Extremization principles for BPS black holes in AdS

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Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes [Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory
- Remarkable precision tests including higher derivatives (counting of microstates, Sen's entropy functional, bulk localization...)

Introduction

No similar result for AdS black holes in $d \geq 4$ was known until very recently. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. This can be done by using localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

We will discuss how to solve the problem for BPS AdS_4 black holes and give intriguing suggestions for AdS_5.

Based on

F. Benini-AZ; arXiv 1504.03698 and 1605.06120

F. Benini-K.Hristov-AZ; arXiv 1511.04085 and 1608.07294

S. M. Hosseini-AZ; arXiv 1604.03122

S. M. Hosseini-A. Nedelin-AZ; arXiv 1611.09374

S. M. Hosseini-K. Hristov -AZ; arXiv 1705.05383

F. Azzurli-N.Bobev-M. Crichigno-V. Min-AZ; 1707.xxxxx

Outline

- **Part I: Entropy of AdS_4 Black Holes**
- **Part II: An extremization principle for AdS_5 Black Holes**
- **Conclusions and Open Problems**

PART I : Entropy of AdS_4 Black Holes

AdS_4 black holes

Consider BPS asymptotically AdS_4 static black holes

$$\mathrm{d}s^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left(dr^2 + V(r)^2\mathrm{d}s^2_{S^2}\right)$$

- vacua of Einstein-Maxwell theories

$$\mathcal{L} = \sqrt{g}(-\frac{1}{2}R + \mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} + g_{ij}\partial X_i\partial X_j + \cdots)$$

- characterized by electric and magnetic charges

$$\int_{S^2} F^{\Lambda} = \mathrm{Vol}(S^2)\, p^{\Lambda}\,, \quad \int_{S^2} *F^{\Lambda} = \mathrm{Vol}(S^2)\, q_{\Lambda}\,,$$

- supersymmetry preserved by a twist

$$\nabla\epsilon = \partial\epsilon + \frac{1}{4}\omega^{ab}\gamma^{ab}\epsilon + iA_R\epsilon = \partial\epsilon$$

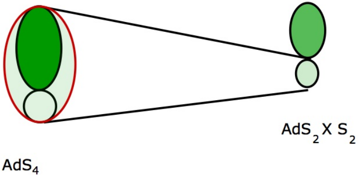
[Cacciatori,Klemm; Gnecchi,Dall'agata; Hristov,Vandoren;Halmagyi;Katmadas]

AdS_4 black holes

These extremal static black holes

$$\mathrm{d}s^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left(dr^2 + V(r)^2 \mathrm{d}s^2_{S^2}\right)$$

are asymptotic to AdS_4 for $r \gg 1$ and with horizon $AdS_2 \times S^2$ at some $r = r_h$



$$ds^2 \sim \frac{dr^2}{r^2} + r^2(-dt^2 + d\theta^2 + \sin\theta^2 d\phi^2)) \qquad ds^2 \sim -(r-r_h)^2 dt^2 + \frac{dr^2}{(r-r_h)^2} + (d\theta^2 + \sin\theta^2 d\phi^2)$$

Dual Field Theory Perspective

General vacua of a bulk effective action

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + F_{\mu\nu}F^{\mu\nu} + V \ldots$$

with a metric

$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A = A_{M_d} + O(1/r)$$

and a gauge fields profile, correspond to CFTs on a d-manifold M_d and a non trivial background field for the R- or global symmetry

$$L_{CFT} + J^\mu A_\mu$$

Dual Field Theory Perspective

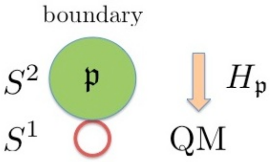
The boundary is $S^2 \times \mathcal{R}$ or $S^2 \times S^1$ in the Euclidean, with a non vanishing background gauge field for the global symmetries on S^2

$$A^\Lambda = -\frac{p^\Lambda}{2} \cos\theta d\phi$$

- The magnetic charges p^Λ corresponds to a deformation of the boundary theory, with a magnetic background for the global symmetries of the theory. In particular, the magnetic charge for the R-symmetry is fixed by requiring $\nabla\epsilon = \partial\epsilon + \frac{1}{4}\omega^{ab}\gamma^{ab}\epsilon + iA_R\epsilon = \partial\epsilon$
- The electric charges q_Λ gives sub-leading contributions at the boundary. They are a VEV in the boundary theory, meaning the the average electric charge of the CFT states is non zero.

Dual Field Theory Perspective

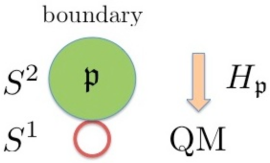
It is useful to think in terms of a dimensional reduction on S^2



to a Quantum Mechanics. The magnetic charges $\mathfrak{p}$ give rise to Landau levels.

Dual Field Theory Perspective

Consider the gran-canonical partition function



$$Z_{S^2 \times S^1}(\Delta, \mathfrak{p}) = \mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{p}}} \right)$$

- magnetic charges $\mathfrak{p}$ are not vanishing at the boundary and appear in the Hamiltonian
- electric charges $\mathfrak{q}$ can be introduced using chemical potentials Δ
- this is known as the topologically twisted index and can be computed using supersymmetric localization [Benini,AZ]

Dual Field Theory Perspective

What's about $(-1)^F$?

$$Z(\Delta, \mathfrak{p}) = \mathrm{Tr}_{\mathcal{H}} \left(e^{iJ\Delta} e^{-\beta H_{\mathfrak{p}}} \right)$$

is too hard to compute!

Assume no cancellation between boson and fermions. Luckily, this is true in the limit of large charges.

Dual Field Theory Perspective

With this assumption, the partition function can be expanded as

$$Z_{S^2\times S^1}(\Delta,\mathfrak{p}) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{p}}}\right) = \sum_{\mathfrak{q}} e^{S(\mathfrak{q},\mathfrak{p})} e^{i\mathfrak{q}\Delta}$$

The entropy $S(\mathfrak{q},\mathfrak{p}) =$ log number of states can be extracted as

$$e^{S(\mathfrak{q},\mathfrak{p})} = \int_{\Delta} Z_{S^2\times S^1}(\Delta,\mathfrak{p}) e^{-i\mathfrak{q}\Delta}$$

or, in the limit of large charges, by a saddle point, a Legendre Transform

$$S_{BH}(\mathfrak{q},\mathfrak{p}) \equiv \mathcal{I}(\Delta) = \log Z(\mathfrak{p},\Delta) - i\Delta\mathfrak{q}\,, \qquad \frac{d\mathcal{I}}{d\Delta} = 0$$

[similar to Sen's formalism, OSV, etc]

Main Example - Black holes in $AdS_4 \times S^7$

- There is a class of dyonic BPS static black holes with an explicit string embedding: vacua of $N = 2$ gauged supergravities arising from M theory truncation on $AdS_4 \times S^7$

[Cacciatori,Klemm; Gnecchi,Dall'Agata; Hristov,Vandoren;Halmagyi;Katmadas]

- The dual field theory to $\mathrm{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$

$$\begin{tikzcd} N_k \arrow[rr, bend left, "A_i"] & & N_{-k} \arrow[ll, bend left, "B_j"] \end{tikzcd}$$

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

Black holes in $AdS_4 \times S^7$

There are electric $\mathfrak{q}_a$ and magnetic $\mathfrak{p}_a$ charges under $U(1)^4 \subset SO(8)$.

- The BPS dyonic black holes depends only on three electric and magnetic charges. Supersymmetry requires [Halmagyi;Katmadas]

$$\sum_{a=1}^{4} \mathfrak{p}_a = 2\,, \qquad\qquad f(\mathfrak{q}_a, \mathfrak{p}_a) = 0$$

- The partition function $\mathrm{Tr}(-1)^F e^{iJ_a\Delta_a} e^{-\beta H_{\mathfrak{p}}}$ similarly depends on three magnetic fluxes and three chemical potentials for the flavor symmetries

$$\sum_{a=1}^{4} \mathfrak{p}_a = 2\,, \qquad\qquad \prod_{a=1}^{4} y_a = 1 \Longrightarrow \sum_{a=1}^{4} \Delta_a \in 2\pi\mathbb{Z}$$

where $y_a = e^{i\Delta_a}$.

ABJM twisted index

The ABJM topologically twisted index can be computed using localization

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}}\in\mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i}\, x_i^{k\mathfrak{m}_i}\, \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)\times$$

$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\, y_1}}{1-\frac{x_i}{\tilde{x}_j}\, y_1}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_1+1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\, y_2}}{1-\frac{x_i}{\tilde{x}_j}\, y_2}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_2+1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\, y_3}}{1-\frac{\tilde{x}_j}{x_i}\, y_3}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_3+1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\, y_4}}{1-\frac{\tilde{x}_j}{x_i}\, y_4}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_4+1}$$

$$\prod_a y_a = 1\,, \qquad \sum \mathfrak{p}_a = 2$$

ABJM twisted index

Luckily enough, the index can be evaluated in the large N limit. Strategy:

- Re-sum geometric series in $\mathfrak{m}, \widetilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^{N}(e^{iB_i} - 1) \prod_{j=1}^{N}(e^{i\widetilde{B}_j} - 1)}$$

- Step 1: find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N

- Step 2: evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

[Benini-Hristov-AZ]

[extended to other models Hosseini-AZ; Hosseini-Mekareeya]

ABJM twisted index

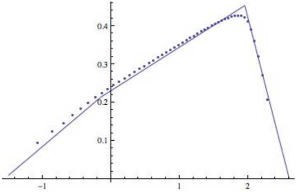
solve the large N Limit of algebraic equations giving the positions of poles

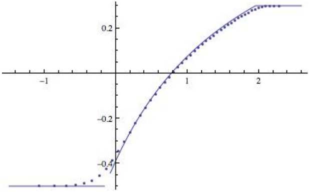
$$1 = x_i^k \prod_{j=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)}$$

Bethe Ansatz Equations - derived by a potential $\mathcal{V}_{BA}(x_i, \tilde{x}_i)$

with an ansatz

$$\log x_i = i\sqrt{N} t_i + v_i \,, \qquad \log \tilde{x}_i = i\sqrt{N} t_i + \tilde{v}_i$$





ABJM twisted index

The final result is surprisingly simple

$$\log Z(\Delta, \mathfrak{p}) = \sum_a i\mathfrak{p}_a \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_a}$$

where

$$\mathcal{V}_{BA} = \frac{2}{3} i N^{3/2} \sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}$$

and

$$\sum_{a=1}^{4} \Delta_a = 2\pi$$

[Benini-Hristov-AZ]

ABJM twisted index

The index is obtained as:

$$\mathcal{I}(\Delta) = \log Z(\Delta, \mathfrak{p}) - \sum_a i\Delta_a \mathfrak{q}_a = \sum\nolimits_a i\mathfrak{p}_i \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_a} - i\Delta_a \mathfrak{q}_a$$

This function can be extremized with respect to the Δ_a and

$$\boxed{S(\mathfrak{p}_a, \mathfrak{q}_a) = \mathcal{I}(\Delta, \mathfrak{p})|_{crit}}$$

$$\boxed{\Delta_a|_{crit} \sim X^a(r_h)}$$

[Benini-Hristov-AZ]

ABJM twisted index

- Notice that the explicit expression for the entropy of the $AdS_4 \times S^7$ black hole is quite complicated. In the case of purely magnetical black holes with just

  $$\mathfrak{p}^1 = \mathfrak{p}^2 = \mathfrak{p}^3$$

  is given by

$$S = \sqrt{-1 + 6\mathfrak{p}^1 - 6(\mathfrak{p}^1)^2 + (-1 + 2\mathfrak{p}^1)^{3/2}\sqrt{-1 + 6\mathfrak{p}^1}}$$

Attractor mechanism in $\mathcal{N}=2$ supegravity

The extremization is equivalent to the attractor mechanism:

$$\mathcal{R} = \frac{F_\Lambda \mathfrak{p}^\Lambda - X^\Lambda \mathfrak{q}_\Lambda}{g_\Lambda X^\Lambda - g^\Lambda F_\Lambda}\,, \qquad F_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}$$

is extremized with respect to the scalar fields at the horizon and its critical value gives the entropy. [Ferrara,Kallosh,Strominger; Dall'Agata,gnecchi]

It is convenient to use the gauge fixing $g_\Lambda X^\Lambda - g^\Lambda F_\Lambda = 1$. The FI terms are $g^\Lambda = 1, g_\Lambda = 0$. Under $X^\Lambda \to \Delta_a/2\pi$

$$\sum_\Lambda X^\Lambda = 1 \qquad \Longleftrightarrow \qquad \sum_a \Delta_a = 2\pi$$

$$\mathcal{F} = \sqrt{X^0 X^1 X^2 X^3} \qquad \Longleftrightarrow \qquad \mathcal{V}_{BA}(\Delta) \sim \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$$

$$\mathcal{R} = \sum_\Lambda \mathfrak{p}^\Lambda \frac{\partial \mathcal{F}}{\partial X^\Lambda} - X^\Lambda \mathfrak{q}_\Lambda \qquad \Longleftrightarrow \qquad \mathcal{I}(\Delta) \sim \sum\nolimits_a \mathfrak{p}_i \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_a} - \Delta_a \mathfrak{q}_a$$

[Benini-Hristov-AZ; Hosseini-AZ]

Extensions

- The large N computation can be extended to many other quivers dual to M theory on $\mathrm{AdS}_4 \times SE_7$ ($N^{3/2}$ scaling) and massive type IIA on warped $\mathrm{AdS}_4 \times Y_6$ ($N^{5/3}$) [Hosseini-AZ; Hosseini-Mekareeya]
- Universal twist. Constant scalar black hole with horizon $\mathrm{AdS}_2 \times \Sigma_g$, $g > 1$ in minimal gauged supergravity can be embedded in all models in M theory or massive type IIA.

$$S_{\mathrm{BH}} = \log Z = (g-1) F_{S^3}$$

  Fluxes along the exact R-symmetry of the 3d CFT. [Azzurli-Bobev-Chicigno-Min-AZ]
- Full matching for magnetically charged BPS black holes in massive IIA on $\mathrm{AdS}_4 \times S^6$ [Hosseini-Hristov-Passias - to appear]

PART II : An extremization principle for AdS_5 Black Holes

Rotating AdS$_5$ BPS black holes

There is a class of electrically charged and rotating BPS black holes asymptotic to AdS$_5 \times S^5$ depending on

two angular momenta J_ψ, J_ϕ in AdS$_5$ $\qquad U(1)^2 \subset SO(4)$

three electric charges Q_I in S^5 $\qquad U(1)^3 \subset SO(6)$

with a constraint $F(J_i, Q_I) = 0$.

[Gutowski-Reall]

The boundary metric is $S^3 \times \mathbb{R}$. The microstates correspond to states of given angular momentum and electric charge in $\mathcal{N} = 4$ SYM. People tried hard to reproduce the entropy in QFT in the limit of large charges

Entropy can be found by an extremization?

Unfortunately the attractor mechanism for 5d rotating black holes is not know.

However one can explicitly check that the entropy is obtain as a Legendre transform

$$\mathcal{S}(Q_I, J_i) = -E(\Delta_I, \omega_i) + 2\pi i\left(\sum_{I=1}^{3} Q_I \Delta_I - \sum_{i=1}^{2} J_i \omega_i\right)\Big|_{\bar{\Delta}_I, \bar{\omega}_i}$$

where

$$E = -i\pi N^2 \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2}$$

subject to the constraint

$$\Delta_1 + \Delta_2 + \Delta_3 + \omega_1 + \omega_2 = 1$$

For $J_\phi = J_\psi$ the 5d rotating black hole can be reduced to a static AdS_4 dyonic black hole. The Legendre transform of E becomes the attractor mechanism in 4d.

Rotating AdS_5 BPS black holes

Has this result a field theory interpretation?

- Legendre transform of 1/16 BPS states partition function: too hard

$$Z(\omega_i, \Delta_I) = \mathrm{Tr} e^{i\Delta_I R_I + i\omega_i J_i}$$

R_I R-symmetry generators in $SO(6)$; J_i angular momenta

- The superconformal index

$$I(\omega_i, \Delta_I) = \mathrm{Tr}(-1)^F e^{i\Delta_I R_I + i\omega_i J_i}\,, \qquad \sum \omega_i + \sum \Delta_I \in 2\pi\mathbb{Z}$$

  number of fugacities equal to the number of conserved charges. This time, unfortunately, $I = O(1)$.

- It is known that the supersymmetric partition function on $S^3 \times S^1$

$$Z_{\mathrm{susy}}(\omega_i, \Delta_I) = e^{-E_{\mathrm{susy}}} I(\omega_i, \Delta_I)$$

  Casimir energy $E_{\mathrm{susy}} = O(N^2)$.

[Martelli,diPietro,Lorentzen,Komargosky,Cassani, Assel]

Entropy can be found by an extremization?

The expression

$$E_{\rm susy} = -i\pi N^2 \frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2}$$

subject to the constraint

$$\Delta_1 + \Delta_2 + \Delta_3 + \omega_1 + \omega_2 = 0$$

is the supersymmetric Casimir energy of $\mathcal{N} = 4$ SYM. [Bobev]

This analogy calls for an explanation

- A different regularization of $Z_{\rm susy}(\omega_i, \Delta_I)$ would reproduce E?
- Degeneracy related to a vacuum energy?
- What is the role of angular ambiguities? they were crucial also for ABJM and AdS_4 black holes

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions.
- it would be interesting to investigate $1/N$ corrections
- it can be extended to other models

  [Hosseini,AZ; Hosseini-Mekareeya; Azzurli,Bobev,Crichigno,Min,AZ]
- the twisted index can be evaluated also for black strings in AdS_5

  [Bobev-Benini; Hosseini,Nedelin,AZ]

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The main message of this talk is that you can related the entropy of a class of AdS black holes to a microscopic counting of states.

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- the twisted index can be evaluated also for black strings in AdS_5

  [Bobev-Benini; Hosseini,Nedelin,AZ]
- the AdS_5 still requires an interpretation

Thank you for the attention !