Higher-Spin Theory in the Local Frame and Holography

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String Theory and Quantum Gravity

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Plan

- Motivation
- Free HS fields
- HS symmetries versus locality
- Unfolded dynamics
- Perturbative analysis and resolution operator for nonlinear HS equations
- Local HS cubic interactions and holography
- Functional class for HS fields
- Conclusion

Challenge: Quantum Gravity

Conjecture that the regime of ultra high (transPlanckian) energies exhibits some high symmetries

The idea of HS gauge theory is to understand what kind of higher symmetries can be introduced in relativistic theory

It was argued (Weinberg, Coleman-Mandula, Weinberg-Witten) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries are incompatible with GR if expanding around Minkowski space

Green light: $(A)dS$ background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

HS Theories and String Theory

HS theories: $\Lambda \neq 0$, $m = 0$

symmetric fields $s = 0, 1, 2, \ldots \infty$

String Theory: $\Lambda = 0$, $m \neq 0$ **except for a few zero modes**

mixed symmetry fields $\vec{s} = 0, 1, 2, \ldots \infty$

String theory has much larger spectrum:

HS Theory: first Regge trajectory

Pattern of HS gauge theory is determined by HS symmetry

What is a string-like extension of a global HS symmetry underlying a string-like extension of HS theory?

MV 2012, Gaberdiel and Gopakumar 2014-2017

- **String Theory as spontaneously broken HS theory?!** $(s > 2, m > 0)$

Fronsdal Fields

Fronsdal fields 1978

All $m = 0$ HS fields are gauge fields

$\varphi_{n_1 \dots n_s}$ is a rank s symmetric tensor obeying $\varphi^k{}_k{}^m{}_{m n_5 \dots n_s} = 0$

Gauge transformation:

$$\delta \varphi_{n_1 \dots n_s} = \partial_{(n_1} \varepsilon_{n_2 \dots n_s)} \,, \qquad \varepsilon^m{}_{m n_3 \dots n_{s-1}} = 0$$

Fronsdal action $S(\varphi)$ implies field equations: $R_{n_1 \dots n_s}(x) = 0$

$R_{n_1 \dots n_s}(x)$: Ricci-like tensor

$$R_{n_1 \dots n_s}(x) = \Box \varphi_{n_1 \dots n_s}(x) - s \partial_{(n_1} \partial^m \varphi_{n_2 \dots n_s m)}(x) + \frac{s(s-1)}{2} \partial_{(n_1} \partial_{n_2} \varphi^m{}_{n_3 \dots n_s m)}(x)$$

Main principle fixing the structure of HS theory: HS gauge symmetry

HS Symmetries Versus Riemann Geometry

HS symmetries do not commute with space-time symmetries

$$[T^n, T^{HS}] = T^{HS}, \qquad [T^{nm}, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS fields

$$\delta_{HS}\varphi_{nm} \sim \varphi_{HS}$$

Consequence:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

Cartan formalism of differential forms preserves coordinate independence without metric.

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to \mathsf{d}\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1 \ldots n_p}(x)$$

$$\mathsf{d}W^\Omega(x) = G^\Omega(W(x))\,, \qquad \mathsf{d} = dx^n \partial_n$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \ldots \Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

Properties

- General applicability
- Manifest (HS) gauge invariance under the gauge transformation

$$\delta W^{\Omega} = \mathsf{d}\varepsilon^{\Omega} + \varepsilon^{\Phi} \frac{\partial G^{\Omega}(W)}{\partial W^{\Phi}} \,,$$

  gauge parameter $\varepsilon^{\Omega}(x)$ is a $(p_{\Omega} - 1)$-form.
- Invariance under diffeomorphisms

  Exterior algebra formalism
- Interactions: nonlinear deformation of $G^{\Omega}(W)$

Independence of ambient space-time: geometry is encoded by $G^{\Omega}(W)$

Key observation: unfolded equation makes sense in any space-time

$$\mathsf{d}W^{\Omega}(x) = G^{\Omega}(W(x)) \,, \quad x \to X = (x, z) \,, \quad \mathsf{d}_x \to \mathsf{d}_X = \mathsf{d}_x + \mathsf{d}_z \,, \quad \mathsf{d}_z = dz^u \frac{\partial}{\partial z^u}$$

X-dependence is reconstructed in terms of fields $W^{\Omega}(X_0) = W^{\Omega}(x_0, z_0)$ at any X_0. To take $W^{\Omega}(x_0, z_0)$ in space M_X with coordinates X_0 is the same as to take $W^{\Omega}(x_0)$ in the space $M_x \in M_X$ with coordinates x

Frame-Like HS Fields

$$g_{nm} \longrightarrow h_n^{\alpha\dot{\alpha}} \longrightarrow \{h_n^{\alpha\dot{\alpha}}, \omega_n^{\alpha\beta}, \bar{\omega}_n^{\dot{\alpha}\dot{\beta}}\} \qquad \alpha, \dot{\alpha} = 1, 2$$

admits a natural generalization to $s \geq 2$

$$\varphi_{n_1 \ldots n_s} \to h_n{}^{\alpha_1 \ldots \alpha_{s-1}\,\dot{\alpha}_1 \ldots \dot{\alpha}_{s-1}} \to \qquad \omega_{\alpha_1 \ldots \alpha_n\,,\dot{\beta}_1 \ldots \dot{\beta}_m}\,, \qquad n + m = 2(s-1)$$

$$s = 1: \quad \omega(x) = dx^n \omega_n(x)$$

$$s = 2: \quad \omega_{\alpha\dot{\beta}}(x)\,, \qquad \omega_{\alpha\beta}(x)\,, \qquad \bar{\omega}_{\dot{\alpha}\dot{\beta}}(x)$$

$$s = 3/2: \quad \omega_{\alpha}(x)\,, \qquad \bar{\omega}_{\dot{\alpha}}(x)$$

Frame-like fields: $|n - m| = 0$ **(bosons) or** $|n - m| = 1$ **fermions**

By virtue of constraints $t = [\frac{1}{2}|n - m|]$ **is an order of derivatives**

$$\omega_{\alpha_1 \ldots \alpha_n\,,\dot{\alpha}_1 \ldots \dot{\alpha}_m} = \Pi\Big(\partial^t h_{\alpha_1 \ldots \alpha_{n_0}\,\dot{\alpha}_1 \ldots \dot{\alpha}_{m_0}}\Big)\,, \qquad |n_0 - m_0| < 2$$

First-Order Unfolded Equations: Spin Two

Fields: $h^{\alpha\dot\alpha}$ and $\omega^{\alpha\beta}$, $\bar\omega^{\dot\alpha\dot\beta}$. Zero-torsion and Einstein equations

$$R_{\alpha\dot\alpha}=0\,,\qquad R_{\alpha_1\alpha_2}=h^{\alpha_3}{}_{\dot\alpha}\wedge h^{\alpha_4\dot\alpha}C_{\alpha_1\ldots\alpha_4}\,,\qquad \overline{R}_{\dot\alpha_1\dot\alpha_2}=h_\alpha{}^{\dot\alpha_3}\wedge h^{\alpha\dot\alpha_4}\overline{C}_{\dot\alpha_1\ldots\dot\alpha_4}$$

$C_{\alpha_1\ldots\alpha_4}$ **and** $\overline{C}_{\dot\alpha_1\ldots\dot\alpha_4}$: **Weyl tensor**

Bianchi identities + Einstein equations imply

$$D^L C_{\alpha_1\ldots\alpha_{n+4}\,,\dot\alpha_1\ldots\dot\alpha_n}+h^{\alpha_{n+5}\dot\alpha_{n+1}}C_{\alpha_1\ldots\alpha_{n+5}\,,\dot\alpha_1\ldots\dot\alpha_{n+1}}=O(C^2)$$

$$D^L\overline{C}_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_{n+4}}+h^{\alpha_{n+1}\dot\alpha_{n+5}}\overline{C}_{\alpha_1\ldots\alpha_{n+1}\,,\dot\alpha_1\ldots\dot\alpha_{n+5}}=O(C^2)$$

$C_{\alpha_1\ldots\alpha_{n+4}\,,\dot\alpha_1\ldots\dot\alpha_n}$ **and** $\overline{C}_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_{n+4}}$: **order-$n$ on-shell nontrivial derivatives of the Weyl tensor**

Analogously, for general-spin 0-forms $\quad C_{\alpha_1\ldots\alpha_n\,,\dot\beta_1\ldots\dot\beta_m}\,,\qquad |n-m|=2s$

$s=0:\quad C(x)$

$s=1/2:\quad C_\alpha(x)\,,\qquad \bar C_{\dot\alpha}(x)$

$s=1:\quad C_{\alpha\beta}\,,\qquad \bar C_{\dot\alpha\dot\beta}$

$s=3/2:\quad C_{\alpha\beta\gamma}\,,\qquad \bar C_{\dot\alpha\dot\beta\dot\gamma}$

$s=2:\quad C_{\alpha_1\ldots\alpha_4}\,,\qquad \bar C_{\dot\alpha_1\ldots\dot\alpha_4}$

Central On-Shell Theorem

Infinite set of integer spins

$$\omega(y,\bar y\mid x)\,,\qquad C(y,\bar y\mid x)\qquad f(y,\bar y)=\sum_{n,m=0}^{\infty}\frac{1}{n!m!}f_{\alpha_1\ldots\alpha_n,\dot\alpha_1\ldots\dot\alpha_m}y^{\alpha_1}\ldots y^{\alpha_n}\bar y^{\dot\alpha_1}\ldots\bar y^{\dot\alpha_m}$$

The full unfolded system for free bosonic fields is **1989**

$$\star\qquad R_1(y,\bar y\mid x)=\frac{i}{4}\left(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar y^{\dot\alpha}\partial\bar y^{\dot\beta}}\,C(0,\bar y\mid x)+\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C(y,0\mid x)\right)$$

$$\star\star\qquad \tilde D_0 C(y,\bar y\mid x)=0$$

Vacuum: $sp(4)\sim o(3,2)$

$$\mathbf{R}_{\alpha\beta}:=\mathbf{d}\omega_{\alpha\beta}+\omega_{\alpha\gamma}\omega_{\beta}{}^{\gamma}-\mathbf{H}_{\alpha\beta}=\mathbf{0}\,,\qquad \mathbf{R}_{\alpha\dot\beta}:=\mathbf{d}+\omega_{\alpha\gamma}\mathbf{h}^{\gamma}{}_{\dot\beta}+\overline{\omega}_{\dot\beta\dot\delta}\mathbf{h}_{\alpha}{}^{\dot\delta}=\mathbf{0}$$

$$\mathbf{H}^{\alpha\beta}:=\mathbf{h}^{\alpha\dot\alpha}\wedge\mathbf{h}^{\beta}{}_{\dot\alpha}\,,\qquad \overline{\mathbf{H}}^{\dot\alpha\dot\beta}:=\mathbf{h}^{\alpha\dot\alpha}\wedge\mathbf{h}_{\alpha}{}^{\dot\beta}$$

$$R_1(y,\bar y\mid x)=D_0^{ad}\omega(y,\bar y\mid x)\qquad D_0^{ad}=D^L-h^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar y^{\dot\beta}}+\frac{\partial}{\partial y^{\alpha}}\bar y_{\dot\beta}\right)$$

$$\tilde D_0=D^L+h^{\alpha\dot\beta}\left(y_\alpha\bar y_{\dot\beta}+\frac{\partial^2}{\partial\mathbf{y}^{\alpha}\partial\bar{\mathbf{y}}^{\dot\beta}}\right)\qquad D^L=\mathrm{d}_x-\left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta}+\bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial\bar y^{\dot\beta}}\right)$$

$\star\star$ **implies that higher-order terms in** y **and** $\bar y$ **describe higher-derivative descendants of the primary HS fields**

Fields of the Nonlinear System

Closed formulation of nonlinear equations demands the doubling of spinors and Klein operator

$$\omega(Y|x) \longrightarrow W(Z;Y;k|x)\,, \qquad C(Y|x) \longrightarrow B(Z;Y;k|x)$$

Some of the nonlinear HS equations determine the dependence on the additional variables Z_A in terms of "initial data"

$\omega(Y;k|x) := W^{dyn}(0;Y|x) + W^{top}(0;Y|x)k$

$C(Y;k|x) := B^{dyn}(0;Y|x)k + B^{top}(0;Y|x)$

$S(Z;Y;k|x) = dZ^A S_A(Z;Y;k|x)$ is a connection along Z^A

Topological fields: finite # d.o.f.: tensors

Klein operator k generates chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \qquad A = (a_\alpha\,, \bar{a}_{\dot\alpha}) : \qquad \tilde{A} = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

$$P(Y) = P^{\alpha\dot\alpha} y_\alpha \bar{y}_{\dot\alpha} \quad \longrightarrow \quad \tilde{P}(Y) = -P(Y)\,, \qquad \tilde{M}(Y) = M(Y)$$

Nonlinear HS Equations

HS star product

$$(f \star g)(Z,Y) = \int dSdT \, \exp iS_A T^A \, f(Z+S, Y+S) g(Z-T, Y+T)$$

$[Y_A, Y_B]_\star = -[Z_A, Z_B]_\star = 2iC_{AB}\,,$ $\qquad Z-Y \,:\, Z+Y$ **normal ordering**

Inner Klein operators:

$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar\kappa = \exp i\bar z_{\dot\alpha} \bar y^{\dot\alpha}\,, \qquad \kappa \star f = \tilde f \star \kappa\,, \qquad \kappa \star \kappa = 1$

$$\begin{cases} \mathrm{d}W + W \star W = 0 \\ \mathrm{d}B + W \star B - B \star W = 0 \\ \mathrm{d}S + W \star S + S \star W = 0 \\ \mathbf{S \star B - B \star S = 0} \\ \mathbf{S \star S = i(dZ^A dZ_A + \eta dz^\alpha dz_\alpha B \star k \star \kappa + \bar\eta d\bar z^{\dot\alpha} d\bar z_{\dot\alpha} B \star k \star \bar\kappa)} \end{cases}$$

1992

Real physics is localized in the x-independent twistor sector

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = dZ^A Z_A\,, \qquad W_0 = \frac{1}{2}\omega_0^{AB}(x) Y_A Y_B$$

$$\mathrm{d}W_0 + W_0 \star W_0 = 0$$

$\omega_0^{AB}(x)$: **describes** AdS_4.

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

$$[S_0\,, f]_\star = -2i\mathrm{d}_Z f\,, \qquad \mathrm{d}_Z = dZ^A \frac{\partial}{\partial Z^A}$$

Reconstruction of Z^A Variables

Perturbatively, equations containing S have the form

$$\mathsf{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|dZ) \qquad \mathsf{d}_Z V[U_{<n}](Z;Y|dZ) = 0$$

can be solved as

$$U_n(Z;Y|dZ) = \mathsf{d}_Z^* V[U_{<n}](Z;Y|dZ) + \mathbf{h(Y)} + \mathsf{d}_Z \epsilon(Z;Y|dZ)$$

For instance

$$\mathsf{d}_Z^* V(Z;Y|dZ) = Z^A \frac{\partial}{dZ^A} \int_0^1 \frac{dt}{t} V(tZ;Y|tdZ)$$

Alternative d_Z^* that differ by d_Z-closed forms can also be used.

Proper choice of boundary conditions in Z-variables is most important in the context of locality beyond the free field level!

Nontrivial space-time equations on $\omega(Y|x)$ and $C(Y|x)$ are in the sector of d_Z-cohomology

Central On-Shell Theorem is reproduced in the lowest order

Conserved Currents and Current Deformation

Gauge invariant conserved currents $J(Y_1, Y_2|x)$ are represented by the bilinears of $C(Y|x)$

$$J(Y_1, Y_2|x) := C(Y_1|x)\tilde{C}(Y_2|x)\,, \qquad \tilde{C}(y, \bar{y}|x) = C(-y, \bar{y}|x)$$

As a consequence of the rank-one equation for $C(Y|x)$, $J(Y_1, Y_2|x)$ obeys the current equation Gelfond, MV (2003)

$$\tilde{D}_2 J(Y_1, Y_2|x) = 0\,, \qquad \tilde{D}_2 := D^L - i\lambda h^{\alpha\dot{\beta}}\Big(y_{1\alpha}\bar{y}_{1\dot{\beta}} - y_{2\alpha}\bar{y}_{2\dot{\beta}} - \frac{\partial^2}{\partial y_1^\alpha \partial \bar{y_1}^{\dot{\beta}}} + \frac{\partial^2}{\partial y_2^\alpha \partial \bar{y_2}^{\dot{\beta}}}\Big)$$

Current deformation has a form of a linear system

$$D\omega - L(w, C) + \Gamma(w, J) = 0\,,$$

$$\tilde{D}C + \mathcal{H}(w, J) = 0\,, \qquad \tilde{D}_2 J(Y_1, Y_2|x) = 0$$

$$L(w, C) := \frac{i}{4}\Big(\eta \overline{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \overline{y}^{\dot{\alpha}} \partial \overline{y}^{\dot{\beta}}}\ \bar{C}(0, \overline{y}|x) + \bar{\eta} H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\ C(y, 0|x)\Big)$$

Linear functionals Γ and $\mathcal{H}$ obey the compatibility conditions

Locality and Nonlocality in HS Gauge Theory

Having infinitely many HS fields with higher derivatives in interactions, the HS Gauge Theory is not local

$\lambda^{-1}D \sim 1$ since $[\lambda^{-1}D\,, \lambda^{-1}D] \sim 1$

A different mass parameter like α' is needed for a low-energy expansion

In HS equations, nonlocality is due to infinite tails of contractions

$$\int \frac{d^4Sd^4T}{(2\pi)^4} \exp i[s_\beta t^\beta + \bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] J(y+s, \bar{y}+\bar{s}; y+t, \bar{y}+\bar{t})$$
$$= \exp -i[\partial_{1\alpha}\partial_{2\beta}\epsilon^{\alpha\beta} + \bar\partial_{1\dot\alpha}\bar\partial_{2\dot\beta}\epsilon^{\dot\alpha\dot\beta}] J(y, \bar{y}; y, \bar{y})$$

# of derivatives in y and $\bar{y}$ infinitely increases for given helicities $\Rightarrow$ infinite tails of space-time derivatives and hence nonlocality

True locality: absence of integration over s and t or $\bar{s}$ and $\bar{t}$

$$\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] f(y, \bar{y}+\bar{s}) g(y, \bar{y}+\bar{t}) = f(y,\bar{y}) \exp[-i \overleftarrow{\partial}_{\dot\alpha} \overrightarrow{\partial}_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}] g(y,\bar{y})$$

For given helicities carried by g and f, only a single term in the sum contributes hence containing a finite number of derivatives

Resolution Redefinition

Original homotopy operator gives in the η-sector

$$B_{2\eta}=\frac{\eta}{2}\int d_{+}^{4}\tau\delta(1-\sum_{i=1}^{4}\tau_i)y^{\beta}\Big(\delta(\tau_1)\partial_{2\beta}+\delta(\tau_2)\partial_{1\beta}\Big)\exp(X)C(Y_1;K)C(Y_2;K)*k*\kappa|_{Y_1}$$

$$X=i(1-\tau_3)z_\alpha y^\alpha+\tau_3 y^\alpha(\partial_{1\alpha}+\partial_{2\alpha})+z^\alpha(\tau_2\partial_{2\alpha}-\tau_1\partial_{1\alpha})+i(\tau_3+\tau_4)\partial_{1\alpha}\partial_2^\alpha+i\bar{\partial}_{1\dot{\alpha}}\bar{\partial}_2^{\dot{\alpha}}\,,$$

$$\partial_{i\alpha}:=\frac{\partial}{\partial y_i^\alpha}\,,\qquad \bar{\partial}_{i\dot{\alpha}}:=\frac{\partial}{\partial \bar{y}_i^{\dot{\alpha}}}$$

Since $\tau_4\geq 0$, **the expansion coefficients** $\tau_3+\tau_4$ **in powers of** $\partial_{1\alpha}\partial_2^\alpha$ **are larger than those** (τ_3) **of** $y^\alpha\partial_{i\alpha}$.

Using the Schoutens identity

$$z_\alpha y^\alpha\partial_{1\beta}\partial_2^\beta+z^\alpha\partial_{1\alpha}y^\beta\partial_{2\beta}-z^\alpha\partial_{2\alpha}y^\beta\partial_{1\beta}=0$$

redefinition of the resolution operator

$$\mathrm{d}^*_{loc}B:=\mathrm{d}^*_Z B-\Delta C_{2\eta}$$

Final result

$$B^{loc}_{2\eta} = \frac{1}{2}\eta \int d^3_+\tau \Big(\delta'(1-\sum_{i=1}^{3}\tau_i) - i y_\alpha z^\alpha \delta(1-\sum_{i=1}^{3}\tau_i)\Big)$$
$$\int d^4U d^4V \exp i(U_A V^A + (1-\tau_3) z_\alpha y^\alpha)$$
$$C(\tau_3 y - \tau_1 z + \tau_3 u, \bar{y}+\bar{u}; K) C(\tau_3 y + \tau_2 z + v, \bar{y}+\bar{v}; K) * k * \kappa\,.$$

The coefficient responsible for the index contraction between the first and second factors of C equals to those in front of the y variables, analogously to the star product of Z-independent functions.

$\Delta C_{2\eta}$ represents the nonlinear shift of 1605.02662 reducing the nonlocal bilinear corrections to the local form in the sector of x, y-variables.

$$\tilde{D}C + \frac{\eta}{4}\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] \int_0^1 d\tau h^{\alpha\dot\alpha} y_\alpha (\tau \bar{s}_{\dot\alpha} + (1-\tau)\bar{t}_{\dot\alpha})$$
$$J(\tau y, (\tau-1)y; \bar{y}+\bar{s}, \bar{y}+\bar{t}; K) * k = 0$$

Current Contribution to the Gauge Sector

Contribution to spin-s equations Gelfond, MV (2017)

$$\mathrm{d}\omega + \widetilde{\mathcal{J}}_{\mathrm{ss}} + \ldots = 0\,,$$

$$\widetilde{\mathcal{J}}_{\mathrm{ss}} = i\eta\bar{\eta}\frac{(\mathrm{s}-2)!}{8(2\mathrm{s})!}\sum_{k,m\in[0,\mathrm{s}]}\frac{(m+k)!(2\mathrm{s}-m-k)!}{(\mathrm{s}-k)!k!(\mathrm{s}-m)!m!}\Big(\mathcal{N}_1\Big)^m\Big(-\mathcal{N}_2\Big)^{\mathrm{s}-m}\Big(-\overline{\mathcal{N}}_2\Big)^k\Big(\overline{\mathcal{N}}_1\Big)^{\mathrm{s}-k}$$
$$\Big\{\sum_{0\le\mathrm{n}\le\mathrm{s}}\frac{1}{(\mathrm{s}+\mathrm{n}-1)!}(i\partial_{1\gamma}\partial_2{}^{\gamma})^{\mathrm{n}}\sum_{j,l=0,1}C^{j,1-j}(Y^1|x)k^j\bar{k}^{1-j}C^{l,1-l}(Y^2|x)k^l\bar{k}^{1-l}$$
$$+\sum_{0<\mathrm{n}\le\mathrm{s}}\frac{1}{(\mathrm{s}+\mathrm{n}-1)!}(i\bar{\partial}_{1\dot{\gamma}}\bar{\partial}_2{}^{\dot{\gamma}})^{\mathrm{n}}\sum_{j,l=0,1}C^{j,1-j}(Y^1|x)k^j\bar{k}^{1-j}C^{l,1-l}(Y^2|x)k^l\bar{k}^{1-l}\Big\}\Big|_{Y^j=0}$$
$$\mathcal{N}_j = y^{\alpha}\partial_{j\alpha}\,,\qquad\qquad \overline{\mathcal{N}}_j = \bar{y}^{\dot{\alpha}}\bar{\partial}_{j\dot{\alpha}}$$

Although $\widetilde{\mathcal{J}}_{\mathrm{s,s}} \sim \eta\bar{\eta}$ the current contribution to Fronsdal equtions depends on the phase of η through Central on-shell theorem.

Matching with the vertex obtained by Metsaev in 1991 from the higher-order analysis and by Taronna and Sleight in 2016 from the holographic analysis at $\eta = 1$ Misuna (2017)

HS holography

General idea of HS duality Sundborg (2001), Witten (2001)

$AdS_4/CFT3$: Klebanov, Polyakov (2002); Giombi, Yin (2009,2010)

Extension to duality between CS boundary theory and HS theories in AdS_4 for various $\eta = \exp i\frac{\pi}{2}\lambda_b$ with $\lambda_b = \frac{N}{k}$

Aharony, Gur-Ari, Yacoby (2011); Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin

Explicit check was only partially successful Giombi and Yin (2012)

Local frame allows further HS holography checks:

zero-form sector Sezgin, Skvortsov, Zhu (May 2017), Didenko, MV (May 2017)

one-form sector N.Misuna (June 2017)

Functional Class

Appropriate space $V_{0,0}$ is spanned (2015) by functions of the form

$$f(Z;Y) = \int_0^1 d\tau \phi(\tau Z;(1-\tau)Y;\tau) \exp i\tau Z_A Y^A$$

with $\phi(W;U;\tau)$ regular in W and U and integrable in τ.

Analysis of locality in the zero-form sector clarifies the structure of the pre-exponential factor on τ_i in the nonlinear case

$$\int d^5_+\tau\rho(\tau)\int d^2u d^2v \exp i(u_\alpha v^\alpha + t z_\alpha y^\alpha) C(\tau_1 y - \tau_3 z + \tau_5 u) C(-\tau_4 z - \tau_2 y + v) * k$$

$$\tau_3 \le t\,, \qquad \tau_4 \le t\,, \qquad \tau_1 \le 1-t\,, \qquad \tau_2 \le 1-t\,, \qquad \tau_5 \le \mathbf{t}$$

The restriction on τ_5 extends the 2015 results to the nonlinear case.

Such functions induce minimally nonlocal higher-order corrections!

Special form of the proper nonlinear correction is not visible from the analysis of quadratic corrections at $Z=0$!

Conclusion

To get local (or minimally nonlocal) results at higher orders one has to use a particular resolution operator d^*_{loc}.

HS equations in the local frame reproduce correctly local corrections and anticipated AdS/CFT results: Gelfond, Didenko, Misuna (2017)

Classes of functions in the twistor space distinguishing between local and nonlocal field redefinitions in HS theory are identified

To do

Further computations in standard HS holography for $\forall\eta$

Via one-form sector

On-shell invariant functionals MV 2015

Analysis of d^*_{loc}, (non?)locality and HS holography at higher orders