Branes wrapped on spindles

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Ferrero, JPG, Ipina,Martelli, Sparks x2

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Cassani, JPG,Martelli, Sparks

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Wrapped branes and susy AdS/CFT

[Maldacena, Nunez 00]

- Basic examples of AdS/CFT:

D3-branes $\mathbb{R}^{1,3}$: $AdS_5 \times S^5 \quad \leftrightarrow \quad d = 4,\ \mathcal{N} = 4\ \mathrm{SYM}$

M5-branes $\mathbb{R}^{1,5}$: $AdS_7 \times S^4 \quad \leftrightarrow \quad d = 6,\ \mathcal{N} = (0,2)$

M2-branes $\mathbb{R}^{1,2}$: $AdS_4 \times S^7 \quad \leftrightarrow \quad d = 3,\ \mathrm{ABJM}$

- Wrap the brane world volume: $\mathbb{R}^{1,k} \times \Sigma_p$
  - Preserve susy?
  - Follow RG to IR - new examples of AdS/CFT?

- Susy can be preserved by a "topological twist" [Witten 88]

  Couple the SCFT on $\mathbb{R}^{1,k} \times \Sigma_p$ to background R-symmetry currents

$$(\partial_\mu + \tfrac{1}{4}\omega_\mu \cdot \Gamma + A_\mu \cdot \gamma)\epsilon = 0$$

  constant spinors on Σ_p

- Geometric viewpoint: [Bershadksy,Sadov,Vafa 95]

  Wrap branes about a calibrated cycle in a special holonomy manifold

- e.g. M5 wrapping holomorphic two-cycles: [Maldacena, Nunez 00]

[Bah,Beem,Bobev,Wecht 11][…]

M5 wrapped on $\Sigma_2 \subset CY_2$:

$$AdS_7 \times S^4 \to AdS_5 \times \text{“}\Sigma_2 \times S^4\text{”}$$

IR dual to : $d = 4,\ \mathcal{N} = 2$ SCFT

M5 wrapped on $\Sigma_2 \subset CY_3$:

$$AdS_7 \times S^4 \to AdS_5 \times \text{“}\Sigma_2 \times S^4\text{”}$$

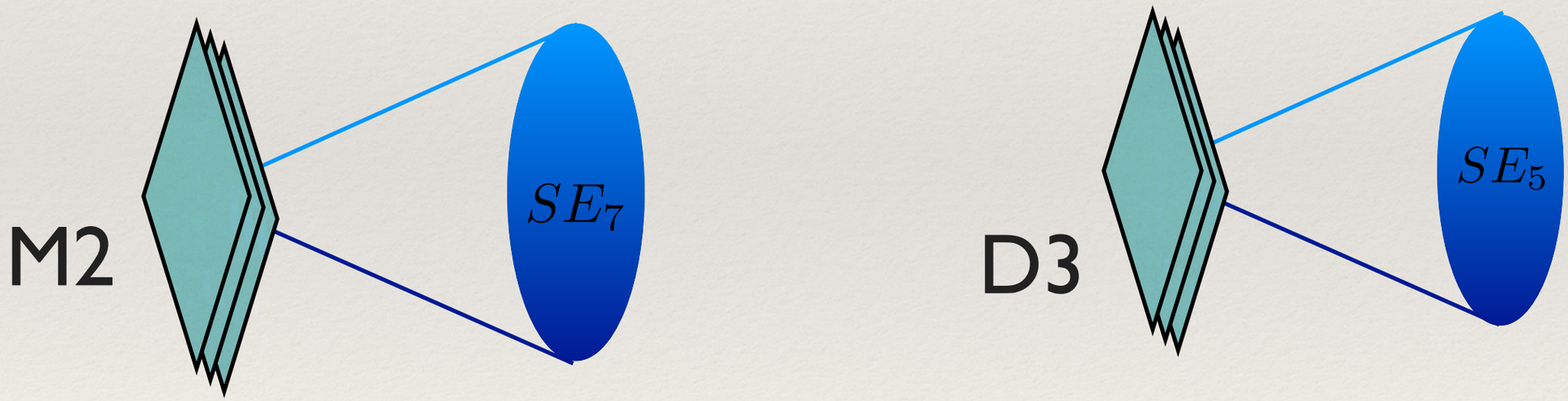
IR dual to : $d = 4,\ \mathcal{N} = 1$ SCFT

- Analogous story for D3-branes and M2-branes wrapped on Σ_2

  [Maldacena, Nunez 00][JPG,Kim,Waldram 01][Benini,Bobev 13]….

- For M5-branes and D3-branes can also wrap on higher dimensional cycles [Gauntlett,Kim,Waldram 00][…]

- Many other developments - eg involving SE spaces



and then wrap world-volume on a two-cycle with a topological twist [Benini,Bobev,Crichigno 15][…]

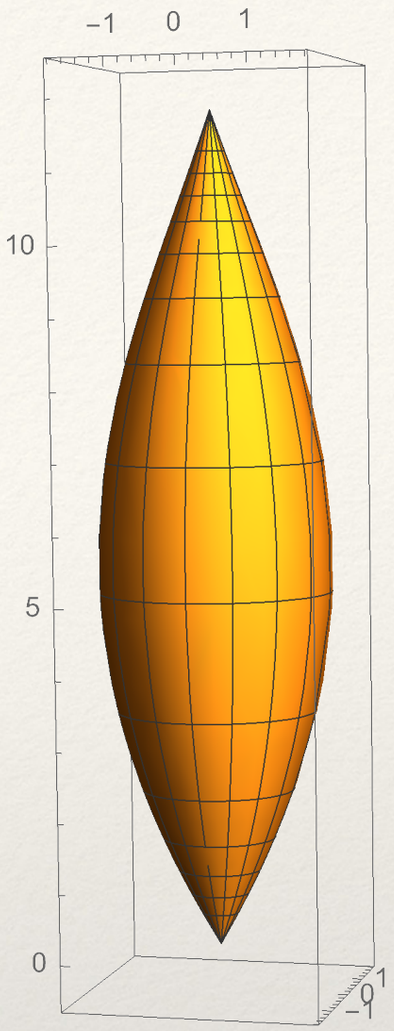
Here: D3, M2 and M5-branes wrapped on a spindle

Spindle has orbifold singularities!

Susy NOT via the usual topological twist!

Spindle $\Sigma(n_-, n_+)$ $n_\pm \in \mathbb{N}$

$hcf(n_+, n_-) = 1$



- Topologically a two-sphere
- Has conical deficits at poles $2\pi(1 - \frac{1}{n_\pm})$
- $\Sigma = \mathbb{WCP}^1(n_-, n_+)$
- Constant curvature metrics don't exist on Σ
- Euler character $\chi = \frac{1}{n_-} + \frac{1}{n_+}$

Plan

- D3-branes wrapped on a spindle - dual SCFT $d = 2, \mathcal{N} = (0, 2)$

  IR physics: new interpretation of type IIB AdS_3 solutions first found in 2006 and precision AdS/CFT test of conjectured dual!

- M2-branes wrapped on spindle - dual SCQM $\mathcal{N} = 2$

  IR: new AdS_2 solutions

  UV: near horizon limit of accelerating AdS4 black holes!

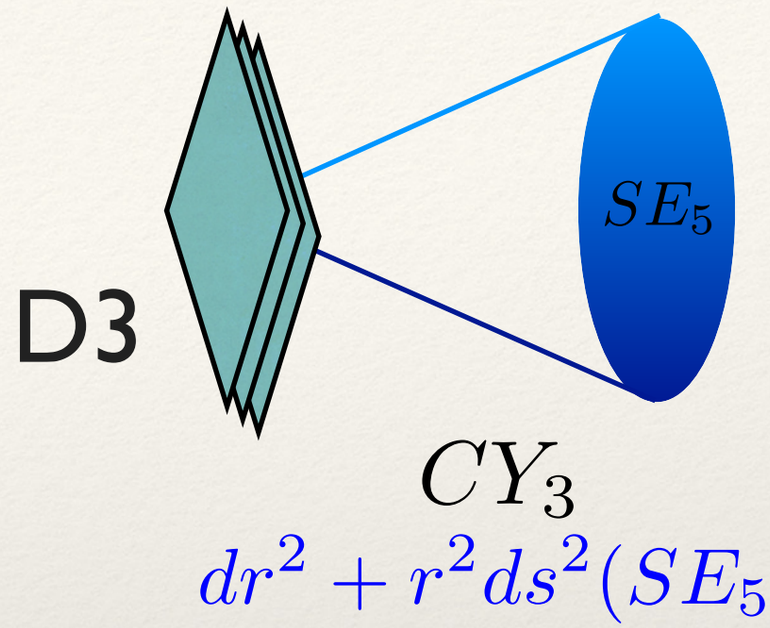
- (M5-branes)

AdS/CFT and Sasaki-Einstein manifolds

Type IIB

$$ds^2 = ds^2(AdS_5) + ds^2(SE_5)$$

$$F_5 = vol(AdS_5) + vol(SE_5)$$

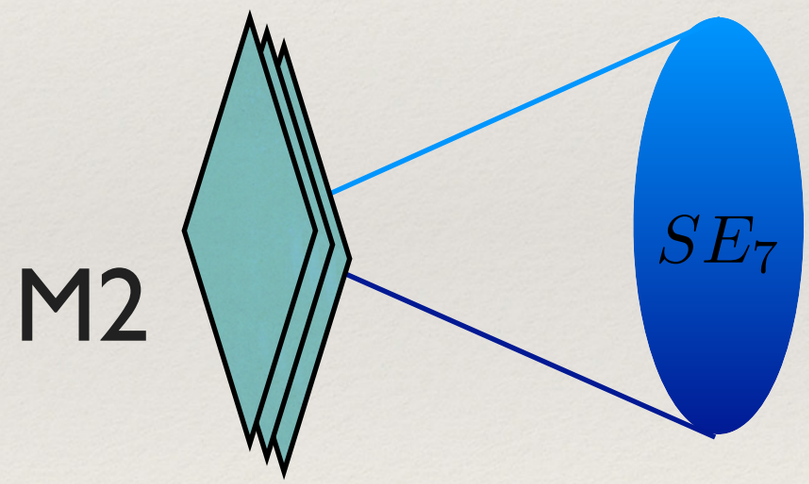


Dual to N=1SCFT in d=4

D=11

$$ds^2 = ds^2(AdS_4) + ds^2(SE_7)$$

$$G_4 = vol(AdS_4)$$



Dual to N=2 SCFT in d=3

Much is known about dual SCFTs…

Geometry:

SE have canonical Killing vector ξ dual to R-symmetry of SCFT

Let η be one-form dual to ξ (i.e. $\eta \cdot \xi = 1$)

$$ds^2(SE_{2n+1}) = \eta^2 + ds^2(KE_{2n})$$

with $d\eta = \rho$, the Ricci form of $ds^2(KE_{2n})$

Regular class: U(1) fibration over KE manifold

Quasi-Regular class: U(1) fibration over KE orbifold

Irregular class: R-symmetry orbits don't close

Regular class:

D=5 - full list

$\mathbb{CP}^2 \quad \rightarrow \quad S^5$ and $S^5/\mathbb{Z}_3$

$\mathbb{CP}^1 \times \mathbb{CP}^1 \quad \rightarrow \quad T^{1,1}$ and $T^{1,1}/\mathbb{Z}_2$

$dP_m, \quad m = 3, \ldots, 8$

D=7 examples

$\mathbb{CP}^3 \quad \rightarrow \quad S^7$ and $S^7/\mathbb{Z}_2$ and $S^7/\mathbb{Z}_4$

$\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1 \quad \rightarrow \quad Q^{1,1,1}$ and $Q^{1,1,1}/\mathbb{Z}_2$

$\mathbb{CP}^2 \times \mathbb{CP}^1 \quad \rightarrow \quad M^{3,2}$

Want to construct SUGRA solutions describing:

D=4 SCFTs dual to $AdS_5 \times SE_5$ placed on $\mathbb{R}^{1,1} \times \Sigma$

D=3 SCFTs dual to $AdS_4 \times SE_7$ placed on $\mathbb{R} \times \Sigma$

with $\Sigma = \mathbb{WCP}^1(n_-, n_+)$ a spindle

How? Use consistent KK truncations:

Type IIB: KK reduce on SE_5 to D=5 minimal gauged SUGRA

[Buchel,Liu 06]

Any (susy) D=5 solution uplifts, locally, to a (susy) D=10 solution e.g. AdS_5 vacuum uplifts to $AdS_5 \times SE_5$

D=11: KK reduce on SE_7 to D=4 minimal gauged SUGRA

[JPG,Varela 07]

D3-branes wrapped on a spindle

[Ferrero,JPG,Ipina,Martelli,Sparks 20]

D=5 gauged SUGRA: $\mathcal{L} \sim \sqrt{-g}(R + 12 - F^2) + A \wedge F \wedge F$

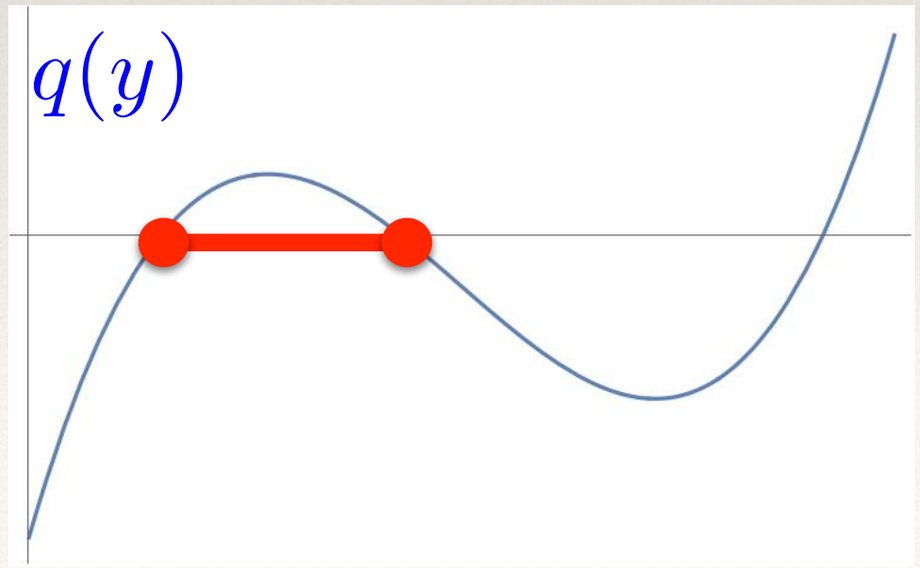
$AdS_3 \times \Sigma(n_-, n_+)$ susy solution - dual to $d = 2, \mathcal{N} = (0, 2)$ SCFT

$$ds_5^2 = \frac{4y}{9} ds^2_{AdS_3} + ds^2_\Sigma \,, \qquad A = \frac{1}{4}\left(1 - \frac{a}{y}\right) dz$$

$$ds^2_\Sigma = \frac{y}{q(y)} dy^2 + \frac{q(y)}{36y^2} dz^2 \qquad q(y) = 4y^3 - 9y^2 + 6ay - a^2$$

$$a = \frac{(n_- - n_+)^2 (2n_- + n_+)^2 (n_- + 2n_+)^2}{4(n_-^2 + n_- n_+ + n_+^2)^3} \,,$$

$$\Delta z = \frac{2(n_-^2 + n_- n_+ + n_+^2)}{3 n_- n_+ (n_- + n_+)} 2\pi \,,$$



R-symmetry flux through spindle:

$$Q_m = \frac{1}{2\pi}\int_\Sigma F = \frac{n_- - n_+}{2n_-n_+}$$

- Not a topological twist! $\quad Q_m \neq \chi = \frac{1}{n_+} + \frac{1}{n_-}$

  Killing spinors are not constant on the spindle

- Uplift to type IIB using SE5 in regular class

  With care: the uplifted solution is completely smooth!

  Subtlety: which SE5 one uplifts on depends on $n_\pm$

  e.g. $n_+ = 2$ $\qquad n_- = 3, 7, \ldots \qquad \to \qquad S^5/\mathbb{Z}_3$

  $\mathbb{CP}^2$ $\qquad\qquad n_- = 5, 9, \ldots \qquad \to \qquad S^5$

Analogy: consider metric on S^3

$$ds^2 = d\theta^2 + \cos^2\vartheta d\phi_1^2 + \sin^2\vartheta d\phi_2^2$$

$$\vartheta \in [0, \pi/2]$$

$$\Delta\phi_i = 2\pi$$

Do a KK reduction using the Killing vector

$$V = n_+\partial_{\phi_1} + n_-\partial_{\phi_2}$$

Find a U(1) fibration over the spindle:

New coordinates: $\phi_1 = n_+\nu$

$$\phi_2 = n_-\nu + \frac{1}{n_+}\mu$$

$$\Delta\mu, \Delta\nu = 2\pi$$

$$V = \partial_\nu$$

$$ds^2 = \Lambda(d\nu + Ad\mu)^2 + d\vartheta^2 + \frac{\cos^2\vartheta\sin^2\vartheta}{\Lambda}d\mu^2$$

$$\Lambda = n_+^2\cos^2\vartheta + n_-^2\sin^2\vartheta$$

In fact the regular $AdS_3 \times$ "$\Sigma \times SE_5$" solutions were already constructed from a very different point of view in 2006!

[Gauntlett,Kim,Waldram 06]

Central charge of $d = 2, \mathcal{N} = (0,2)$ SCFT

$$c = \frac{4(n_- - n_+)^3}{3n_-n_+(n_-^2 + n_-n_+ + n_+^2)} a_{4d}$$

For $d = 4, \mathcal{N} = 1$ SCFT dual to $AdS_5 \times SE_5$

We now have a precise conjecture for d=2 SCFT: take d=4 SCFT dual to $AdS_5 \times SE_5$ wrap on a spindle and then flow to the IR

Precision check: central charge from a field theory computation

Use c-extremisation principle of [Benini,Bobev 12]

Idea: Extremise $c_{trial}(R)$ over possible 2-d R symmetries

- Allow for mixing with internal symmetry of spindle
- The 2d anomaly polynomial is obtained by integrating the 4d anomaly polynomial over the spindle, taking into account that there is flux through the spindle.

c.f. [Bah,Bonetti,Minasian,Nardoni 19]
[Hristov, Tachikawa,Zaffaroni 20]

$$R_{\rm trial} = \tilde{R} + \varepsilon\, J$$

Find $\varepsilon_* = \dfrac{3n_-n_+(n_- + n_+)}{n_-^2 + n_-n_+ + n_+^2}$ and $c(R_*) = c_{SUGRA}$

Many open questions eg

- Is there a black string solution of the form:

$$AdS_5 \to AdS_3 \times \Sigma$$

M2-branes wrapped on a spindle [Ferrero,JPG,Ipina,Martelli,Sparks 20]

D=4 gauged SUGRA: $\mathcal{L} \sim \sqrt{-g}(R + 6 - F^2)$

$AdS_2 \times \Sigma(n_-, n_+)$ Solutions:

- NOT a topological twist $Q_m \neq \chi(\Sigma)$
- Can uplift on SE7 in regular class - completely regular solutions

[Gauntlett,Kim,Waldram 06]

New features:

- Rotation, consistent with AdS2 symmetries

$$ds^2 = f_1(y)\Big(-\rho^2\, d\tau^2 + \frac{d\rho^2}{\rho^2}\Big) + f_2(y) dy^2 + f_3(y)(dz + j\,\rho\, d\tau)^2$$

- UV completion $AdS_4 \to AdS_2 \times \Sigma$

This is precisely a two parameter family of supersymmetric dyonic, rotating and accelerating back holes! [Plebanski,Demianski 76]

- Acceleration gives rise to conical singularities

Long history of dealing with them. Here we find that are completely removed by uplifting SE_7 !

- Conformal boundary has a spindle for $J \neq 0$

Final Comments

- Have wrapped D3,M2-branes on spindles

  Supersymmetry without usual topological twist

  Generalisations [Hosseini,Hristov, Zaffaroni][Boido,Ipina,Sparks]
  [Fererro,Inglese,Martelli,Sparks][Couzens,Stemerdink,van de Heisteeg]

- For wrapped D3,M2s uplifted with certain regular Sasaki-Einstein the upstairs metric is regular

  When does this happen?

- Connects with broader programme of understanding

  $AdS_3 \times GK_7$ and $AdS_2 \times GK_9$ solutions

  [Kim 05][Kim,Park 06][Gauntlett,Kim 07]
  [Couzens,Gauntlett,Martelli,Sparks 18][Gauntlett,Martelli,Sparks19, 19][…]

- Field theory:

  What rules for fields at orbifold points? eg N=4 SYM

  SUGRA suggest some SCFTs are obstructed- why?

- Can also wrap M5-branes on spindles [Ferrero,JPG,Martelli,Sparks 21]

No $AdS_5 \times \Sigma$ solutions in minimal D=7 gauged supergravity (why?) but can construct in non-minimal case

Dual to $d = 4, \mathcal{N} = 1$ SCFTs

Susy NOT via usual topological twist but now $Q_m = \chi(\Sigma)$: "topologically a topological twist"!?

Can carry out a precision test of AdS/CFT by comparing with d=6 (0,2) SCFT reduced on a spindle - exact agreement

D=11 solution has orbifold singularities…rules?

- UV completions

  M2: accelerating black holes - new interpretation

  Recover entropy using new developments involving complex saddle points [Cabo-Bizet,Cassani,Martelli,Murthy 18][Cassani,Papini 19] [Cassani,Gauntlett,Martelli,Sparks 21]

  D3,M5?

- Wrap branes on higher dimensional orbifolds?

- Rich new landscape to explore!