The protected closed string spectrum of AdS_3/CFT_2 from integrability

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Based on work with R. Borsato, O. Ohlsson Sax, A. Sfondrini, A. Torrielli, A. Babichenko, M. Baggio, T. Lloyd, K. Zarembo

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Plan

1. **Introduction to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4/\mathrm{CFT}_2$**
2. **Green-Schwarz strings**
3. **Exact worldsheet S matrix**
4. **Exact closed string spectrum**
5. **Protected closed string spectrum**
6. **Mixed Flux background**
7. **$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$**
8. **Outlook**

Introduction to AdS_3/CFT_2

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- CFT_2 dual of $AdS_3 \times S^3 \times S^3 \times S^1$ remains poorly understood

$AdS_3 \times S^3 \times T^4$

Supergravity solution for D1- and D5-branes

	0	1	2	3	4	5	6	7	8	9
$N_c \times$ D1	•	•								
$N_f \times$ D5	•	•	•	•	•	•				

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$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ can be supported by R-R + NS-NS 3-form flux.

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The corresponding flat space modulus becomes massive.

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We are left with a 20-dimensional moduli space

$$\mathcal{H}\backslash \mathrm{so}(4,5;\mathbf{R})/\mathrm{so}(4) \times \mathrm{so}(5)\,.$$

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States with $D_L = J_L$ and $D_R = J_R$ are $\frac{1}{2}$-BPS.
We label them as $(\frac{J_L}{2}, \frac{J_R}{2})_S$

$AdS_3 \times S^3 \times T^4$ protected sugra spectrum

[de Boer '98] found the sugra spectrum using a KK reduction on S^3

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The multiplets and their multiplicities are

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)_S^{\oplus 5} \quad \oplus \quad \bigoplus_{m\geq 2}\left(\tfrac{m}{2},\tfrac{m}{2}\right)_S^{\oplus 6} \oplus \left(\tfrac{m-1}{2},\tfrac{m+1}{2}\right)_S \oplus \left(\tfrac{m+1}{2},\tfrac{m-1}{2}\right)_S$$

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The first 5 multiplets contain the moduli.
The first/second line have bosonic/fermionic h.w. states

Green-Schwarz strings

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and

$$ds^2_{\mathrm{AdS}^3} = -\Big(\frac{1 + \frac{z_1^2+z_2^2}{4}}{1 - \frac{z_1^2+z_2^2}{4}}\Big)^2 dt^2 + \Big(\frac{1}{1 - \frac{z_1^2+z_2^2}{4}}\Big)^2 (dz_1^2 + dz_2^2) \,,$$

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Well-suited for expansion around BMN ground state.

GS action in general background

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Explicit expressions up to $\mathcal{O}(\theta^4)$ [Cvetic, Lü, Pope, Stelle '99, Wulff '14]

$$L = L_{\text{bos}} + L_{\text{kin}} + L_{\text{WZ}}$$

where, for example

$$L_{\text{kin}} = -i\sqrt{h}h^{\alpha\beta}\bar{\theta}^I \not{E}_\alpha \left(\delta^{IJ} D_\beta + \frac{\sigma_3^{IJ}}{24} \not{F} \not{E}_\beta\right) \theta_J$$

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$$M_0 = \frac{4}{\sqrt{4-z^2}\sqrt{4-y^2}} \left(1 - \tfrac{1}{2} \not{z} \Gamma^{012}\right) \left(1 - \tfrac{1}{2} \not{y} \Gamma^{345}\right) \,.$$

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χ are massless. η are massive.

Gauge fixing the GS action

Impose BMN light-cone kappa gauge

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In addition one can find $\mathcal{A}$ the algebra that commutes with H

The algebra $\mathcal{A}$

The algebra of charges that commutes with H takes the form

$$\{\mathbf{Q}_{\mathrm{L}}{}^{\dot{a}}, \overline{\mathbf{Q}}_{\mathrm{L}\dot{b}}\} = \frac{1}{2}\delta^{\dot{a}}{}_{\dot{b}}\,(\mathbf{H} + \mathbf{M}), \qquad \{\mathbf{Q}_{\mathrm{L}}{}^{\dot{a}}, \mathbf{Q}_{\mathrm{R}\dot{b}}\} = 0$$

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Central extensions related to worldsheet momentum $\mathbf{P}$

$$\mathbf{C} = +i\frac{h}{2}(e^{+i\mathbf{P}}-1), \qquad \overline{\mathbf{C}} = -i\frac{h}{2}(e^{-i\mathbf{P}}-1),$$

$h \sim 1/\alpha' + \ldots$ is the coupling constant.

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Non-relativistic dispersion relation: Massless particles can scatter.

Exact worldsheet S matrix

Determining the S matrix from $\mathcal{A}$

The 2-body S matrix is determined up to a scalar **dressing factor** by requiring all generators in $\mathcal{A}$ commute with it

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For example, $|\phi_p^L, \psi_q^L\rangle \to |\psi_q^L, \phi_p^L\rangle$ takes the form

$$B_{pq}^{\mathrm{LL}} = \left(\frac{x_p^-}{x_p^+}\right)^{1/2} \frac{x_p^+ - x_q^+}{x_p^- - x_q^+}\,,$$

Exact closed string spectrum

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S non-diagonal: nested Bethe Equations and **auxiliary roots**

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S non-diagonal: nested Bethe Equations and **auxiliary roots**

The excitations come in two levels. Level I excitations are

	Left massive	Right massive	Massless
Level-I excitation	Y^{L}	Z^{R}	χ^1, χ^2
Bethe root	$x^{\pm}$	$\bar{x}^{\pm}$	$z^{\pm}$
Excitation number	N_2	$N_{\bar{2}}$	N_0

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Acting with supercharge generate level-II excitations

	$\mathbf{Q}^{\mathrm{L}1}$, $\overline{\mathbf{Q}}^{\mathrm{R}1}$	$\mathbf{Q}^{\mathrm{L}2}$, $\overline{\mathbf{Q}}^{\mathrm{R}2}$
Bethe root	y_1	y_3
Excitation number	N_1	N_3

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The massive momentum-carrying roots satisfy the equations

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{\substack{j=1\\ j\neq k}}^{N_2} \nu_k^{-1}\nu_j \frac{x_k^+ - x_j^-}{x_k^- - x_j^+} \frac{1-\frac{1}{x_k^+ x_j^-}}{1-\frac{1}{x_k^- x_j^+}} (\sigma_{kj}^{\bullet\bullet})^2$$

$$\times \prod_{j=1}^{N_1} \nu_k^{\frac{1}{2}} \frac{x_k^- - y_{1,j}}{x_k^+ - y_{1,j}} \prod_{j=1}^{N_3} \nu_k^{\frac{1}{2}} \frac{x_k^- - y_{3,j}}{x_k^+ - y_{3,j}}$$

$$\times \prod_{j=1}^{N_{\bar{2}}} \nu_j \frac{1-\frac{1}{x_k^+ \bar{x}_j^+}}{1-\frac{1}{x_k^- \bar{x}_j^-}} \frac{1-\frac{1}{x_k^+ \bar{x}_j^-}}{1-\frac{1}{x_k^- \bar{x}_j^+}} (\widetilde{\sigma}_{kj}^{\bullet\bullet})^2$$

$$\times \prod_{j=1}^{N_0} \nu_k^{-1/2}\nu_j \frac{x_k^+ - z_j^-}{x_k^- - z_j^+} \left(\frac{1-\frac{1}{x_k^- z_j^-}}{1-\frac{1}{x_k^+ z_j^+}}\right)^{\frac{1}{2}} \left(\frac{1-\frac{1}{x_k^+ z_j^-}}{1-\frac{1}{x_k^- z_j^+}}\right)^{\frac{1}{2}} (\sigma_{kj}^{\bullet\circ})^2,$$

frame factors $\nu_k = 1, e^{ip_k}$ in spin-chain, string frame, respectively.

All-loop Bethe equations for closed string spectrum

A physical solution further satisfies the level-matching condition

$$1 = \prod_{j=1}^{N_2} \frac{x_j^+}{x_j^-} \prod_{j=1}^{N_{\bar{2}}} \frac{\bar{x}_j^+}{\bar{x}_j^-} \prod_{j=1}^{N_0} \frac{z_j^+}{z_j^-} .$$

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The energy of a state is given by

$$D - J = N_2 + N_{\bar{2}} + ih \sum_{k=1}^{N_2} \left(\frac{1}{x_k^+} - \frac{1}{x_k^-} \right) + ih \sum_{k=1}^{N_{\bar{2}}} \left(\frac{1}{\bar{x}_k^+} - \frac{1}{\bar{x}_k^-} \right) + ih \sum_{k=1}^{N_0} \left(\frac{1}{z_k^+} - \frac{1}{z_k^-} \right) .$$

Symmetries of the all-loop Bethe equations

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In addition, the Bethe equations have four $U(1)$ symmetries corresponding to the shifts along T^4.

Protected closed string spectrum

Groundstates of the all-loop Bethe equations

The energy of a state is given by

$$D-J = N_2+N_{\bar{2}}+ih\sum_{k=1}^{N_2}\left(\frac{1}{x_k^+}-\frac{1}{x_k^-}\right)+ih\sum_{k=1}^{N_{\bar{2}}}\left(\frac{1}{\bar{x}_k^+}-\frac{1}{\bar{x}_k^-}\right)+ih\sum_{k=1}^{N_0}\left(\frac{1}{z_k^+}-\frac{1}{z_k^-}\right).$$

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Conclusion:
Protected states are massless zero-momentum magnons.
Derivation of [Ohlsson Sax, Torrielli, BS '12]

Protected states of AdS_3/CFT_2 from integrability.

We have conventional BMN groundstate

$$|(\phi^{++})^L\rangle$$

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Adding massless root ($N_0 = 1$, $z^{\pm} = 1$) get two states ($\mathrm{su}(2)_\circ$)

$$|(\phi^{++})^{L-1}\chi_{\mathsf{R}}^{+\pm}\rangle + \text{symmetric permutations},$$

Similarly, for $\chi_{\mathsf{L}}^{+\pm}$. Easy check to see BEs satisfied.

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Similarly, for $\chi_{\mathrm{L}}^{+\pm}$. Easy check to see BEs satisfied.

Next consider state with two right-moving massless fermions,

$$|(\phi^{++})^{L-2}\chi_{\mathrm{R}}^{++}\chi_{\mathrm{R}}^{+-}\rangle + \text{symmetric permutations},$$

$N_0 = 2$, roots sitting at $z^{\pm} = +1$ or $z^{\pm} = -1$. BEs satisfied.

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State	N_0	N_1	N_3	J_L	J_R	$J_\circ$
$(\phi^{++})^L$	0	0	0	$\frac{L}{2}$	$\frac{L}{2}$	0
$(\phi^{++})^{L-1}\chi_R^{+\pm}$	1	0	0	$\frac{L-1}{2}$	$\frac{L}{2}$	$\pm\frac{1}{2}$
$(\phi^{++})^{L}\quad\chi_L^{+\pm}$	1	1	1	$\frac{L+1}{2}$	$\frac{L}{2}$	$\pm\frac{1}{2}$
$(\phi^{++})^{L-2}\chi_R^{++}\chi_R^{+-}$	2	0	0	$\frac{L-2}{2}$	$\frac{L}{2}$	0
$(\phi^{++})^{L-1}\chi_R^{+\pm}\chi_L^{+\pm}$	2	1	1	$\frac{L}{2}$	$\frac{L}{2}$	± 1
$(\phi^{++})^{L-1}\chi_R^{+\pm}\chi_L^{+\mp}$	2	1	1	$\frac{L}{2}$	$\frac{L}{2}$	0
$(\phi^{++})^{L}\quad\chi_L^{++}\chi_L^{+-}$	2	1	1	$\frac{L+2}{2}$	$\frac{L}{2}$	0
$(\phi^{++})^{L-2}\chi_R^{++}\chi_R^{+-}\chi_L^{+\pm}$	3	1	1	$\frac{L-1}{2}$	$\frac{L}{2}$	$\pm\frac{1}{2}$
$(\phi^{++})^{L-1}\chi_R^{+\pm}\chi_L^{++}\chi_L^{+-}$	3	1	1	$\frac{L+1}{2}$	$\frac{L}{2}$	$\pm\frac{1}{2}$
$(\phi^{++})^{L-2}\chi_R^{++}\chi_R^{+-}\chi_L^{++}\chi_L^{+-}$	4	2	2	$\frac{L}{2}$	$\frac{L}{2}$	0

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We find $\frac{1}{2}$-BPS states for $J \geq 2$ we

- 6 bosonic states with charges $\left(\frac{J}{2}, \frac{J}{2}\right)_S$,
- 4+4 fermionic states with charges $\left(\frac{J-1}{2}, \frac{J}{2}\right)_S$ and $\left(\frac{J}{2}, \frac{J-1}{2}\right)_S$,
- 1+1 bosonic states with charges $\left(\frac{J}{2} - 1, \frac{J}{2}\right)_S$ and $\left(\frac{J}{2}, \frac{J}{2} - 1\right)_S$.

Additionally, there are

- 2+2 fermionic states with charges $\left(\frac{1}{2}, 0\right)_S$ and $\left(0, \frac{1}{2}\right)_S$
- 5 bosonic states with charges $\left(\frac{1}{2}, \frac{1}{2}\right)_S$.

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Mplets $\left(\frac{1}{2}, \frac{1}{2}\right)_S$ contain descendants which give the 20 moduli.

Mixed Flux

Mixed Flux $AdS_3 \times S^3 \times T^4$

With NSNS + RR flux integrability still holds

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The dispersion relation now becomes

$$E(p) = \sqrt{(m - \frac{k}{2\pi} p)^2 + 4h^2 \sin\left(\frac{p}{2}\right)^2}\,.$$

where k is the WZW level. [Hoare, Stepanchuk, Tseytlin '14, Lloyd *et al.*'14]

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So $\frac{1}{2}$-BPS spectrum is unchanged

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

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Supergravity solution for D1-, D5- and D5'-branes

	0	1	2	3	4	5	6	7	8	9
$N_1 \times$ D1	•	•								
$N_5 \times$ D5	•	•	•	•	•	•				
$N_5' \times$ D5'	•	•					•	•	•	•

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Near-horizon limit: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{R}^1$. Supersymmetry requires

$$1 = \frac{R^2_{\mathrm{AdS}}}{R^2_{\mathrm{S}^3_1}} + \frac{R^2_{\mathrm{AdS}}}{R^2_{\mathrm{S}^3_2}} \equiv \alpha + (1 - \alpha)$$

For holography we compactify $\mathrm{R}^1 \to S^1$.

Symmetry of $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$

Global symmetry of background is $\mathrm{d}(2,1;\alpha)_L \oplus \mathrm{d}(2,1;\alpha)_R$.

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Short representations of $\mathrm{d}(2,1;\alpha)$ satisfy

$$D = \alpha J_+ + (1-\alpha) J_- \,.$$

D is the conformal dimension and $J_\pm$ are the $\mathrm{su}(2)^2$ spins.

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Reps short on L and R are $\frac{1}{4}$-BPS $\left(\frac{J_{L,+}}{2}, \frac{J_{L,-}}{2}; \frac{J_{R,+}}{2}, \frac{J_{R,-}}{2}\right)_S$

In general, SCFA shortening condition is different to $\mathrm{d}(2,1;\alpha)$ one

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When $J_+ = J_-$ and state is uncharged under circle $\mathrm{U}(1)$
$\mathrm{d}(2,1;\alpha)$ and SCFA shortening conditions coincide

$AdS_3 \times S^3 \times S^3 \times S^1$ protected sugra spectrum

[de Boer, Pasquinucci, Skenderis '99] computed sugra KK spectrum under assumption that all states are in short mplets of $\mathrm{d}(2,1;\alpha)^2$.

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Under this assumption they found

$$\bigoplus_{l^+\geq 0\,,l^-\geq\frac{1}{2}} (l^+,l^-;l^+,l^-)_S \quad\oplus\quad \bigoplus_{l^+\geq 1/2\,,l^-\geq 0} (l^+,l^-;l^+,l^-)_S$$

$$\bigoplus_{l^\pm\geq 0} (l^+,l^-;l^+ + \tfrac{1}{2},l^- + \tfrac{1}{2})_S \quad\oplus\quad (l^+ + \tfrac{1}{2},l^- + \tfrac{1}{2};l^+,l^-)_S$$

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Many states have spins on the two S^3's differing by large amount.

Protected spectrum for $AdS_3 \times S^3 \times S^3 \times S^1$

Mixed flux strings on $AdS_3 \times S^3 \times S^3 \times S^1$ are integrable

[Borsato, Ohlsson Sax, Sfondrini, BS '15]

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$$\bigoplus_J (J,J;J,J)_S \oplus \left(J+\tfrac{1}{2},J+\tfrac{1}{2};J+\tfrac{1}{2},J+\tfrac{1}{2}\right)_S \oplus \left(J,J;J+\tfrac{1}{2},J+\tfrac{1}{2}\right)_S \oplus \left(J+\tfrac{1}{2},J+\tfrac{1}{2};J,J\right)_S .$$

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$$\oplus \left(J,J;J+\tfrac{1}{2},J+\tfrac{1}{2}\right)_S \oplus \left(J+\tfrac{1}{2},J+\tfrac{1}{2};J,J\right)_S \ .$$

This is different from sugra spectrum. Protected states have (almost) the same amount of spin on the two S^3s.

Protected spectrum for $AdS_3 \times S^3 \times S^3 \times S^1$

Mixed flux strings on $AdS_3 \times S^3 \times S^3 \times S^1$ are integrable

[Borsato, Ohlsson Sax, Sfondrini, BS '15]

Protected states are still zero-momentum massless magnons

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Explicit sugra calculation with no shortening assumption

[Eberhardt, Gaberdiel, Gopakumar, Li '17]

Conclusions and Outlook

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An integrable spin-chain has been identified on the Higgs branch CFT in large-N_f calculations. [Ohlsson Sax, Sfondrini, BS '14]

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Understand how integrability enters on CFT_2 side.

Find CFT_2 dual of $\mathrm{AdS}^3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$. [Tong '14]

Thank you