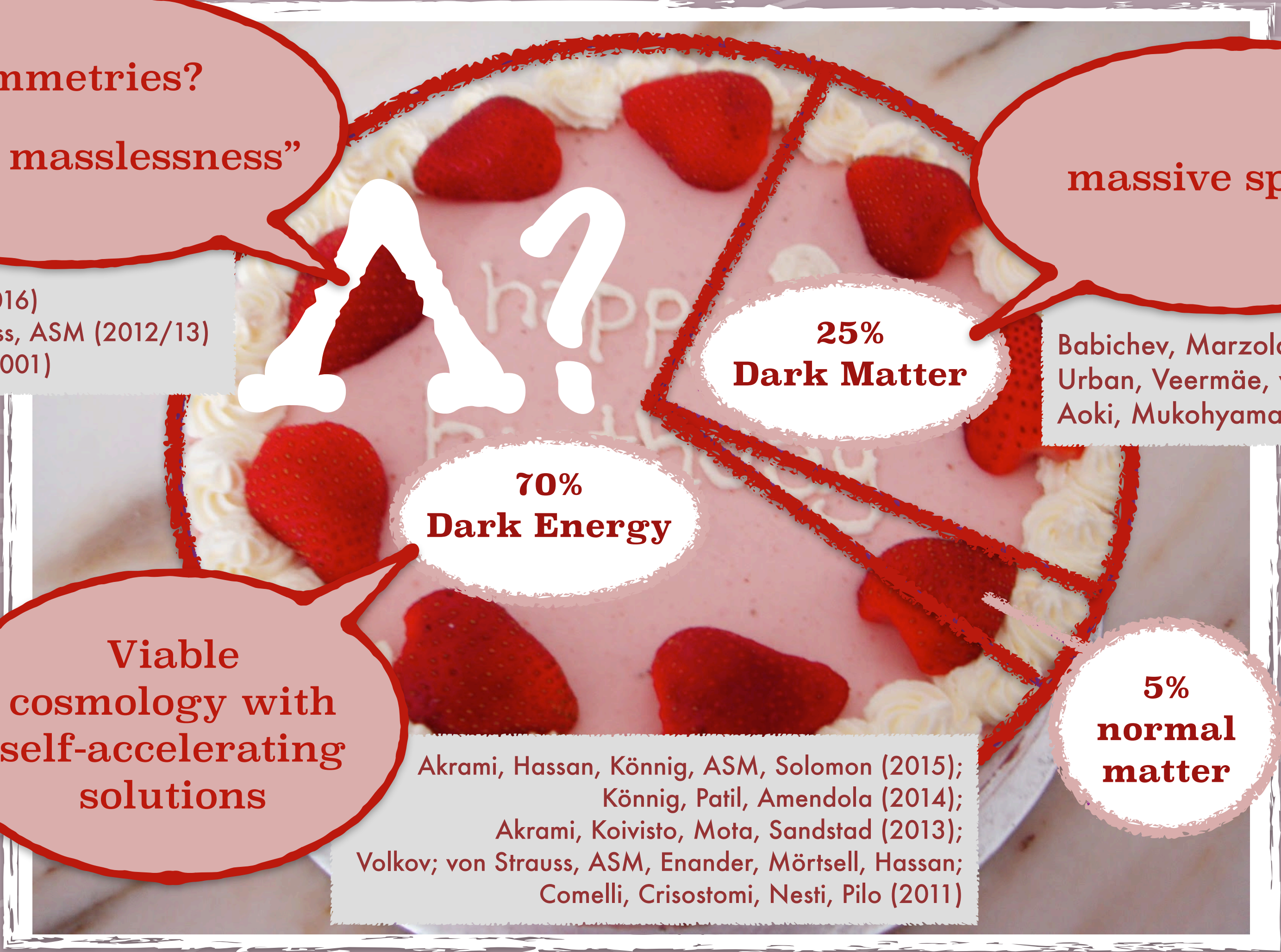
Symmetries?

“partial masslessness”

Apolo, Hassan (2016)
Hassan, von Strauss, ASM (2012/13)
Deser, Waldron (2001)

massive spin-2 ?

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016); Aoki, Mukohyama (2016)



Viable cosmology with self-accelerating solutions

Akrami, Hassan, Könnig, ASM, Solomon (2015);
Könnig, Patil, Amendola (2014);
Akrami, Koivisto, Mota, Sandstad (2013);
Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan;
Comelli, Crisostomi, Nesti, Pilo (2011)

Dark Matter

- unidentified type of matter comprising ~27% of energy in the universe
- very weak interactions with ordinary matter, but gravitates normally
- observed in galaxy rotation curves, cosmic microwave background, gravitational lensing, structure formation, matter power spectra, ...

⇨ Standard paradigm (tons of different models, e.g. MSSM):
cold relic density of weakly interacting massive particle (WIMP)

⇨ possibly directly detectable, but so far null results

Spin-2 Dark Matter

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Recall the (linearised) gravitational metric: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

massless massive

and the General Relativity limit of bimetric theory: $\alpha = m_f / m_g \to 0$

- ⇨ gravity is weak because the physical Planck mass is large ($m_{\rm Pl} = m_g \sqrt{1+\alpha^2}$)
- ⇨ massive spin-2 field decouples from matter, interacts only with gravity

Structure of Vertices

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$

Cubic

δG^3	$\delta G^2 \delta M$	$\delta G \delta M^2$	δM^3
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$	$\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$ $\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$

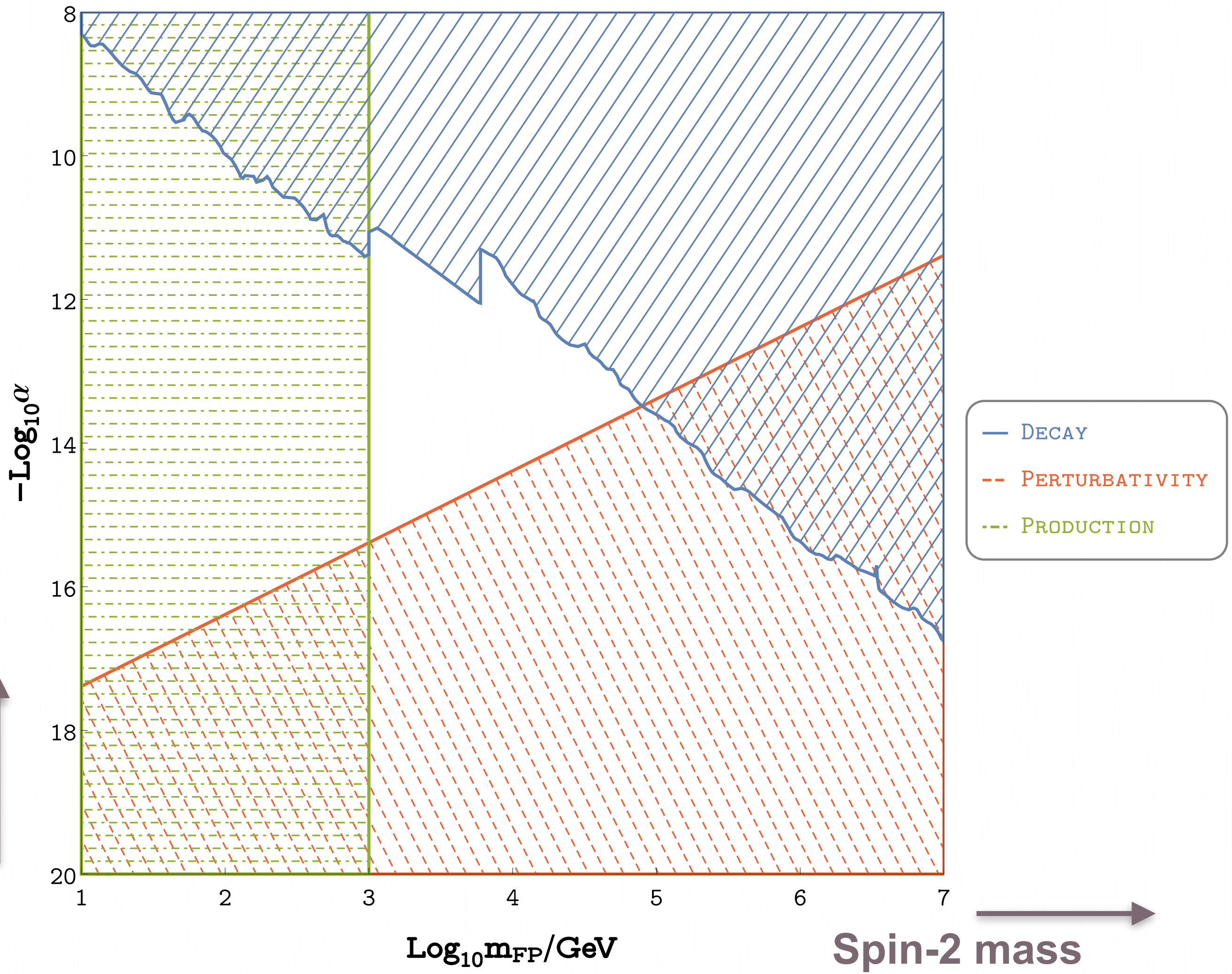
- no vertices giving rise to decay of massive into massless spin-2
- massive field gravitates just like baryonic matter, even in GR limit
- self-interactions of massive spin-2 are enhanced in the GR limit

DM mass

Decay rate:

$$\Gamma(\delta M \to XX) \sim \frac{\alpha^2 m_{FP}^3}{m_{Pl}^2}$$

ratio of
Planck masses



Spin-2 mass

Features of Spin-2 DM

- heavy spin-2 field automatically resembles dark matter when gravity resembles General Relativity
- interactions with baryonic matter are suppressed by the Planck mass
- spin-2 mass and interaction scale are on the order of a few TeV

⇨ no need for extra fields, artificial symmetries or fine tuning

⇨ bimetric theory could explain dark matter in the context of gravity

⇨ massive spin-2 field is a natural addition to the Standard Models

Summary

Ghost- free bimetric theory...

- is one of the few known consistent modifications of General Relativity
- describes nonlinear interactions of massless and massive spin-2 fields
- can be interpreted as gravity in the presence of an extra spin-2 field
- contains an interesting dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale
  (but can we detect it ?)

review: ASM, Mikael von Strauss; 1512.00021

Thank you for your attention!