Advances in Non-relativistic Quantum Gravity

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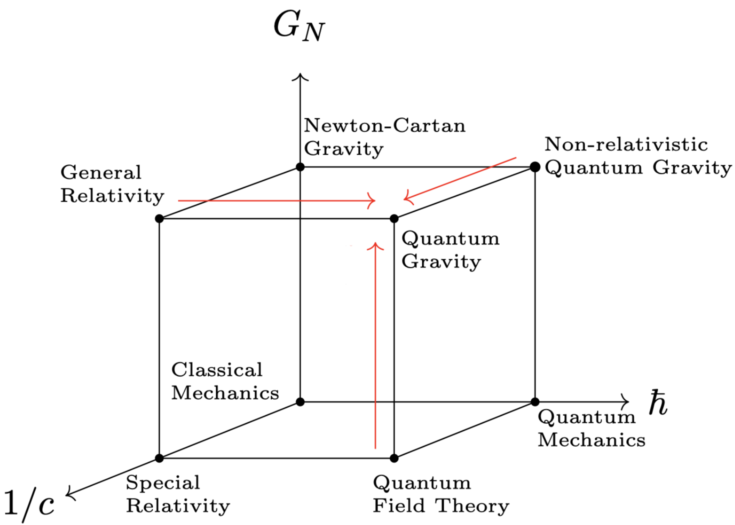
Three Roads to Quantum Gravity



NR Quantum Gravity

Does combining gravity with quantum mechanics require relativity?

Does NR string theory define NR quantum gravity?

Does NR gravity has its own holographic principle?

Some References

NR gravity/string theory involving null-reduction

T. Harmark, J. Hartong and N. A. Obers (2017); Kluson (2018);
T. Harmark, J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018);
Kluson (2019); Roychowdhury (2019); T. Harmark, J. Hartong, L. Menculini,
N. A. Obers and G. Oling (2019); A.D. Gallegos, U. Gürsoy and N. Zinnato (2019);
L. Bidussi, T. Harmark, J. Hartong, N.A, Obers, G. Oling (2021)

NR strings with NR worldsheet

C. Batlle, J. Gomis and D. Not (2017); C. Batlle, J. Gomis, L. Mezincescu and
P. K. Townsend (2017); T. Harmark, J. Hartong and N. A. Obers (2017); T. Harmark,
J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018)

Outline

Non-relativistic Limits

Outline

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Non-relativistic String theory

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Defining a NR Limit

STEP 1: decomposing $E_\mu{}^{\hat{A}} = (E_\mu{}^0, E_\mu{}^{A'}) =$ (clock, ruler) and introducing M_μ, perform an invertable field redefinition involving a parameter c:

$$E_\mu{}^0 = c\tau_\mu + c^{-1}m_\mu\,, \qquad E_\mu{}^{A'} = e_\mu{}^{A'}\,, \qquad M_\mu = c\tau_\mu - c^{-1}m_\mu$$

STEP 2: take the limit $c \to \infty$ and take care of possible divergences

Example: Particles and a 'critical' limit

cp. to Seiberg, Susskind, Toumbas (2000); Gopakumar, Maldacena, Minwalla, Strominger (2000); Danielsson, Guijosa, Kruczenski (2000), Gomis, Ooguri (2001)

Starting from a particle coupled to gravity, the red terms in the above field redefinition lead to divergencies in the kinetic and Wess-Zumino term that cancel against each other.

Infinities

Using a second-order formulation of general relativity, the NR limit of the spin-connection fields $\Omega_\mu{}^{\hat{A}\hat{B}}(E)$ contains a leading divergence that usually is set to zero by imposing the zero torsion constraint

$$\partial_{[\mu}\tau_{\nu]} = 0$$

Given this constraint the NR limit of the Einstein e.o.m. (no action!) yields the NC gravity e.o.m. where the Newton potential Φ can be identified as the time component of the central charge gauge field m_μ:

$$\Phi \sim \tau^\mu m_\mu$$

This NC gravity theory is a reformulation of Newtonian gravity valid in any frame and including strong gravity effects

The Zero Torsion Constraint

$$\partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

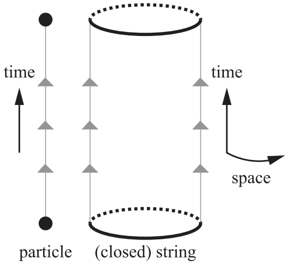
$$\text{Torsional NC gravity}: \quad \partial_\mu \tau_\nu - \Gamma_{\mu\nu}{}^\rho \tau_\rho = 0 \quad \rightarrow \quad \Gamma_{[\mu\nu]}{}^\rho \tau_\rho = \partial_{[\mu}\tau_{\nu]}$$

Geometry with Co-dimension 2 Foliation

The string should be coupled to a 2-form gauge field $B_{\mu\nu}$ with

$$B_{\mu\nu} = -c^2 \epsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B + b_{\mu\nu}$$

defining a geometry with a co-dimension 2 foliation where $\tau_\mu \rightarrow \tau_\mu{}^A$ with $\hat{A} = (A, A') = (0, 1, A')$



The Basic Variables

The decomposition leading to NC gravity

$$\{E_\mu{}^{\hat{A}}, M_\mu\} \quad \rightarrow \quad \{\tau_\mu, e_\mu{}^{A'}, m_\mu\}$$

gets replaced by the following redefinition:

$$\{E_\mu{}^{\hat{A}}, B_{\mu\nu}, \Phi\} \quad \rightarrow \quad \{\tau_\mu{}^A, e_\mu{}^{A'}, b_{\mu\nu}, \phi\}$$

The Newton potential Φ can be identified with the time-space component $\epsilon^{AB}\tau^\mu{}_A\tau^\nu{}_B b_{\mu\nu}$ of the 2-form gauge field $b_{\mu\nu}$

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The NR String Sigma Model

The bosonic closed string sigma model (without Yang-Mills) is given by

J. Gomis, Z. Yan + E.B. (2018); J. Gomis, J. Rosseel, C. Şimşek, Z. Yan + E.B. (2019)

$$S_{\mathrm{NR}\sigma} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha x^\mu \partial_\beta x^\nu\, e_\mu{}^{A'}\, e_\nu{}^{B'}\, \delta_{A'B'} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu\, b_{\mu\nu}\right] + S_{\mathrm{dilaton}}$$

with world-sheet metric $h_{\alpha\beta} \sim \tau_{\alpha\beta} \equiv \partial_\alpha x^\mu \partial_\beta x^\nu \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$

This is the generalization of flat spacetime to a string NC background

Gomis, Ooguri (2001); Danielsson, Guijosa, Kruczenski (2000)

Note: we have not imposed any geometric constraint sofar (they follow later from dynamics)

Special Features

- The KR 2-form field $b_{\mu\nu}$ transforms under string-Galilean boost transformations:

$$\delta b_{\mu\nu} = \partial_{[\mu}\lambda_{\nu]} + 2\epsilon_{AB}\lambda_{A'}{}^{A}\tau_{[\mu}{}^{B}e_{\nu]}{}^{A'}$$

A relativistic matter field $B_{\mu\nu}$ becomes a NR geometric field $b_{\mu\nu}$

- There is an emergent dilatation symmetry:

$$\delta\tau_{\mu}{}^{A} = \lambda_D\tau_{\mu}{}^{A}\,, \qquad \delta\phi = \lambda_D$$

This means that the # of NR background fields is one less than the # of relativistic background fields $\rightarrow$ one 'missing e.o.m.'

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Canceling the Divergences

$$S_{\text{rel}} = \frac{1}{2\kappa^2}\int d^{10}x\, E\left(\mathcal{R} - \frac{1}{12}\mathcal{H}_{\mu\nu\rho}\mathcal{H}^{\mu\nu\rho}\right)$$

with $\mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$. We redefine

$$E_\mu{}^A = c\tau_\mu{}^A\,, \qquad E_\mu{}^{A'} = e_\mu{}^{A'}\,, \qquad B_{\mu\nu} = -c^2\epsilon_{AB}\tau_\mu{}^A\tau_\nu{}^B + b_{\mu\nu}$$

and find

$$S = c^2 \overset{(2)}{S} + \overset{(0)}{S} + c^{-2}\overset{(-2)}{S} + c^{-4}\overset{(-4)}{S}$$

where $\overset{(2)}{S}$ is proportional to the torsion tensor

$$\tau_{\mu\nu}{}^A \equiv \partial_{[\mu}\tau_{\nu]}{}^A$$

Two miracles: (i) the metric and 2-form contributions to $\overset{(2)}{S}$ precisely cancel and (ii) $\overset{(0)}{S}$ is invariant under local dilatations!

Special Features of Non-relativistic Action

$$S_{\rm NR} = \frac{1}{2\,\kappa^2}\int \mathrm{d}^{10}x\, e\,\Big(\mathrm{R}(J) - \frac{1}{12}\,h_{A'B'C'}\,h^{A'B'C'} - 4\,\mathcal{D}_{A'}\,b^{A'} - 4\,b_{A'}\,b^{A'} - 4\,\tau_{A'\{AB\}}\tau^{A'\{AB\}}\Big)\,.$$

- the action has an emergent dilatation symmetry and therefore has one 'missing field' and one 'missing e.o.m.'
  - the dilatation gauge field $b_\mu = b_\mu(\tau,\phi)$ is dependent
- The 'missing' e.o.m. follows from taking the NR limit of the e.o.m. and is precisely the Poisson equation of the Newton potential
  - The full set of e.o.m. form a reducible, but indecomposable representation: the Poisson equation needs the action!
- The e.o.m. of the Newton potential itself gives the following non-linear geometric constraint: $\tau_{B'C'A}\tau^{B'C'A} = 0$
  - This prevents an overdetermined set of equations

Action, E.O.M. and β-Functions

The e.o.m. of non-relativistic string theory are determined by calculating the β-functions

Gomis, Oh, Yan (2019), Yan, Yu (2019), Gomis, Yan, Yu (2020); see also Gallegos, Gürsoy and Zinnatos (2019)

The emergent dilatation symmetry has the following effect:

$$\begin{aligned} \text{NR NS-NS action} \quad &\to \quad \text{common equations} \ + \text{Non-linear} \\ \text{NR } \beta\text{-functions} \quad &\to \quad \text{common equations} \ + \text{Poisson} \\ \text{NR e.o.m.} \quad &\to \quad \text{common equations} \ + \text{Poisson} + \text{Non-linear} \end{aligned}$$

The nonlinear equation is required in order that the NR string σ model does not flow towards a relativistic string σ model

Z. Yan (2021)

Torsional String Newton-Cartan (TSNC) geometry

basic variables: $\{\tau_\mu{}^A, e_\mu{}^{A'}, b_{\mu\nu}, \phi\}$

$\{\omega_\mu\,, \omega_\mu{}^{AA'}\,, \omega_\mu{}^{A'B'}\,, b_\mu\}$ are dependent, e.g., $b_\mu = e_\mu{}^{A'}\,\tau_{A'A}{}^A + \tau_\mu{}^A\partial_A\phi$

$$\nabla_\mu\tau_\nu{}^A \equiv \partial_\mu\tau_\nu{}^A - \omega_\mu\,\epsilon^{AB}\tau_{\nu B} - b_\mu\,\tau_\nu{}^A - \Gamma^\rho_{\mu\nu}\tau_\rho{}^A = 0\,,$$

$$\nabla_\mu e_\nu{}^{A'} \equiv \partial_\mu e_\nu{}^{A'} - \omega_\mu{}^{A'B'}e_{\nu B'} + \omega_\mu{}^{AA'}\tau_{\nu A} - \Gamma^\rho_{\mu\nu}e_\rho{}^{A'} = 0$$

non-zero torsion: $T^\rho_{\mu\nu} = 2\,\Gamma^\rho_{[\mu\nu]} = 2D_{[\mu}(\omega, b)\tau_{\nu]}{}^A\tau_A{}^\rho$

cp. to Geracie, Prabhu, Roberts (2015)

Outline

Taking the NR limit of fermions

Define world-sheet chirality projection operators $\Pi_\pm$ by

$$\Pi_\pm = \frac{1}{2}(1 \pm \gamma_0\gamma_1)$$

and consider projected target space spinors

$$\chi_\pm = \Pi_\pm \chi$$

Gomis, Kamimura, Townsend (2004)

We then redefine the spinor χ as

$$\chi = c^{1/2}\chi_+ + c^{-1/2}\chi_-$$

such that, after taking $c \to \infty$, the projected spinors transform under Galilean boosts as follows:

$$\delta\chi_+ = 0 \quad \text{and} \quad \delta\chi_- = \frac{1}{2}\lambda^{AA'}\Gamma_{AA'}\chi_+$$

New features compared to the bosonic case

- There is no direct connection between a two-dimensional sigma model description and the NR target space effective action
- Taking the naive NR limit leads to divergent terms in the supersymmetry rules

These divergences can be controlled by

- the occurrence of 2 'superconformal' Stueckelberg symmetries beyond dilatations yielding a shortened supergravity multiplet
- imposing by hand the following twistless torsional constraint:

$$T_{\mu\nu}^{-} = 0 \qquad \text{or} \qquad \tau_{[\mu}{}^{-}\partial_\nu \tau_{\rho]}{}^{-} = 0$$

Christensen, Hartong, Obers, Rollier (2013)

defining a 'self-dual' TSNC geometry (invariant under SUSY!)

Minimal Supergravity Action versus E.O.M.

- the action has one emergent dilatation and two emergent superconformal symmetries. It therefore has one 'missing' bosonic and two 'missing' fermionic fields plus corresponding 'missing' e.o.m.

- The 'missing' e.o.m. follow from taking the NR limit of the e.o.m. and are precisely the Poisson equation of the Newton potential plus two fermionic partner equations

- The minimal supergravity action is a pseudo action in the sense that it is only invariant under supersymmetry if one uses the twistless torsional constraint <u>after</u> varying the action. Due to this the e.o.m. that follow from the action transform under supersymmetry to the 'missing' e.o.m.: they belong to the same supermultiplet

cp. to Vanhecke, Van Proeyen (2017)

Outline

Outlook

- how general is our limit technique?
    - invertable field redefinition, cancellation of divergences, local dilatation symmetry
- including Yang-Mills to obtain heterotic gravity
    - sigma model anomaly, T-duality: taking a string NR limit followed by spatial reduction is dual to a null-reduction
- open strings

  see lectures by Z. Yan at 1st School on NR QFT, Gravity and Geometry
- connection to Double Field Theory

  Ko, Melby-Thompson, Meyer and Park (2015); Gallegos, Gürsoy, Verma and Zinnato (2020)
- extension to IIA/IIB supergravity and M-theory

  for bosonic sector of M-theory, see Blair, Gallegos, Zinnato (2021)

Take-Home Message

Our results pave the way for a target space approach to NR string theory:

(supersymmetric) brane solutions, compactifications, NR holography etc.