Black-hole formation in 2d CFT

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String Theory and Quantum Gravity, Ascona
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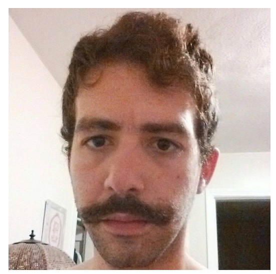
work with



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Setting the stage

AdS/CFT relates gravity (often in AdS) to unitary field theory (often CFT)
Familiar notions of quantum field theory are geometrized

Want to explore CFT ➛ (quantum) gravity
recent revival of interest in low-D toy models (AdS_3/CFT_2, SYK,…)

➙ relevant developments in CFT, many-body physics:

- time evolution and spread of entanglement
- thermalization of closed quantum systems (e.g. via eigenstates)
- non-perturbative methods (e.g. bootstrap)

Thermalization ➛ BH formation (& evaporation)

Unitarity at stake

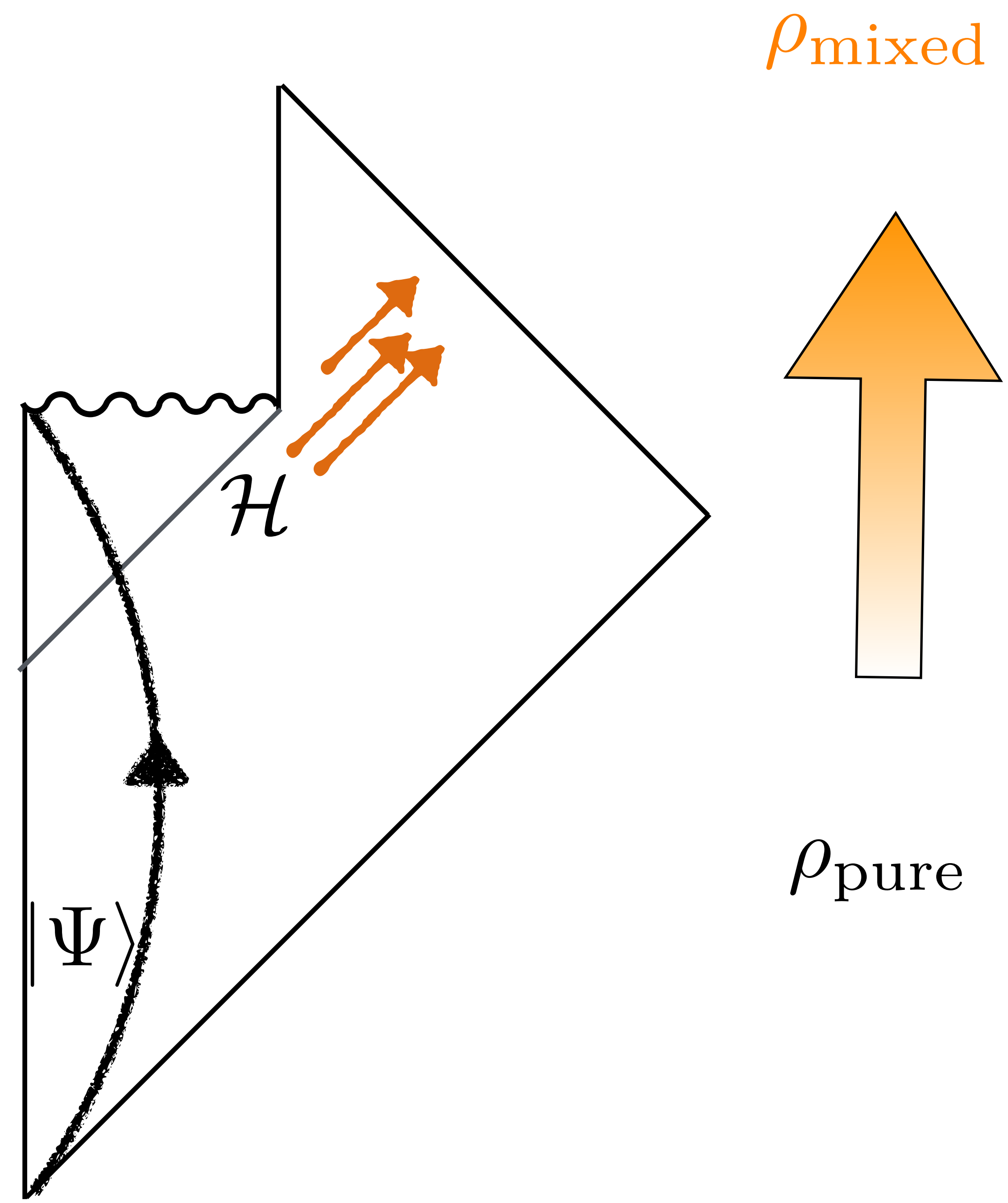
[Hawking, Maldacena]

- gravity as an EFT implies pure to mixed evolution
- fundamentally incompatible with a unitary S-matrix

Use simplified laboratory of AdS_3/CFT_2

1. Signatures of information loss in CFT correlations @ large c

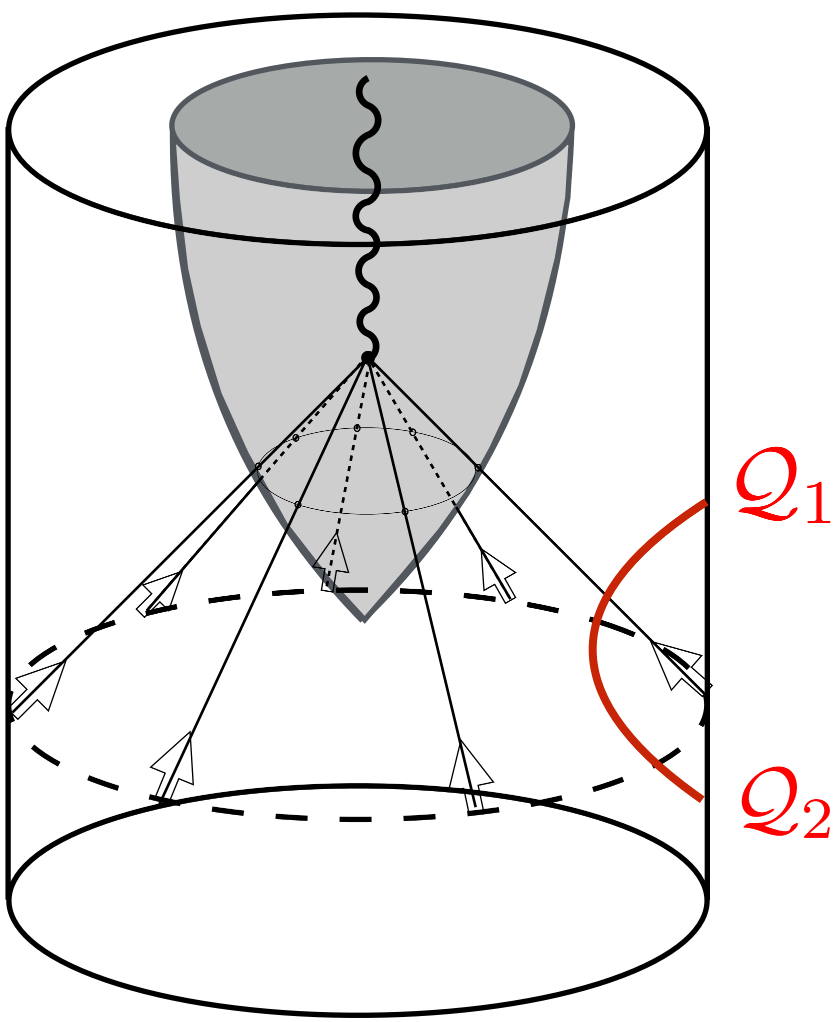
2. New results on bulk-boundary relation in semiclassical limit



Approach

Tension with unitarity is sharpest for collapsing black hole

➞ how do we describe black-hole collapse in CFT?



$$\langle \mathcal{V} | \mathcal{Q}_1(t,0)\mathcal{Q}_2(0) | \mathcal{V} \rangle$$

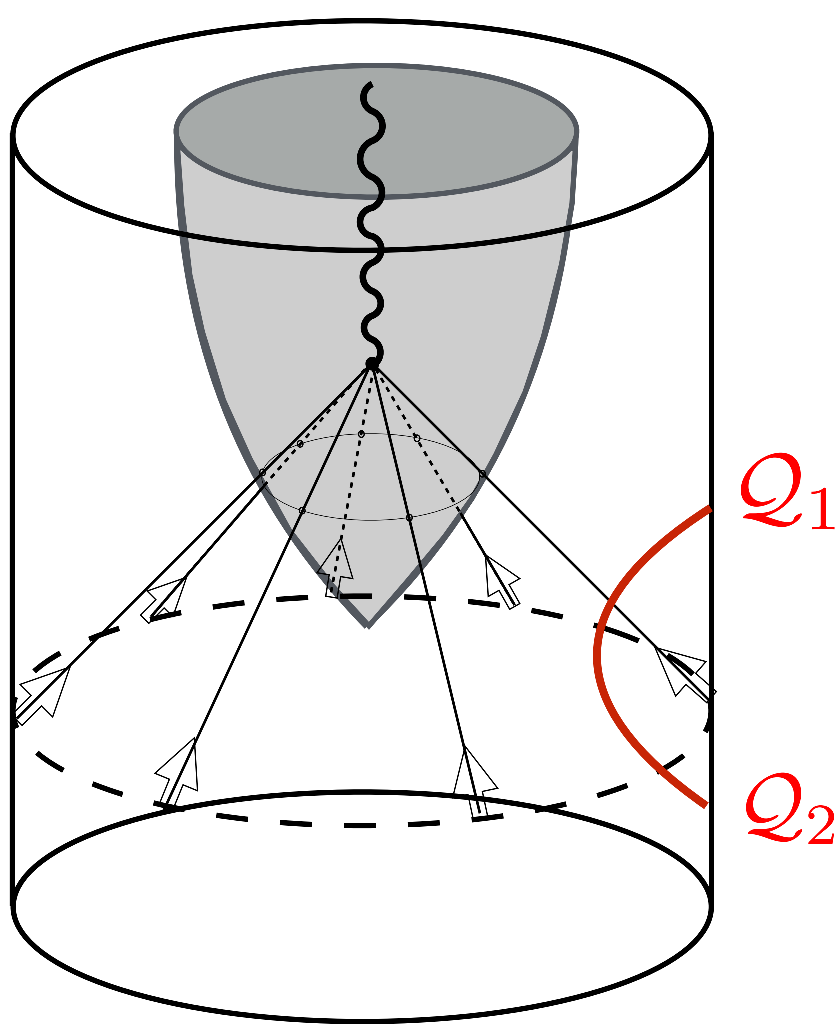
$|\mathcal{V}\rangle$ heavy pure state ➞ BH collapse

measure correlations of light probe operators $\mathcal{Q}$

Results

Follow CFT from quench to thermalisation at large c
[also: Calabrese, Cardy; Hartman, Maldacena]

Calculate Lorentzian physics via continuum monodromy method: entanglement, autocorrelation,…



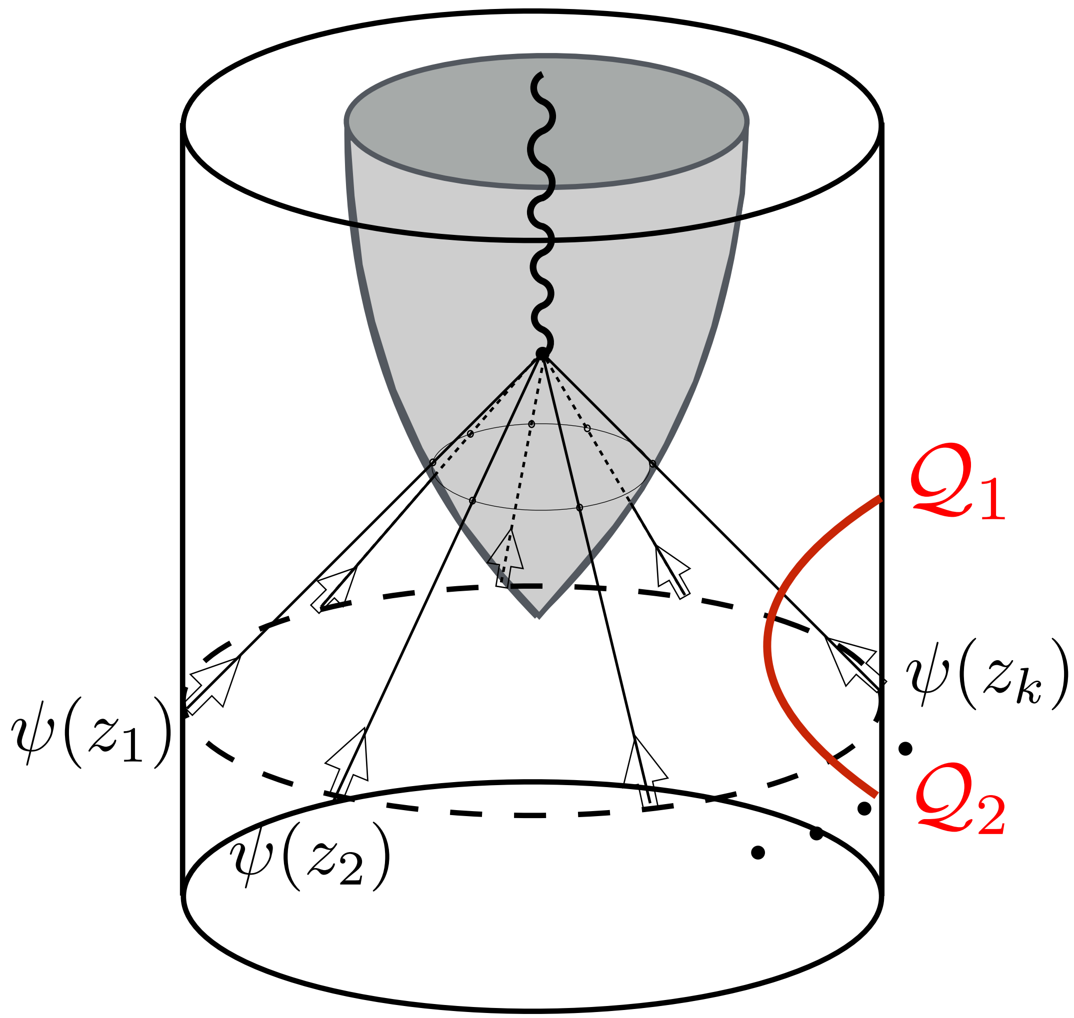
Results at large c: match gravity calculations in Vaidya

Autocorrelation: signs of information loss and retrieval

General correlation function: from conformal blocks to path integral

information loss in CFT

The Black Hole in the Tin Can



global AdS3

Throw in a shell of n dust particles

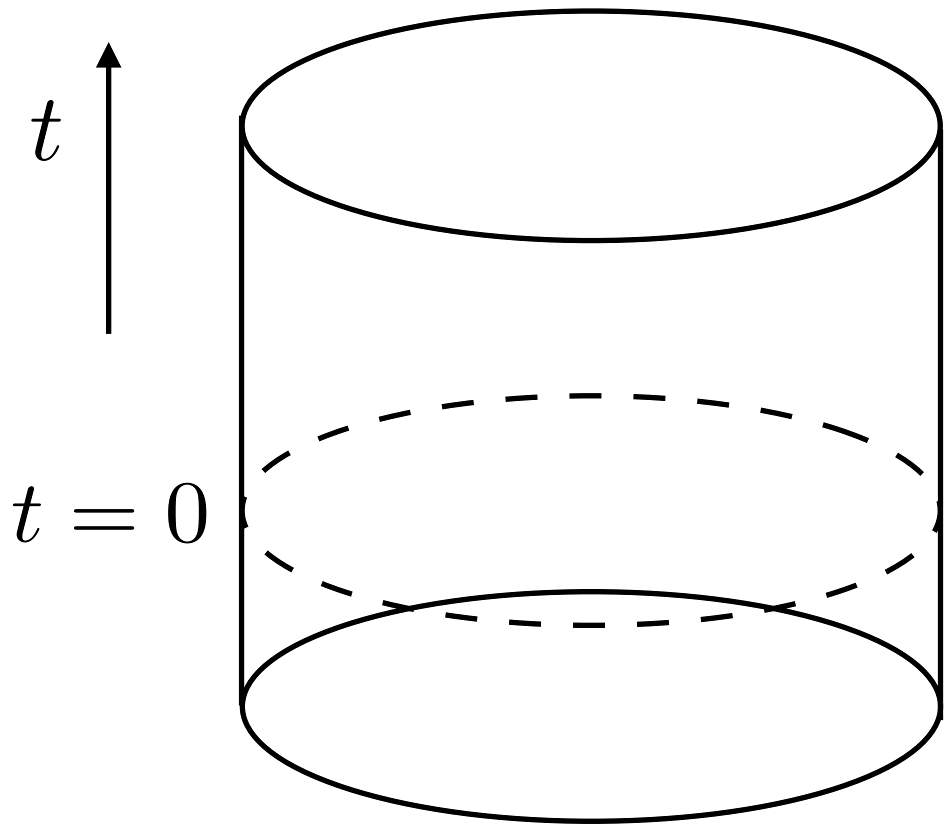
$$S = S_{\rm EH} + \sum_{I=1}^{n} S_i[\text{particle}]$$

smooth limit: $n \to \infty$

bulk BH collapse: Vaidya metric

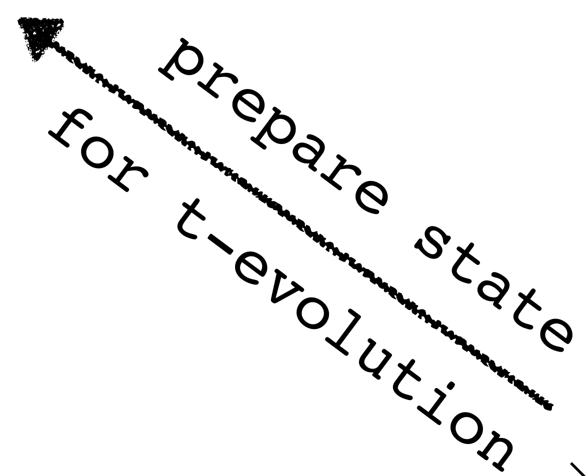
remark: certain quantities such as entanglement entropy are sensitive to behind horizon physics (away from equilibrium)

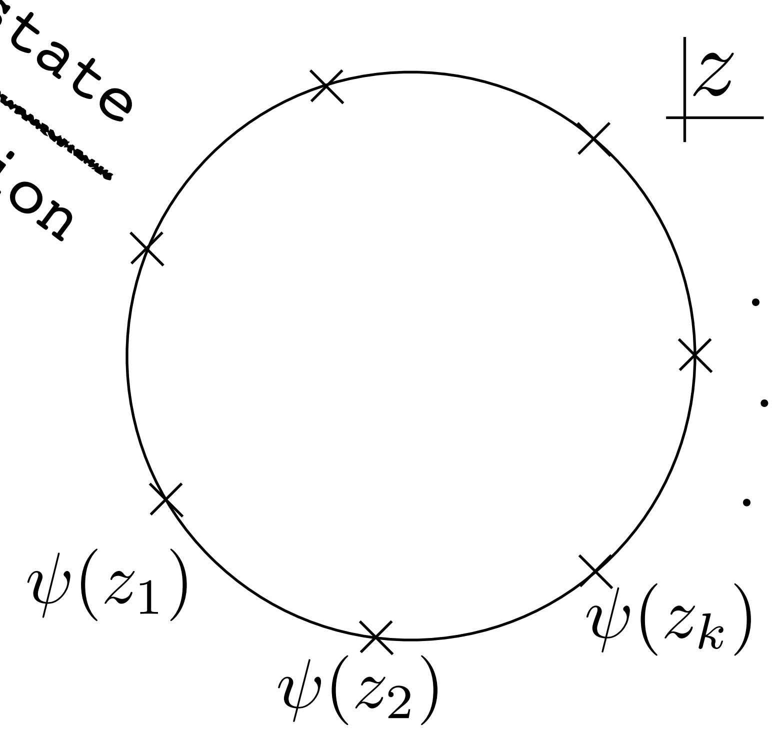
Translating to the CFT



start in excited state at t=0:

prepare by Euclidean path integral
➛ regulator σ





$$|\mathcal{V}\rangle = \frac{1}{\mathcal{N}} \prod_{k=1}^{n} \psi(e_k, \bar{e}_k)|0\rangle$$

ψ is primary

Vacuum dominance

in the semi-classical limit (large c), get sum of exponentials

$$\langle \mathcal{V} | \mathcal{Q}_1(x_1) \mathcal{Q}_1(x_2) | \mathcal{V} \rangle = \sum_{\text{blocks}} a_k e^{-\frac{c}{6} f_k^{(n)}(x_1, x_2)}$$

correlator approximated by largest term, the identity block

"it from **id**"

the dominant contribution comes from the identity Virasoro block, that is the unit operator **id** *and all its descendants $T, \partial T, T^2\ T\partial T\ldots$, (multi-graviton exchange in bulk)*

subleading corrections exponentially suppressed in $e^{-c} \sim e^{-1/G}$

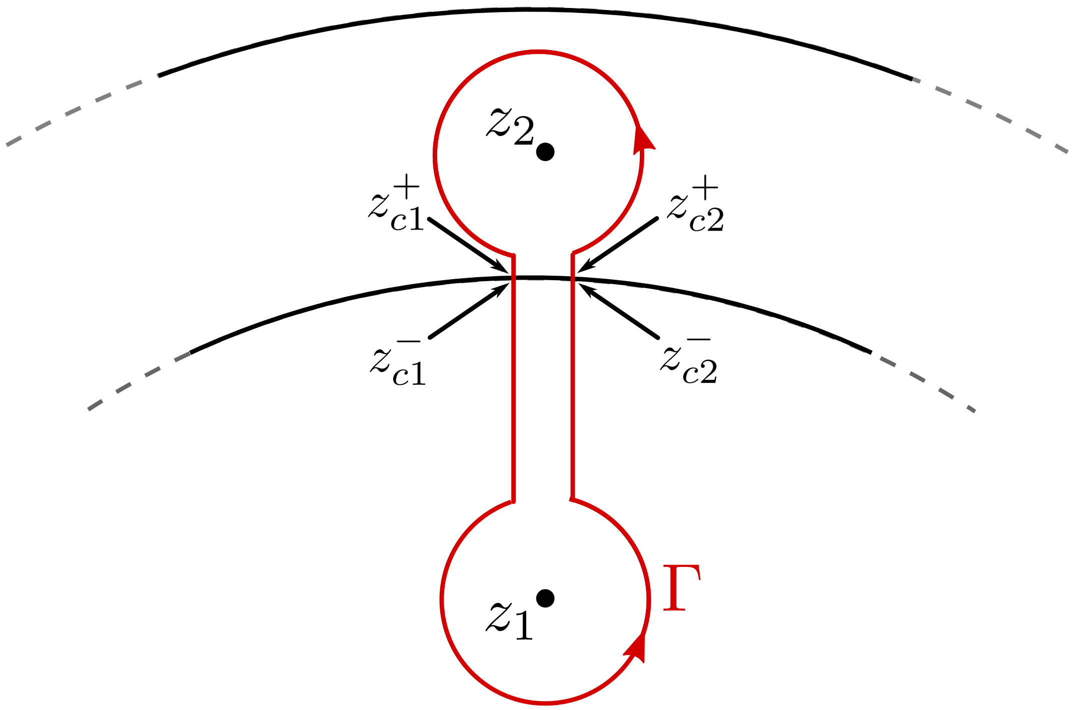
Autocorrelation

let us now return to the black hole and compute

$$G(t_1,t_2) = \langle \mathcal{V} | \mathcal{Q}_1(t_1,0)\mathcal{Q}_2(t_2,0) | \mathcal{V} \rangle$$

Dominated by a **single** id channel

$$\mathcal{F}_0^{\Gamma(0)} = \exp\left[-\frac{c}{6} f_0^{\infty}(t_1,t_2)\right]$$



Determine semiclassical block
from monodromy problem
[Zamolochikov]

$$\boxed{G(t_1,t_2) = \left(\frac{1}{\pi T}\cos\left(\frac{t_1}{2}\right)\sinh\left(\pi T t_2\right) - 2\sin\left(\frac{t_1}{2}\right)\cosh\left(\pi T t_2\right)\right)^{-2\Delta^{\mathcal{Q}}}}$$

Late Lorentzian times

Let us return to the original question of information loss

The correlation function decays without bound at large time

$$\boxed{G(t_1, t_2) \sim \exp\left(-\frac{2\pi\Delta^{\mathcal{Q}} t}{\beta}\right)}$$

Manifestly in conflict with unitarity: **CFT loses information!**

But leading result comes with non-perturbative corrections

$$G(t_1, t_2) = a_0 \underbrace{e^{-\frac{c}{6} f_0^\infty}}_{\text{Vaidya geometry}} + \sum_{k \neq \text{vac}} a_k \underbrace{e^{-\frac{c}{6} f_k^\infty}}_{\text{Other states}}$$

On information loss

This is the anti-information paradox: what happened to unitarity?

$$\overline{|G(t)|} = \overline{\left|\sum_{n,k} e^{i(E_n - E_k)t} \Psi_n^*(\mathcal{V}) \langle n|\mathcal{Q}|k\rangle \langle k|\mathcal{Q}|\mathcal{V}\rangle\right|} \neq 0$$

➙ (average) correlations cannot become arbitrarily small

Neglected non-perturbative corrections. They contribute

$$\sum_{k \neq \text{vac}} a_k e^{-\frac{c}{6} f_k^{\infty}(1,2,\ldots p)} \sim e^{-S}$$

restore unitary at large time ➙ non-perturbative effects in $1/G_N$

Comments

Boundary story is that of thermalization.
Non-unitary truncation, corresponds to leading bulk answer

Can investigate similar questions for heavy eigenstates

$$\langle \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \mathcal{O}_H \rangle \sim \langle \mathcal{O}_L \mathcal{O}_L \rangle_{T_H}$$

Closely related to study of ETH in CFT [Dymarsky et al.; Datta et al.]

[Kaplan et al.] looked at contributions from higher blocks:
non-exponential late time behaviour $t^{-3/2}$

Not good enough: need to sum over all heavy blocks
Similar story for spectral form factor [Dyer & Gur-Ari]

from conformal blocks to path integrals

General correlation function

suppose we would like to compute

$$G(t_1, x_1 | t_2, x_2) = \langle \mathcal{V} | \mathcal{Q}_1(t_1, x_1) \mathcal{Q}_2(t_2, x_2) | \mathcal{V} \rangle$$

no longer dominated by a **single** id channel. Prescription:

$$\boxed{G(t_1, x_1 | t_2, x_2) = \int dx_c \left| \mathcal{F}_0^{\Gamma(x_c)} \right|^2}$$

Sum over **id** in all channels (looks odd from CFT perspective)

(remark: **id** in one channel = sum over heavies in another)

Complex saddle points

consider probe with $1 \ll h_{\mathcal{Q}} \ll c$

evaluate correlator via saddle-point

$$G(t_1, x_1 | t_2, x_2) = \int dx_c \underbrace{\mathcal{F}_0^{\Gamma} \overline{\mathcal{F}_0^{\Gamma}}}_{\in \mathbb{C} \text{ (continuation to Lorentzian)}}$$

we find complex saddle points: $x_c \in \mathbb{C}$

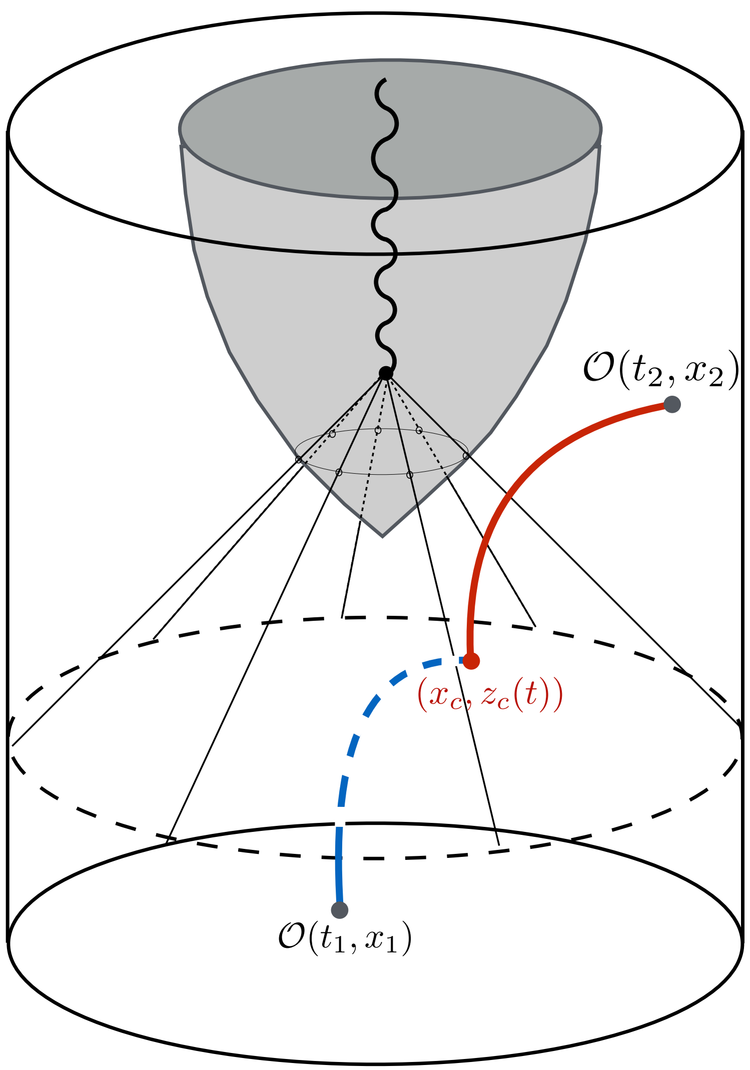
radical change of philosophy of Virasoro id block:

bulk physics is **not** well approximated by **id** in any single channel

Bulk perspective

$$G(t_1, x_1 | t_2, x_2) = \int [Dx(\tau)]\, e^{im \int d\tau}$$

$1 \ll h_{\mathcal{Q}} \ll c$ → geodesic approximation



$$\boxed{G(t_1, x_1 | t_2, x_2) = \int dx_c e^{i\Delta\mathcal{L}(x_1^\mu, x_c) + i\Delta\mathcal{L}(x_c, x_2^\mu)}}$$

Gravity saddle point = CFT saddle point

for same kinematics, get complex saddle point (analytically continued geodesic)

Comments

$\sum_{\Gamma_0}$ vs. $\max_{\Gamma_0}$ prescription:

All previous cases: real exponential, so $\sum_{\Gamma_0} = \max_{\Gamma_0}$

Lorentzian dynamics allows to distinguish, and $\sum_{\Gamma_0}$ comes out on top

Aren't we overcounting?

Usually sum over blocks, not channels

Working assumption: no overlap between **id** in different channels, when dualized in to a single channel (at large c)

Creates subtlety when looking at 1/c corrections wrapping up

Conclusions

time-dependent 3D quantum gravity with matter in 1/c expansion
'it from id' ➙ ideal arena to think about quantum BHs

CFT correlation functions seemingly violate unitarity (naïve).
non-perturbative corrections in c restore unitarity

on gravity side these correspond to non-perturbative effects in G_N.
geometric interpretation? bulk interpretation?

monodromy method identifies off-shell contributions on both sides:

General map from conformal block expansion to bulk path int?

thank you!

more details

A word on limits

We probe the physics via 2n + p correlations

$$G(1, 2, \dots p) = \langle \mathcal{V} | \mathcal{Q}_1 \, , \dots \, \mathcal{Q}_p | \mathcal{V} \rangle$$

we want to approach smooth, semi-classical gravity

$$c \to \infty$$

$$n \to \infty$$

$$\sigma \to 0$$

$$E \sim n h_\psi / \sigma \to \mathcal{O}(c)$$

Furthermore probe operators satisfy

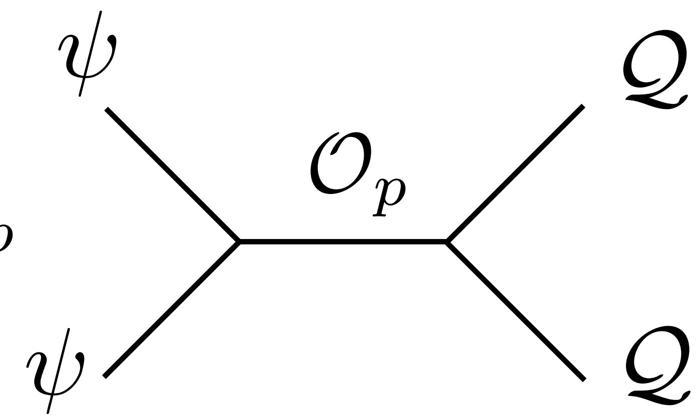
$$1 \ll h_{\mathcal{Q}} \ll c$$

Continuum monodromy method

Choice of channel

recall: 4-pt function:

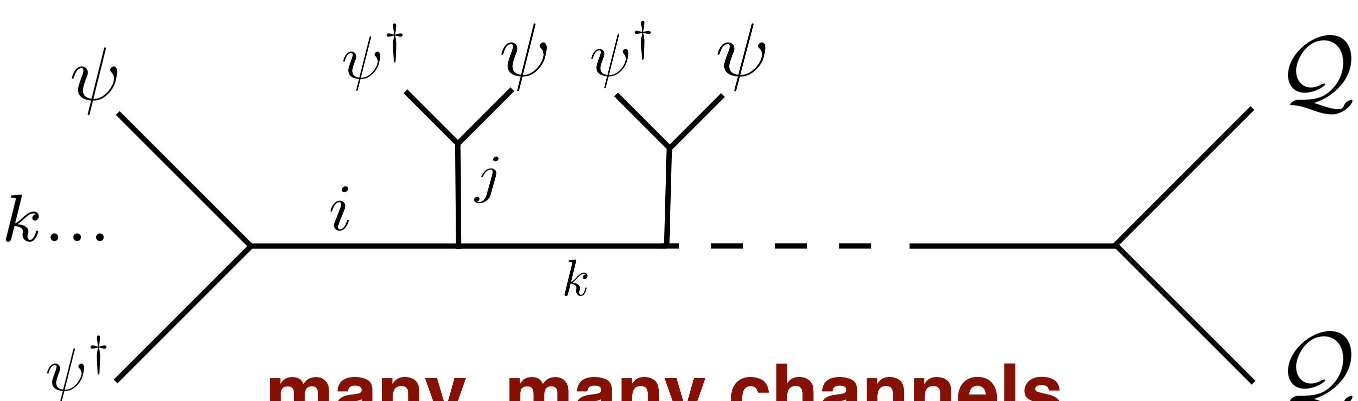
$$\langle \psi(\infty)\mathcal{Q}(z,\bar{z})\mathcal{Q}(1)\psi(0)\rangle = \sum_{\text{primaries p}} c_p$$



s & t channel

similarly can expand our correlators in conformal blocks

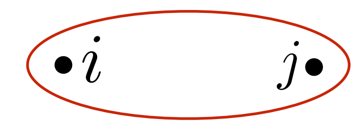
$$G(1,2) = \sum_{i,j,k,\ldots} c_{ijk\ldots}$$



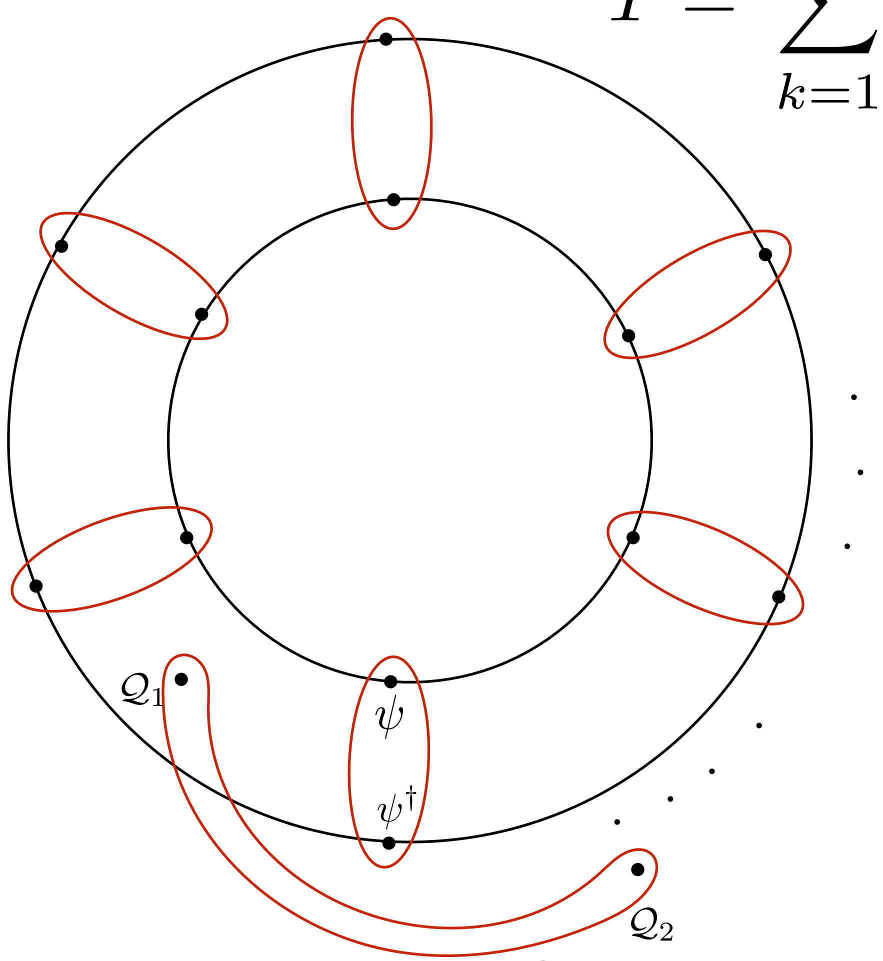
many, many channels

focus on those that propagate **id** on each internal line (=vac)

Choice of contraction

for each OPE contraction, draw a cycle 

$$T = \sum_{k=1}^{2n+p} \left[\frac{6h/c}{(z-z_k)^2} + \frac{c_k}{z-z_k} \right]$$



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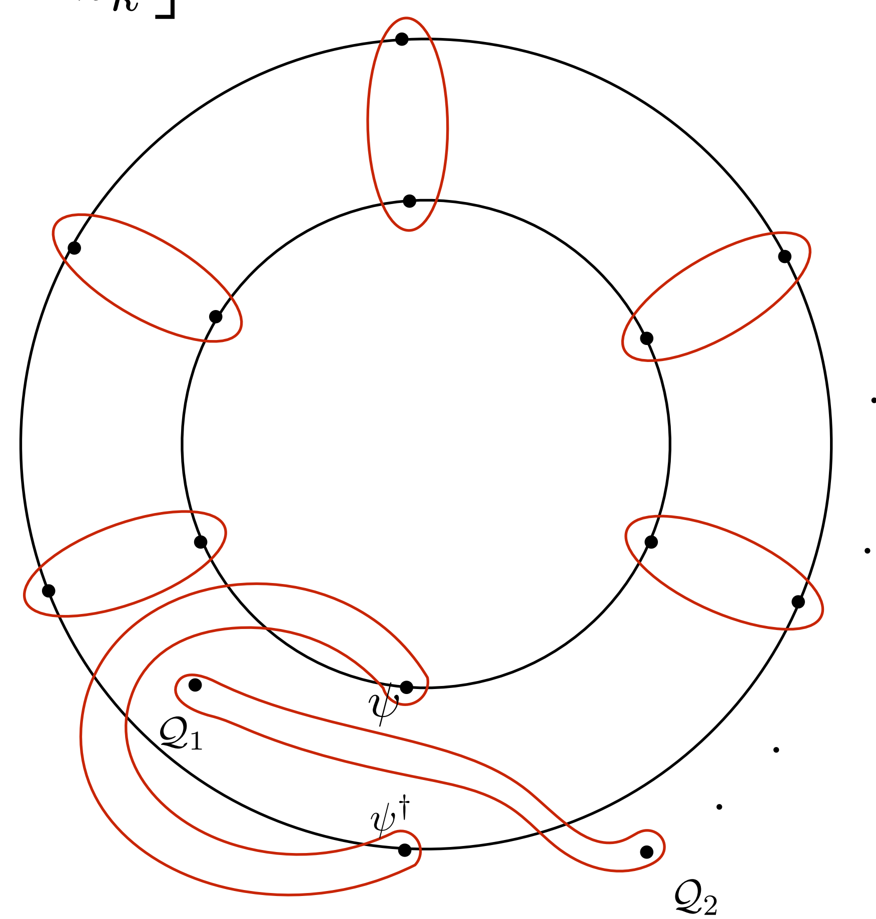


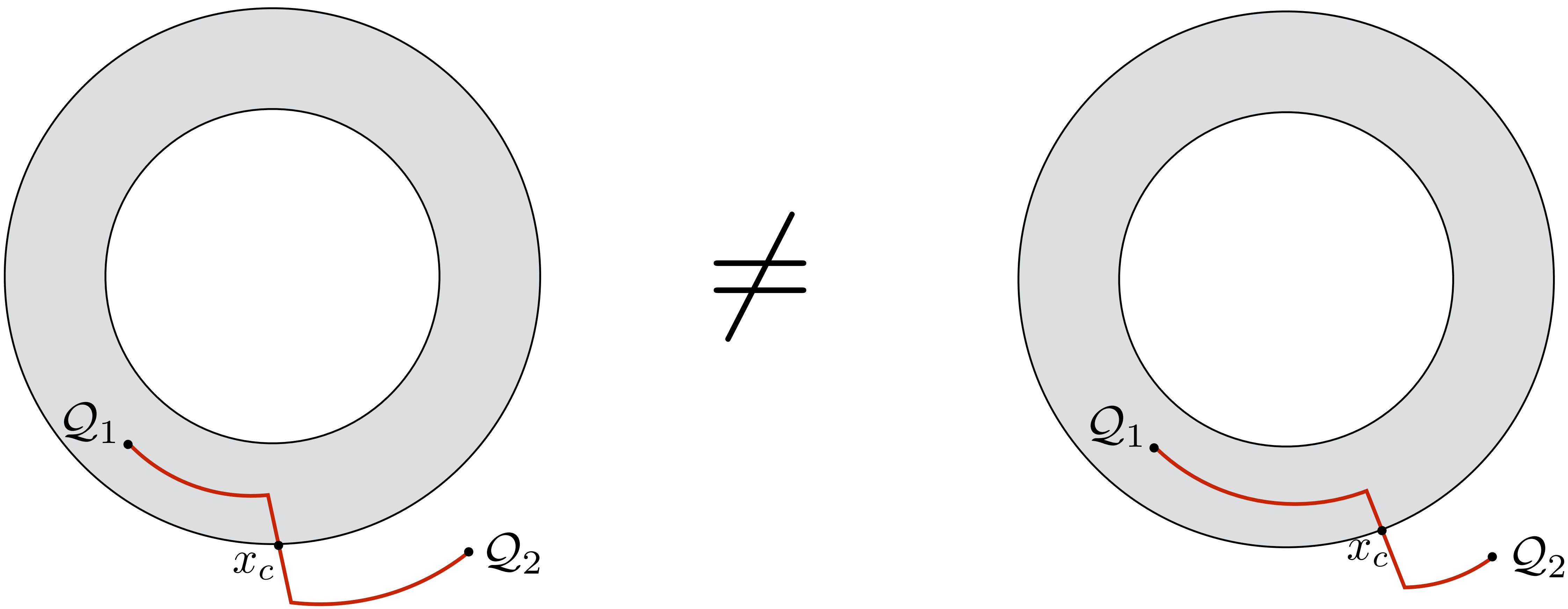
each such choice defines a certain channel Γ

fix monodromies of $y''(z) + Ty(z) = 0 \xrightarrow{c_k} f_k^{(n)}(1, 2, \ldots p)$

Taking the smooth limit

generally a hard problem, big simplification occurs for n $\longrightarrow \infty$

channel is labeled by single continuous parameter x_c



stress tensor $\longmapsto$ distribution

continuum monodromy method

$$f_0^\infty(1,2,\ldots,p)$$

$\longleftrightarrow$

3D semi-classical gravity

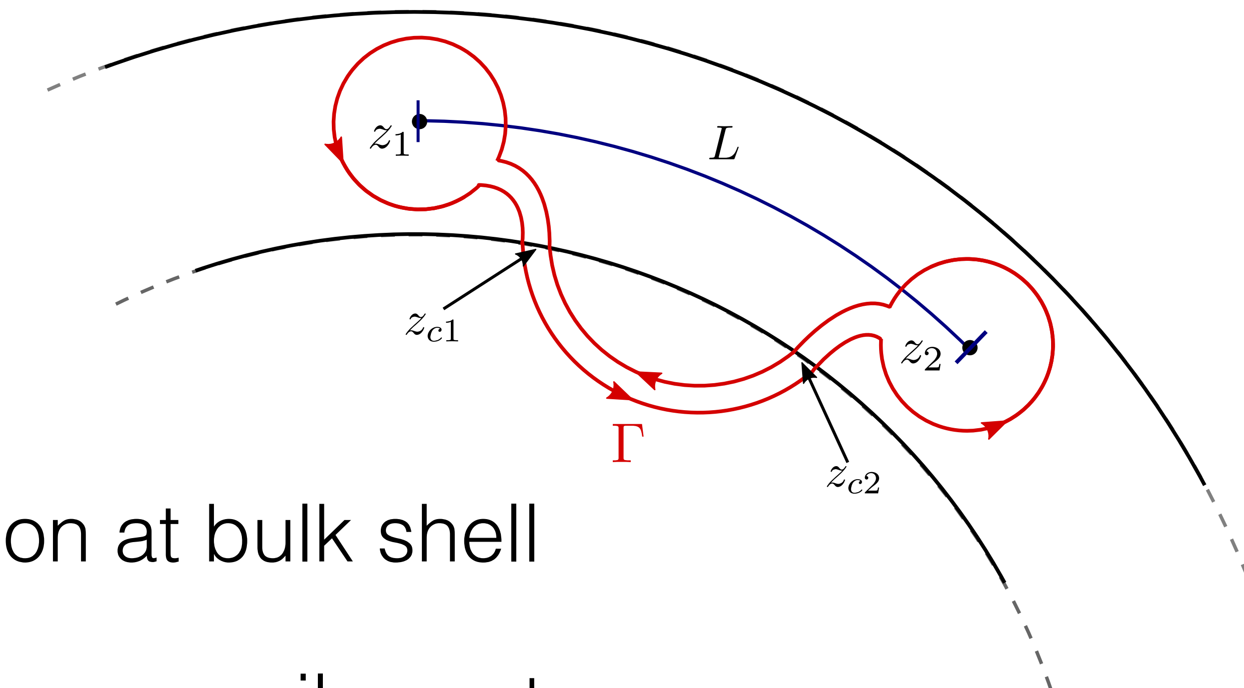
$$\mathcal{L}_{\text{geo}}(1,2,\ldots p)$$

Entanglement entropy

entanglement entropy

Q-type operators ➞ twist insertions: $G_q(t) = \langle \mathcal{V} | \sigma_q(t, \ell_1) \tilde{\sigma}_q(t, \ell_2) | \mathcal{V} \rangle$

$$\boxed{S(A) = \lim_{q \to 1} \frac{1}{1-q} G_q(t)}$$



crossing points z_{c1} & z_{c2} ↔ refraction at bulk shell

it from id ➞ require trivial monodromy on smile contour

write $z_1 = e^{i\theta_1}, z_2 = e^{i(\theta_1 + L)}$ & continue to Lorentzian time $\theta_1 = t$

maximize S(A) over crossing points ➞ parametric equation for S(t)

entanglement entropy

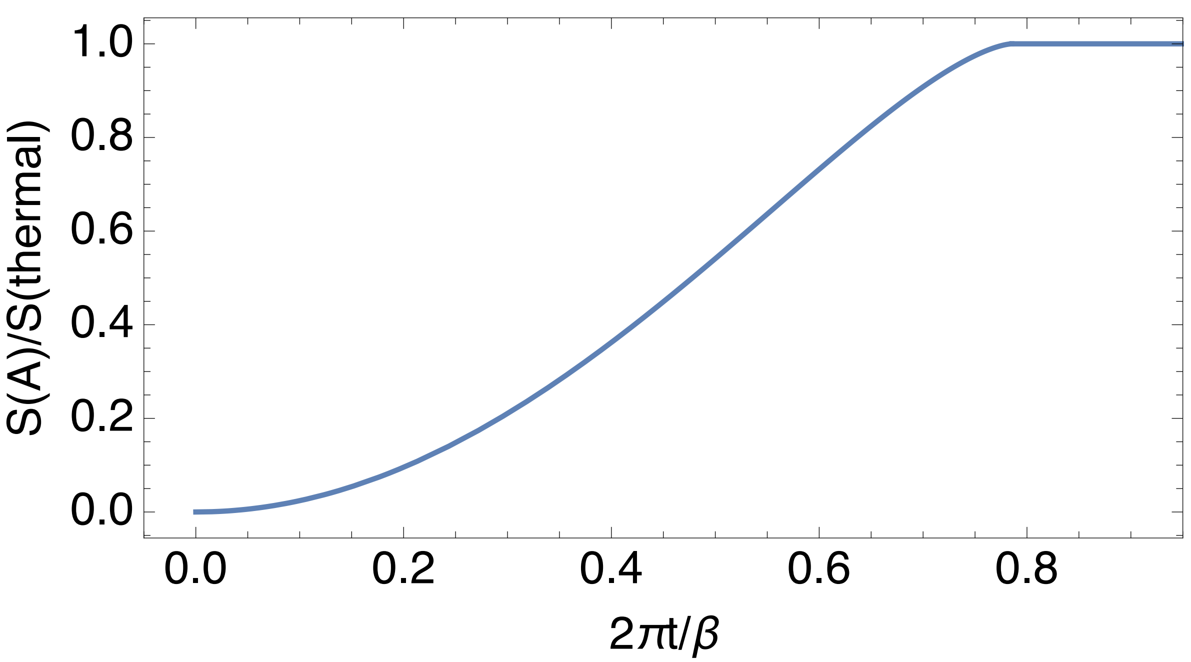
Implicit formula for growth of entanglement entropy:

$$t = \frac{\beta}{2\pi}\cosh^{-1}\left\{\cosh\left(2\pi T q\right) + 2\pi T \tan\left(\frac{L}{2}-q\right)\sinh\left(2\pi T q\right)\right\}$$

$$S_{EE} = \frac{c}{3}\log\left\{\frac{\sin\left(\frac{L}{2}-q\right)\cosh\left(2\pi T q\right) + \frac{1}{2\pi T}\left[1+\frac{1}{2}\left\{1+4\pi^2T^2\right\}\tan^2\left(\frac{L}{2}-q\right)\right]\cos\left(\frac{L}{2}-q\right)\sinh\left(2\pi T q\right)}{\epsilon_{UV}/2}\right\}$$

matches **exactly** global AdS_3 Vaidya:

- thermal at late time
- EE growth = change of channel
- sees **beyond horizon**



CFT calculation shows that purity of state is preserved: $S(A) = S(A^c)$

Unitarity vs. thermalization

Unitarity vs thermalization

(constraints on long-time correlations from unitarity)

Correlations in a closed quantum system, e.g.

$$G(t) = \mathrm{tr}\rho\mathcal{O}(t)\mathcal{O}(0)$$

Time average over a large time T cannot vanish by unitarity

$$\lim_{T\to\infty} \overline{|G(t)|^2} \neq 0$$

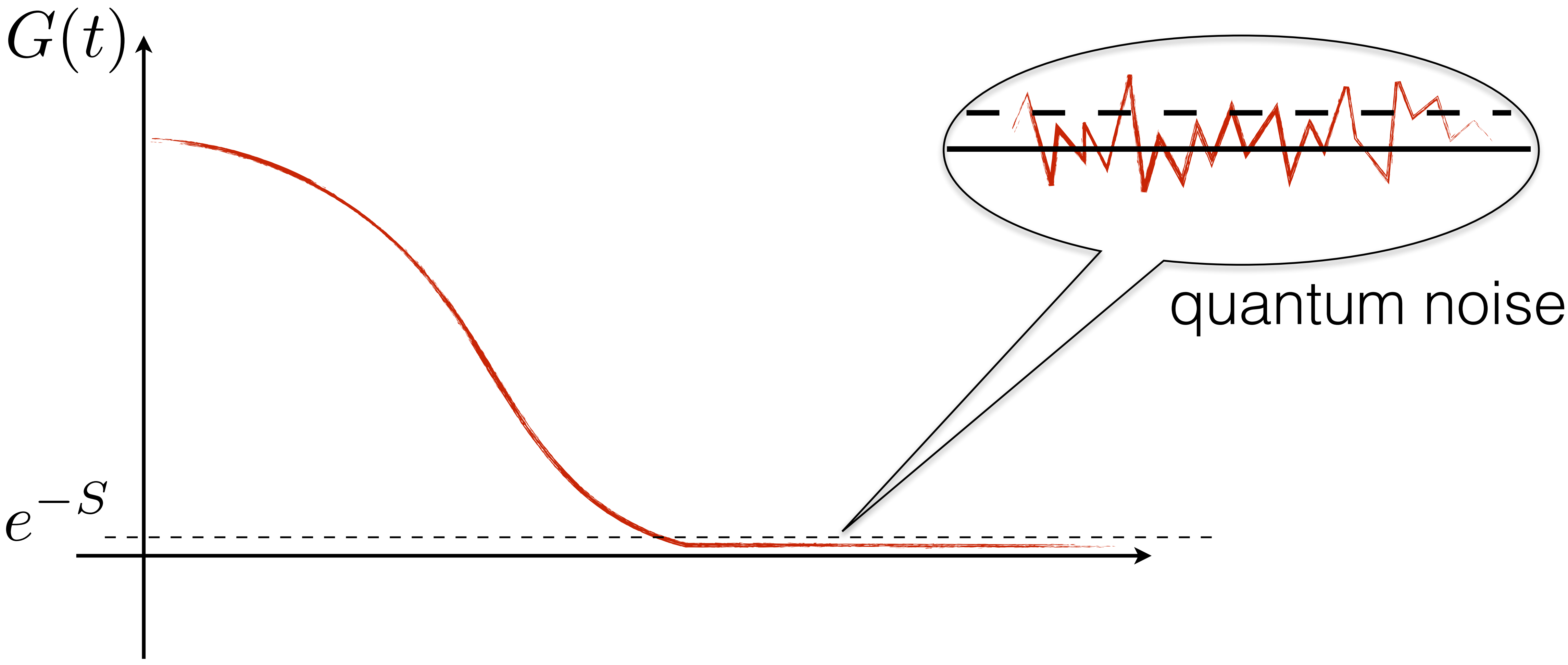
Need to assume spectrum is generic (no specific ordering principle)

➢ connection with ETH

Unitarity vs thermalization

(constraints on long-time correlations from unitarity)

$$\rho = e^{-\beta H}$$



see also [Barbon & Rabonivici]