# Crossover from a fractionalized SYK spin liquid to a confining spin glass

Strings, Fields and Holograms,
Ascona, Switzerland
October 11, 2021

Subir Sachdev

Maine Christos, Felix Haehl, and S.S., arXiv:2110.00007

INSTITUTE FOR ADVANCED STUDY

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Talk online: sachdev.physics.harvard.edu arXiv.org > cond-mat > arXiv:2109.05037

Condensed Matter > Strongly Correlated Electrons

[Submitted on 10 Sep 2021]

# Sachdev-Ye-Kitaev Models and Beyond: A Window into Non-Fermi Liquids

Debanjan Chowdhury, Antoine Georges, Olivier Parcollet, Subir Sachdev

Comments: 72 pages, 25 figures and lots of references. Comments are welcome

1. Classical and quantum Ising spin glass

2. $S=1/2$ SU(2) spins with random exchange

   A. SYK spin liquid

   B. Numerical results

   C. Spin glass: crossover from fractionalization to confinement

3. Entropy and Complexity

4. Holography: $AdS_2$ fragmentation ?

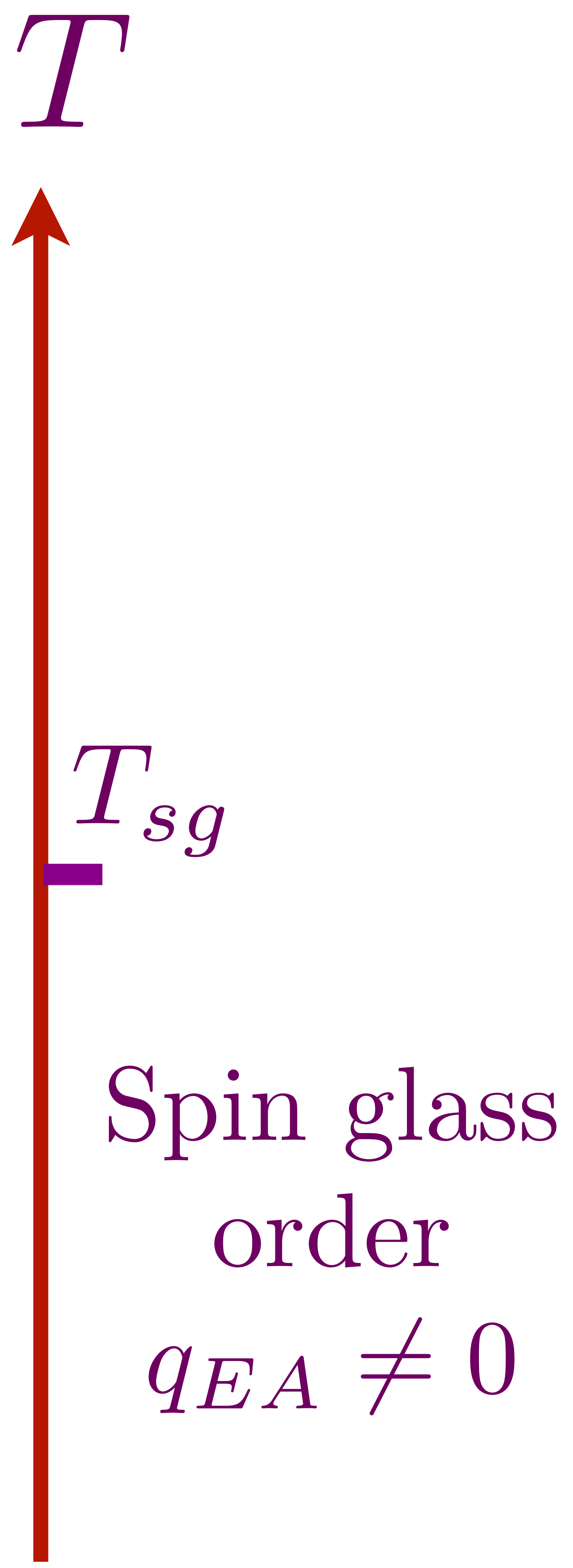
## Sherrington-Kirkpatrick model

$$H = \frac{1}{2\sqrt{N}} \sum_{i,j=1}^{N} J_{ij} \sigma_i \, \sigma_j$$

$$\mathcal{Z} = \sum_{\sigma_i = \pm 1} e^{-H/T}$$

$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad$ Different $J_{ij}$ uncorrelated.

Edwards-Anderson order parameter $q_{EA} = \overline{\langle \sigma_i \rangle^2}$



<u>Parisi solution:</u> introduce $n$ replicas, $a, b = 1 \ldots n$

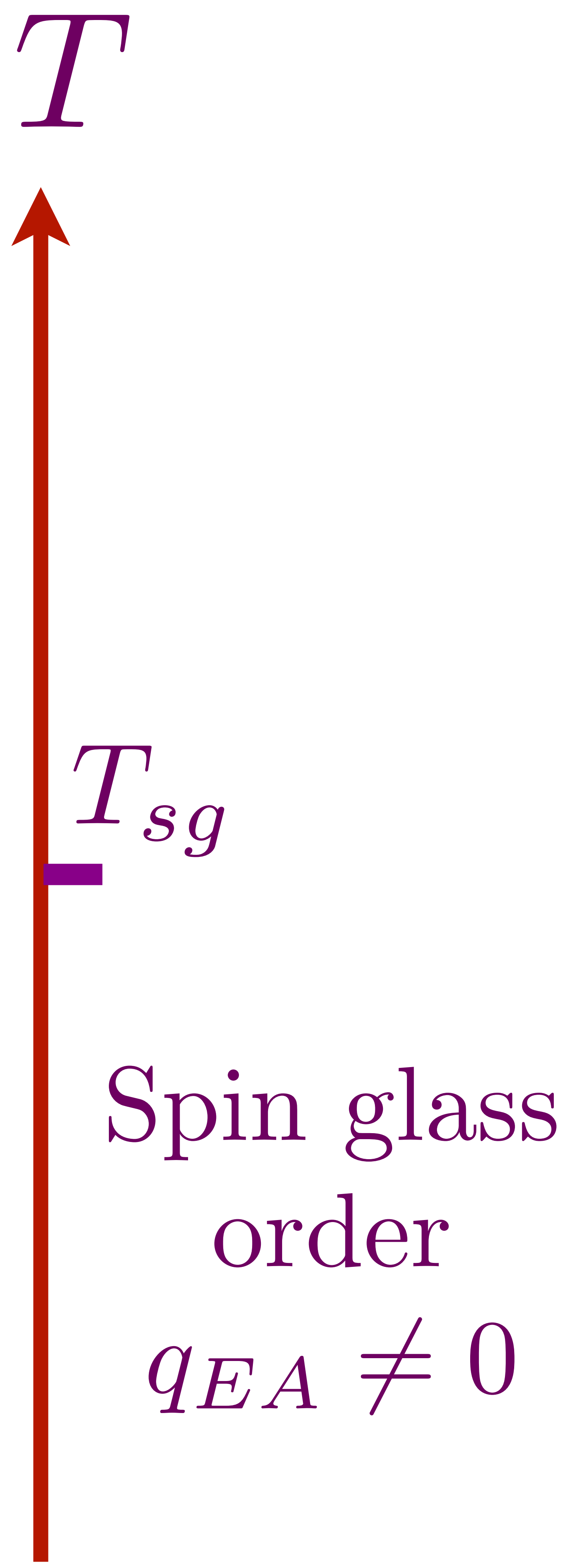
$$\overline{\mathcal{Z}^n} = \sum_{\sigma_i = \pm 1} \exp\left( \frac{J^2}{4T^2 N} \left[ \sum_i \sigma_i^a \sigma_i^b \right]^2 \right)$$

$$= \int dq_{ab} \exp\left( - \frac{N J^2}{2T^2} q_{ab}^2 \right) \left[ \sum_{\sigma^a = \pm 1} \exp\left( \frac{J^2}{T^2} q_{ab} \sigma^a \sigma^b \right) \right]^N .$$

In the large $N$ limit, need saddle points of the free energy $F$ as a function of $q_{ab} = \langle \sigma^a \sigma^b \rangle$.

$$F(q_{ab}) = \frac{J^2}{2T} q_{ab}^2 - T \ln \left[ \sum_{\sigma^a = \pm 1} \exp\left( \frac{J^2}{T^2} q_{ab} \sigma^a \sigma^b \right) \right] .$$

The matrix $q_{ab}$ is characterized by a monotonic function $q(u)$, $u \in [0, 1]$, with $q(1) = q_{EA}$.

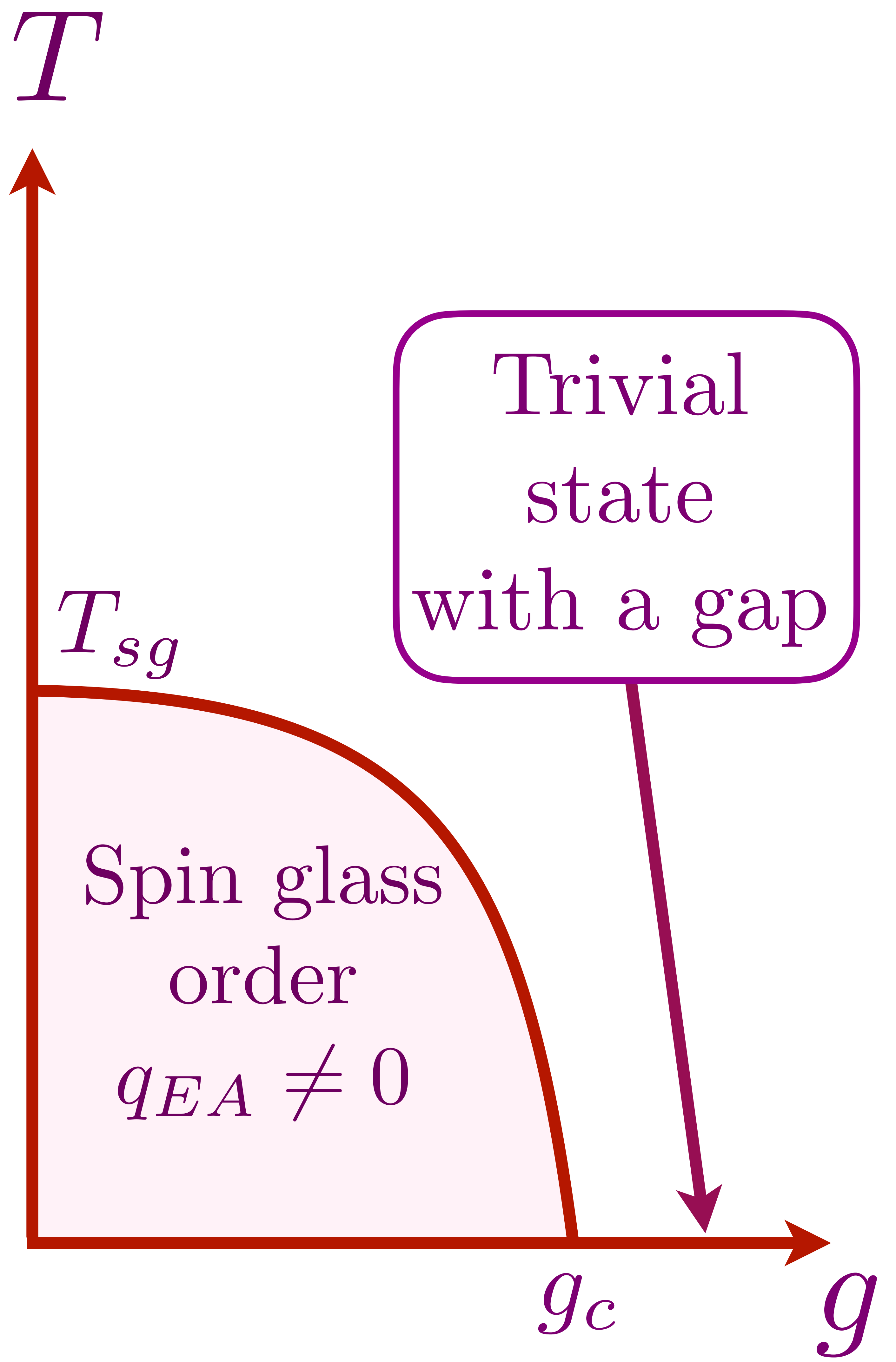


# Quantum Ising model

$$H = \frac{1}{2\sqrt{N}} \sum_{i,j=1}^{N} J_{ij} \sigma_i^z \sigma_j^z - g \sum_i \sigma_i^x$$

For simplicity, promote $\sigma^z$ to a real field $\phi$.
In the large $N$ limit, need saddle points of the action $\mathcal{S}$ as a functional of $Q_{ab}(\tau - \tau') = \langle \phi_a(\tau)\phi_b(\tau')\rangle$ .

$$\mathcal{S}[Q_{ab}] = \frac{J^2}{2} \int d\tau d\tau' [Q_{ab}(\tau - \tau')]^2 - \ln\left[\int \mathcal{D}\phi_a(\tau) \exp\left(-\int d\tau \left[\frac{1}{2g}\left(\frac{\partial \phi_a}{\partial \tau}\right)^2 + V(\phi_a)\right] + J^2 \int d\tau d\tau' Q_{ab}(\tau - \tau')\phi_a(\tau)\phi_b(\tau)\right)\right].$$




Miller, Huse; Read, Sachdev Ye (1993)

# Quantum Ising model

$$Q_{ab}(\tau - \tau') = \frac{1}{N} \sum_i \langle \phi_{ia}(\tau) \cdot \phi_{ib}(\tau') \rangle$$

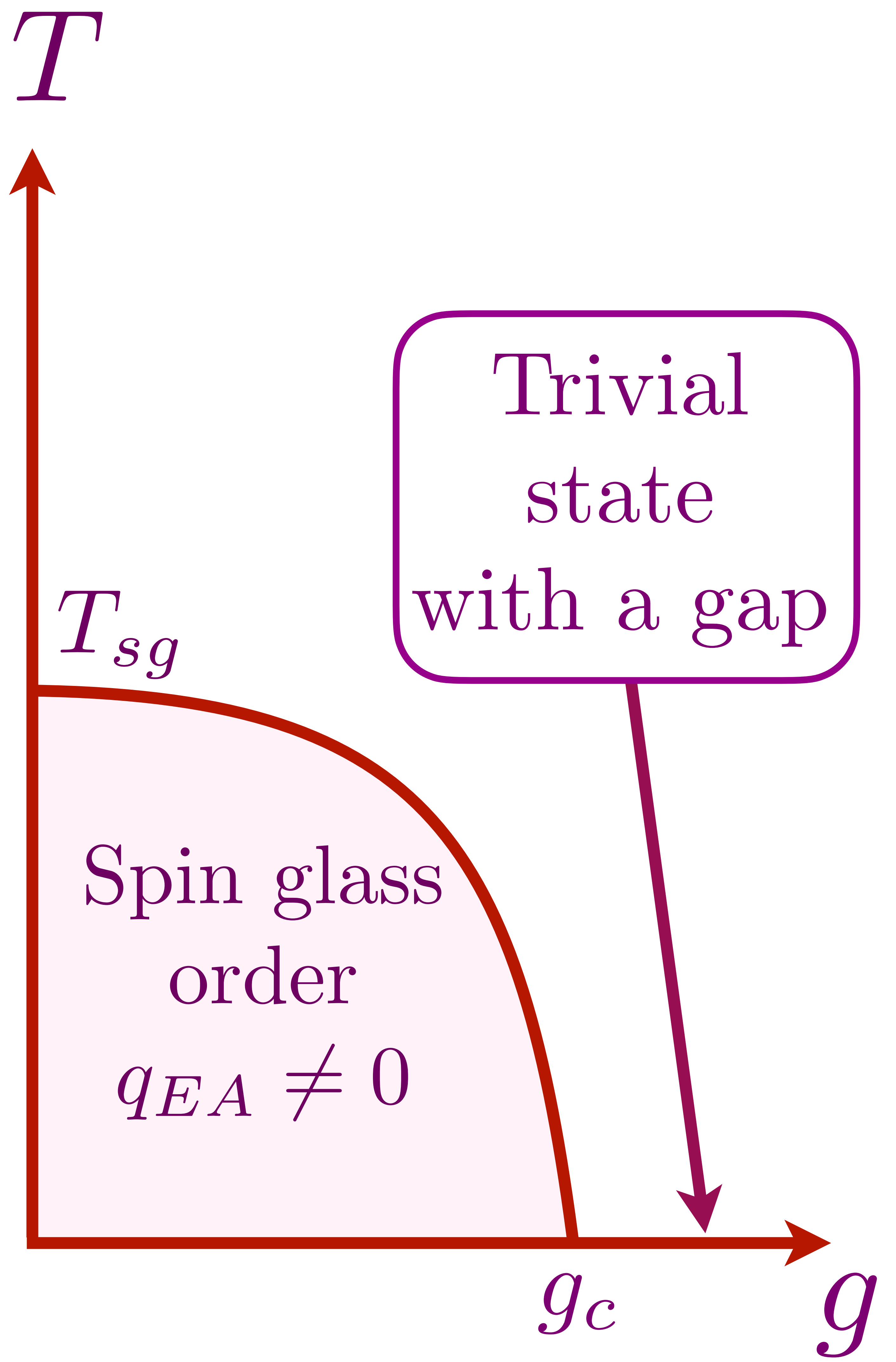
For $a \neq b$, $Q_{ab}(\tau) = q_{ab} = \overline{\langle \phi_i(\tau) \rangle \cdot \langle \phi_i(0) \rangle}$

is $\tau$ independent.

$$q_{EA} = \lim_{n \to 0} \frac{1}{n(n-1)} \sum_{a \neq b} q_{ab}$$

$$q_{EA} = \lim_{\tau \to \infty} \overline{\langle \phi_i(\tau) \cdot \phi_i(0) \rangle} = \lim_{n \to 0} \lim_{\tau \to \infty} \frac{1}{n} \sum_a Q_{aa}(\tau) \,.$$

Replica off-diagonal structure
is very similar to classical model.
Quantum effects are described by $Q_{aa}(\tau)$.

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## Quantum generalization of the Sherrington-Kirkpatrick model to $S = 1/2$ spins with SU(2) symmetry

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$[S_{i\mu}, S_{j\nu}] = i\delta_{ij}\epsilon_{\mu\nu\lambda} S_{i\lambda} \quad , \quad \boldsymbol{S}_i^2 = 3/4$$

$$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad \text{Different } J_{ij} \text{ uncorrelated.}$$

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### Two possible ground states

I. Gapless spin liquid

$$\lim_{\tau \to \infty} \langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle \sim \frac{1}{|\tau|^a}$$

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II. Spin glass order

$$\lim_{\tau \to \infty} \langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle = q_{EA} > 0$$

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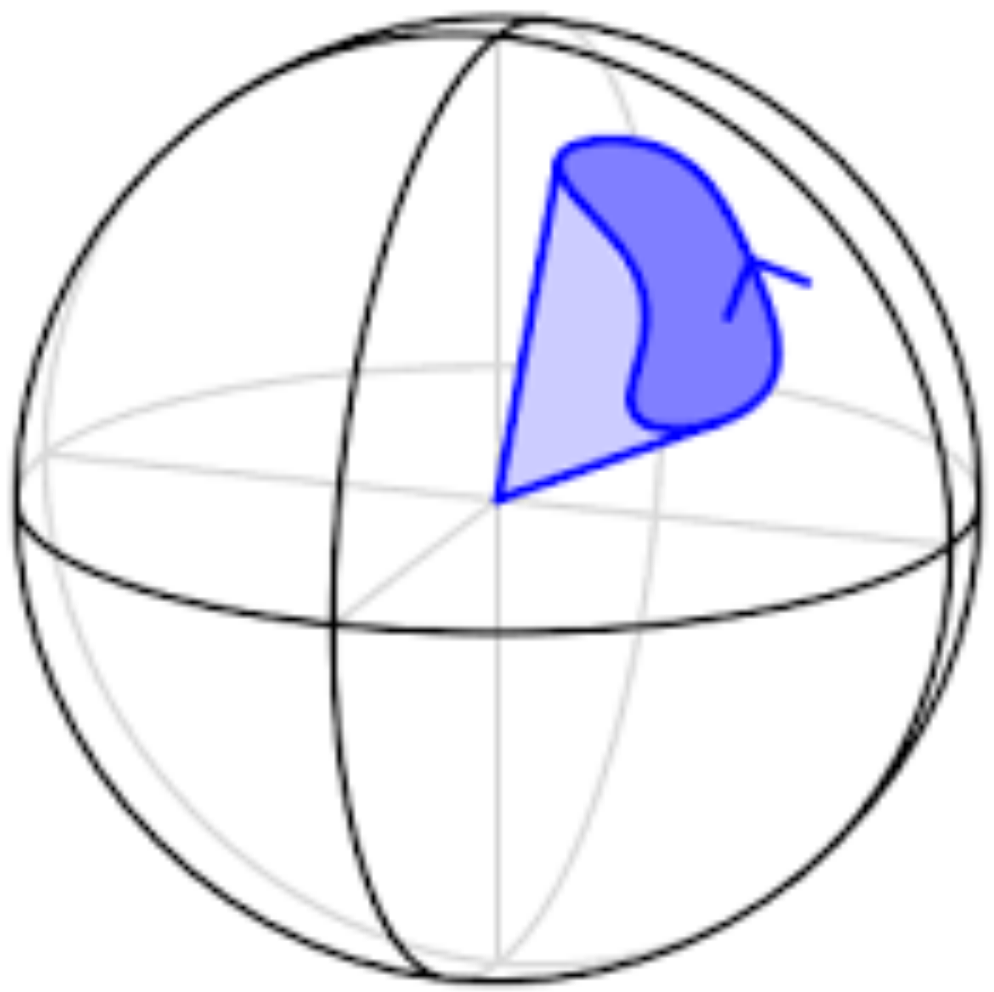
# Action for quantum spin glass order $Q_{ab}(\tau)$

$$\mathcal{S}[Q] = \frac{\beta J^2}{2} \int d\tau \, [Q_{ab}(\tau)]^2 - \ln \mathcal{Z}_f[Q]$$

$$\mathcal{Z}_f[Q] = \int \mathcal{D}\boldsymbol{S}_a(\tau) \delta(\boldsymbol{S}_a^2 - 1) \exp\left[ -\frac{i}{2} \int d\tau \boldsymbol{A}_a(\boldsymbol{S}_a) \cdot \partial_\tau \boldsymbol{S}_a \right.$$

$$\left. - J^2 \int d\tau d\tau' Q_{ab}(\tau - \tau') \boldsymbol{S}_a(\tau) \cdot \boldsymbol{S}_b(\tau') \right]$$

where $\boldsymbol{\nabla}_{\boldsymbol{S}_a} \times \boldsymbol{A}_a(\boldsymbol{S}_a) = \boldsymbol{S}_a$.



## $G$-$\Sigma$-$Q$ theory of SU($M$) spin model

Generalize to SU($M$) spins and introduce fermionic spinons $f_\alpha$, $\alpha = 1, \ldots, M$

$$S_{\alpha\beta} = f_\alpha^\dagger f_\beta - \frac{\delta_{\alpha\beta}}{2}, \quad f_\alpha^\dagger f_\alpha = M/2 \,.$$

The large $N$ equations for any $M$ are

$$\frac{\mathcal{S}[Q]}{N} = \frac{J^2 M}{4} \int d\tau d\tau' [Q_{ab}(\tau - \tau')]^2 - \ln \mathcal{Z}_f[Q]$$

$$\mathcal{Z}_f[Q] = \exp\left(-\frac{J^2}{8} \int d\tau d\tau' \sum_{a,b} Q_{ab}(\tau - \tau')\right) \int \mathcal{D}G_{ab}(\tau,\tau')\mathcal{D}\Sigma_{ab}(\tau,\tau')\mathcal{D}\lambda_a(\tau) \exp\left[-M I[Q]\right]$$

$$I[Q] = -\ln\det\left[-\delta'(\tau-\tau')\delta_{ab} - i\lambda_a(\tau)\delta(\tau-\tau')\delta_{ab} - \Sigma_{ab}(\tau,\tau')\right] - i\frac{1}{2}\int d\tau \lambda_a(\tau)$$

$$+ \int d\tau d\tau' \left[-\Sigma_{ab}(\tau,\tau')G_{ba}(\tau',\tau) + \frac{J^2}{2} Q_{ab}(\tau-\tau')G_{ab}(\tau,\tau')G_{ba}(\tau',\tau)\right] .$$

## SU($M$) spin model

In the limit $M \to \infty$, the saddle point equations for the fermion Green's function, self energy and $Q$ become

$$\Sigma_{ab}(\tau) = J^2 Q_{ab}(\tau) G_{ab}(\tau)$$

$$G_{ab}(i\omega) = \left[ i\omega\delta_{ab} - \Sigma_{ab}(i\omega) \right]^{-1}$$

$$Q_{ab}(\tau) = -G_{ab}(\tau) G_{ba}(-\tau)$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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$$\Sigma(\tau) = J^2 Q_{aa}(\tau) G(\tau)$$
$$G(i\omega) = \left[i\omega - \Sigma(i\omega)\right]^{-1}$$
$$Q_{ab}(\tau) = -G(\tau)G(-\tau)\delta_{ab}$$

It is not possible for a fermion Green's function to have non-zero replica off-diagonal components. Then $Q_{ab}$ must also be replica diagonal, and these equations are precisely those of the SYK model!

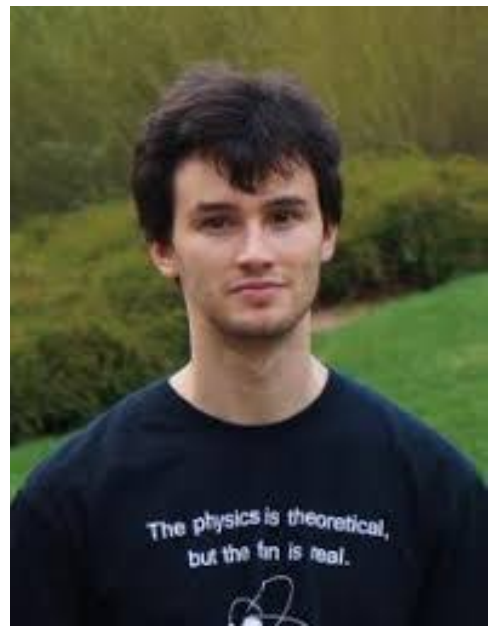
Solution of these equations yield a spin liquid ground state.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

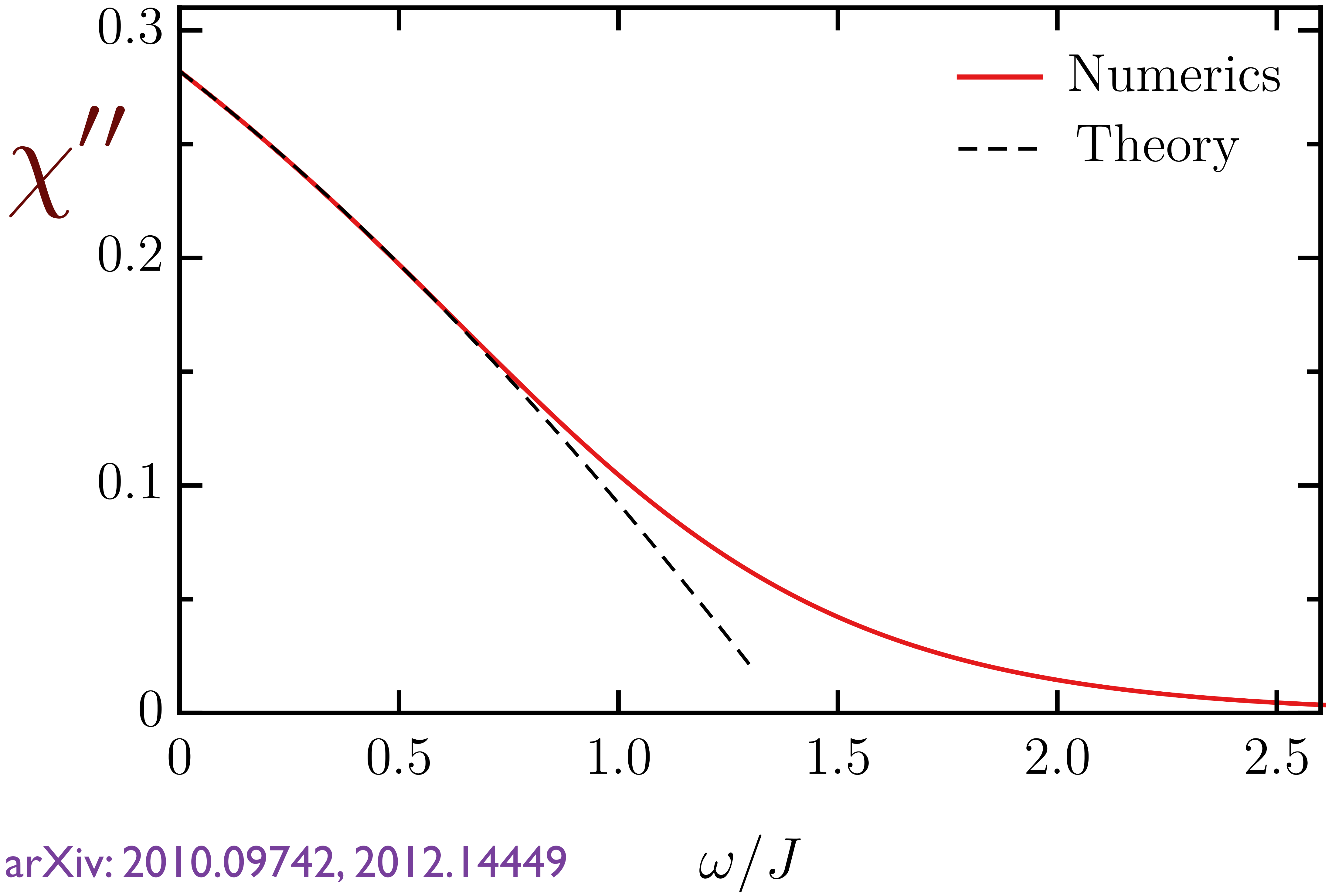
# Dynamic spin susceptibility of the spin liquid at $M = \infty$

$$Q(\tau) = \int_0^\infty \frac{d\omega}{\pi} \chi''(\omega) e^{-\omega\tau}$$

$$\chi''(\omega) \sim \mathrm{sgn}(\omega) \left[ 1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots \right]$$



Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory. $\mathcal{C}$ is a known number, and $\gamma$ is the co-efficient of the action for the 'boundary graviton' in holographic dual.




Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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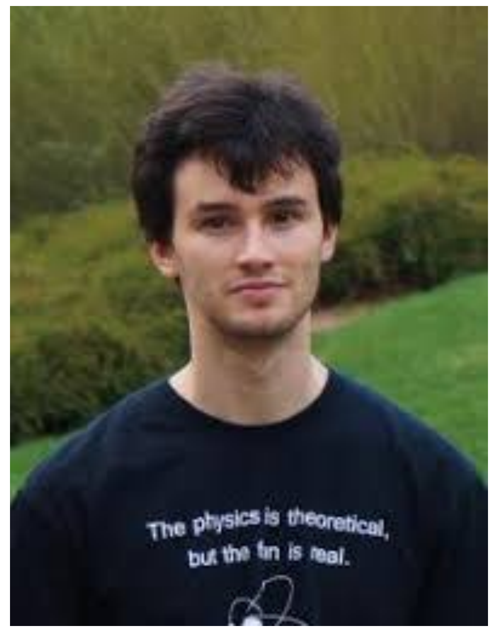

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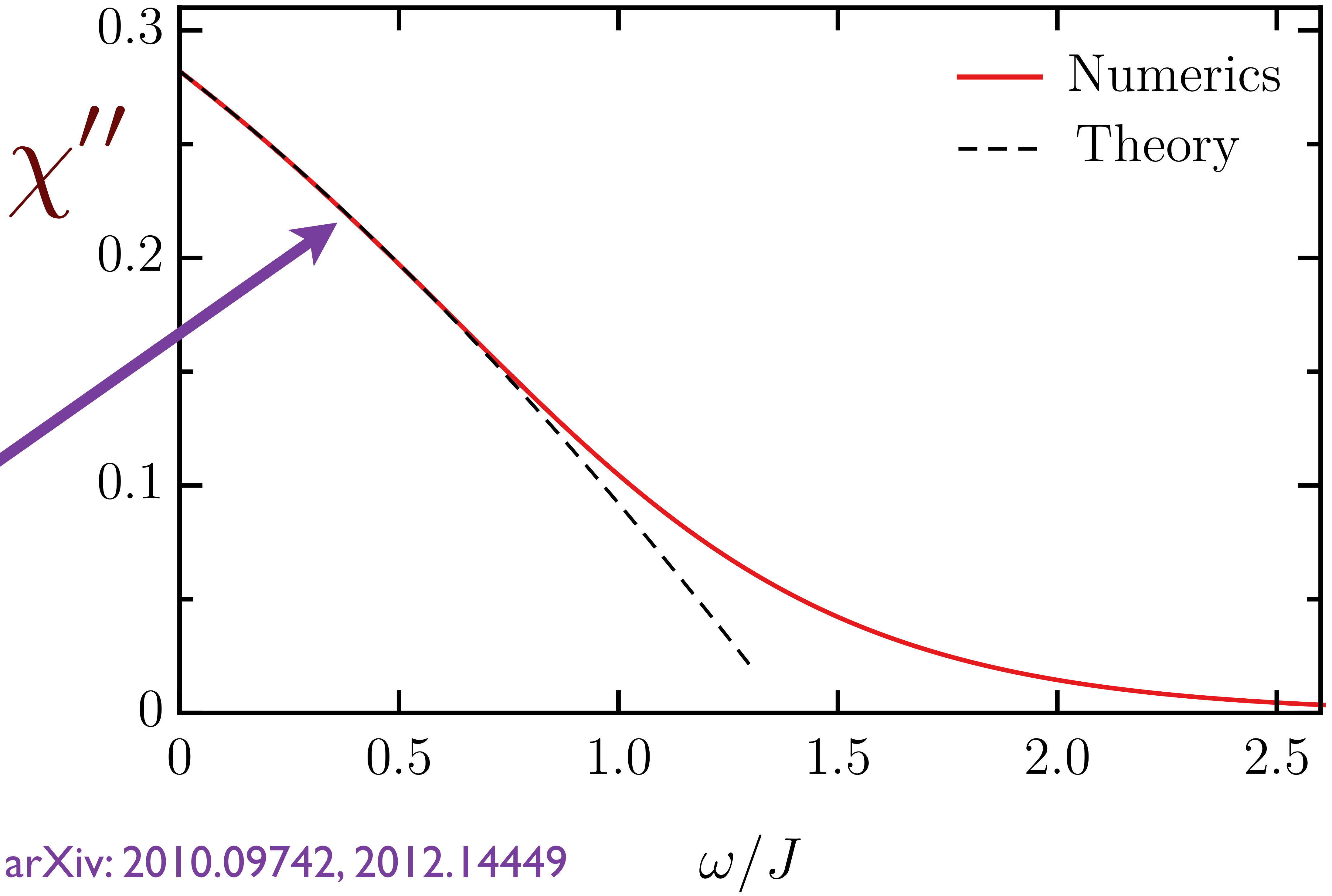
Correction from the boundary graviton

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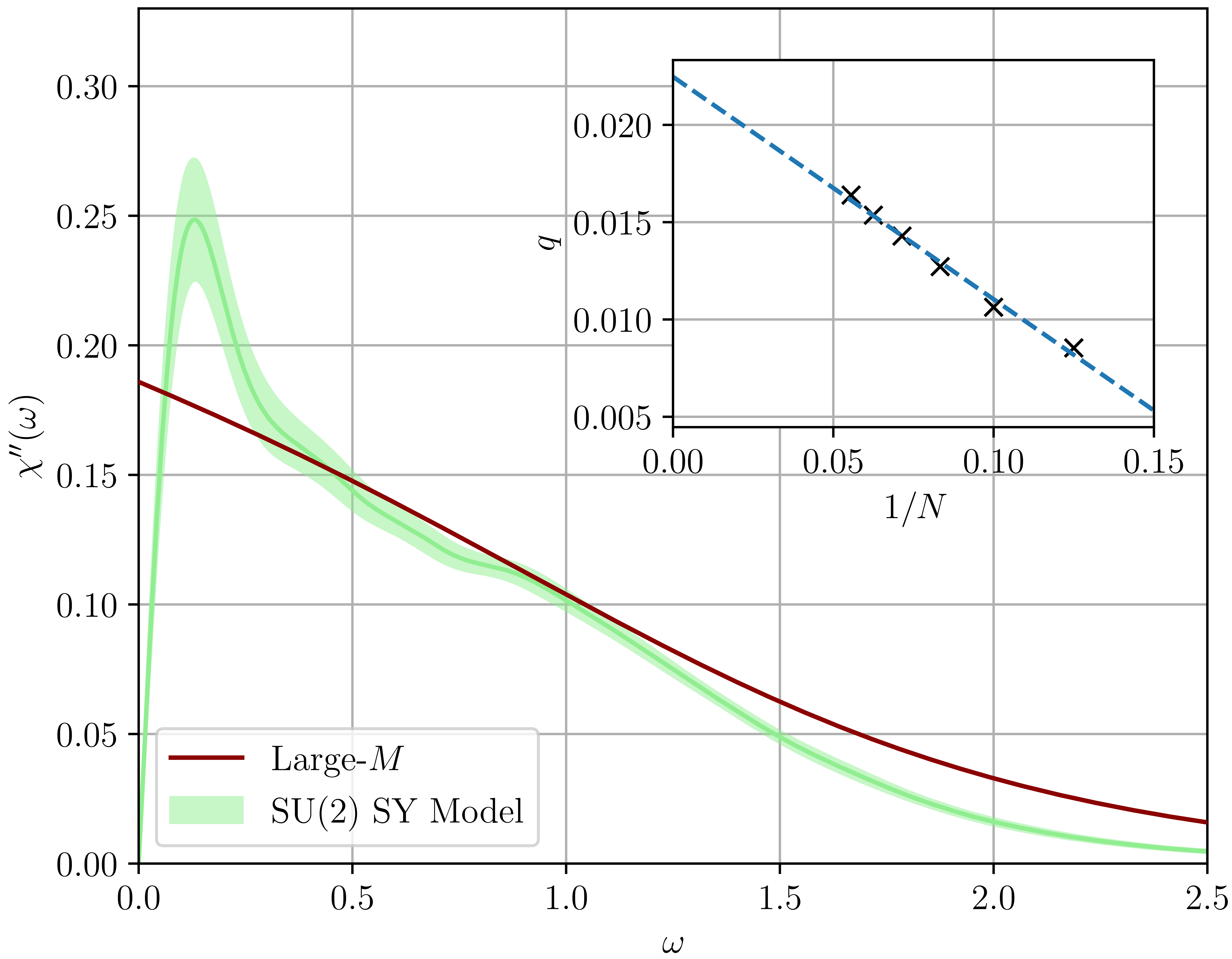
# Exact diagonalization of clusters of SU(2) spins




H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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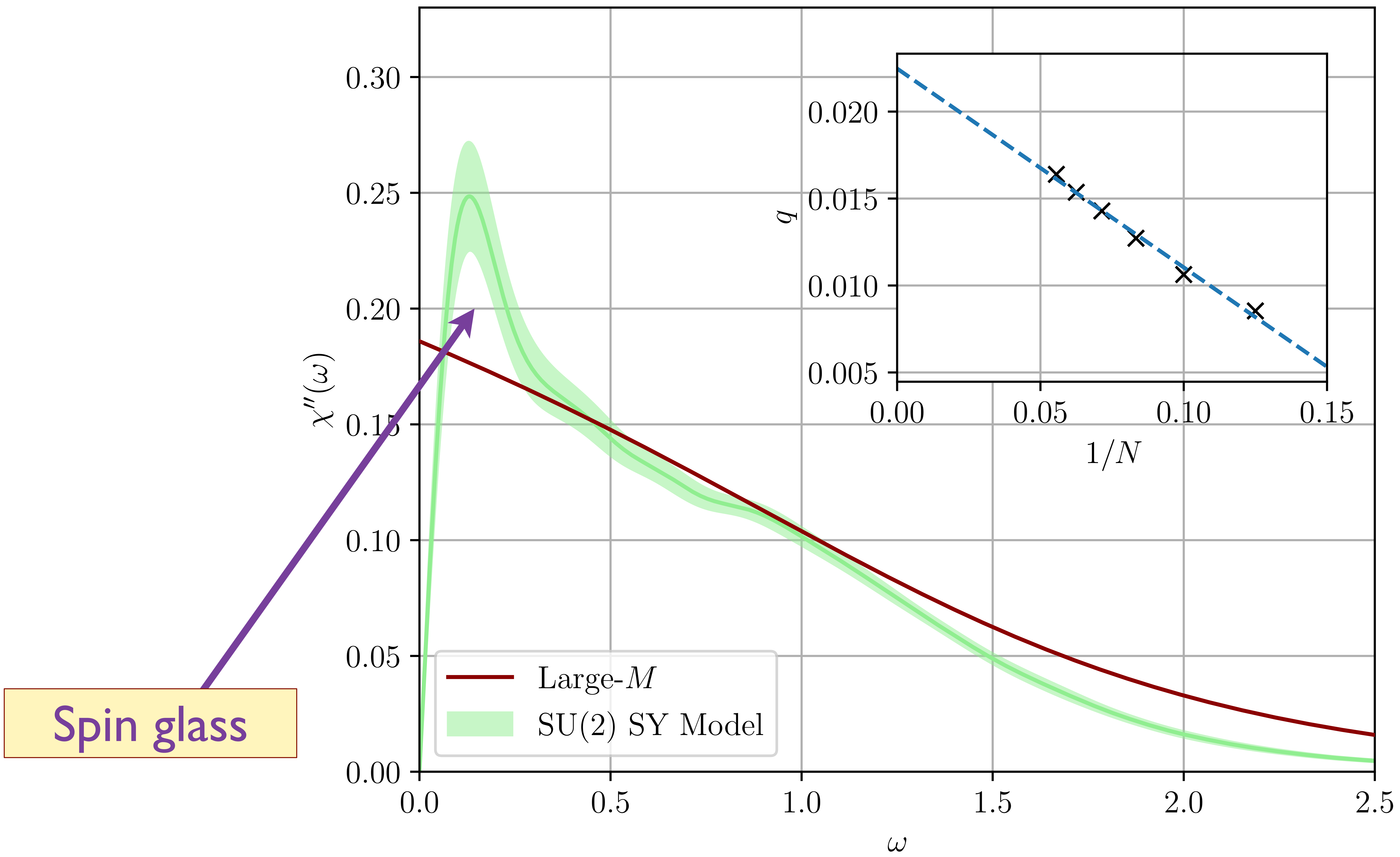


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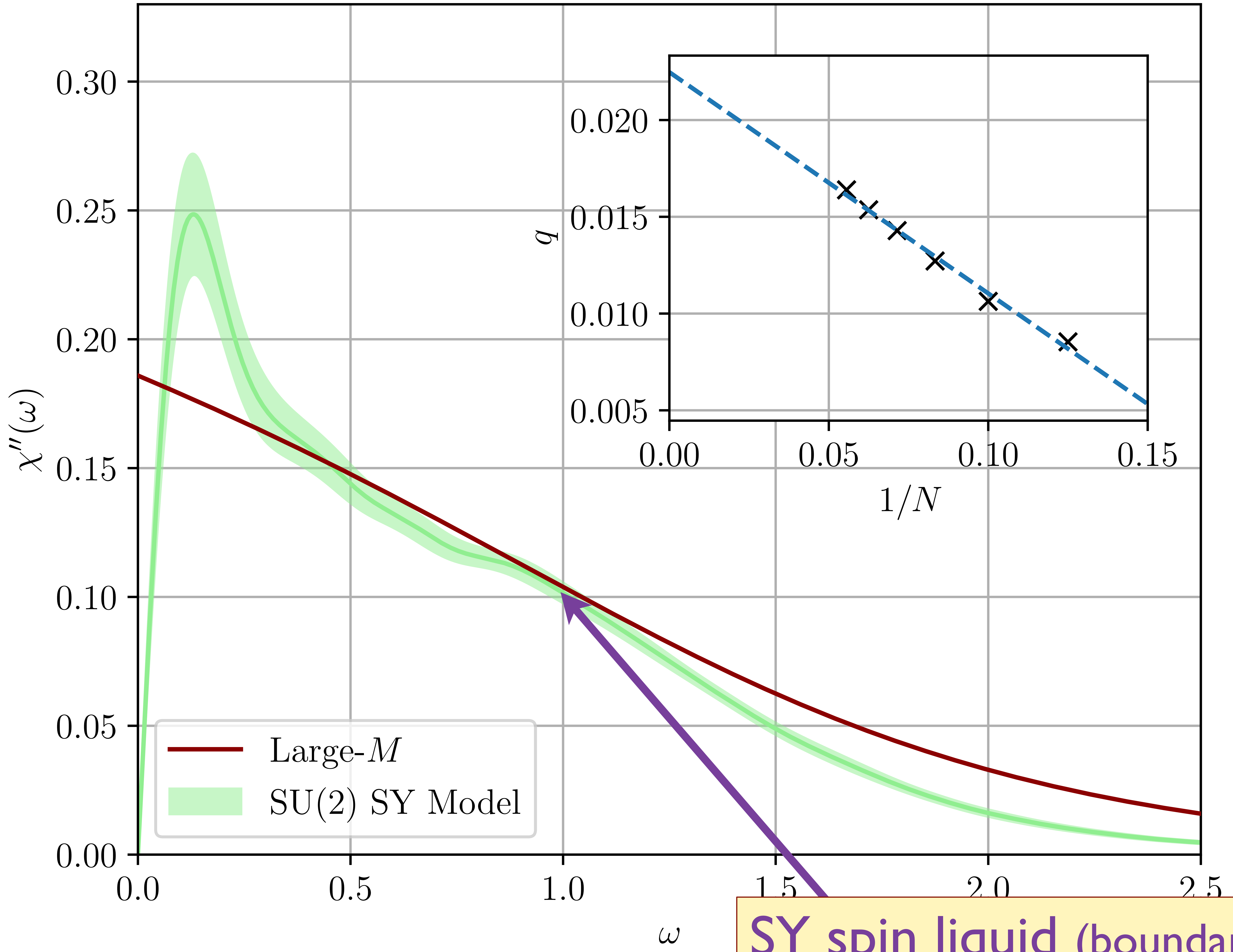


SY spin liquid (boundary graviton)

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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## $G$-$\Sigma$-$Q$ theory of SU($M$) spin model

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$$\mathcal{Z}_f[Q] = \exp\left(-\frac{J^2}{8} \int d\tau d\tau' \sum_{a,b} Q_{ab}(\tau - \tau')\right) \int \mathcal{D}G_{ab}(\tau, \tau')\mathcal{D}\Sigma_{ab}(\tau, \tau')\mathcal{D}\lambda_a(\tau) \exp\left[-M I[Q]\right]$$

$$I[Q] = -\ln\det\left[-\delta'(\tau - \tau')\delta_{ab} - i\lambda_a(\tau)\delta(\tau - \tau')\delta_{ab} - \Sigma_{ab}(\tau, \tau')\right] - i\frac{1}{2}\int d\tau \lambda_a(\tau)$$

$$+ \int d\tau d\tau' \left[-\Sigma_{ab}(\tau, \tau')G_{ba}(\tau', \tau) + \frac{J^2}{2} Q_{ab}(\tau - \tau')G_{ab}(\tau, \tau')G_{ba}(\tau', \tau)\right].$$

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$$+ \int d\tau d\tau' \left[-\Sigma_{ab}(\tau,\tau')G_{ba}(\tau',\tau) + \frac{J^2}{2}Q_{ab}(\tau-\tau')G_{ab}(\tau,\tau')G_{ba}(\tau',\tau)\right].$$

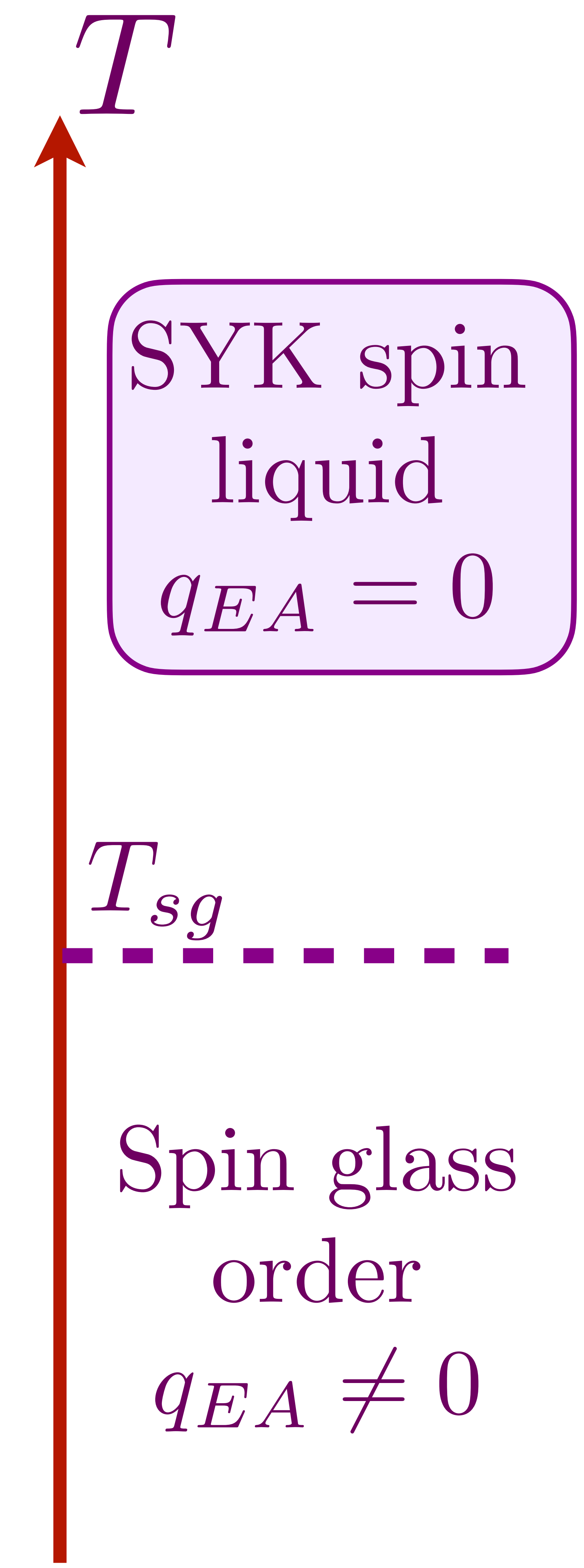
Write $Q_{ab}(\tau) = [Q(\tau) + \overline{q}]\,\delta_{ab} + q_{ab}$ (with $q_{aa} = 0$) and expand action in powers of $q_{ab}$

$$\frac{\mathcal{S}[Q]}{NMn} = \frac{J^2}{4T^2}\left(\overline{q}^2 + \frac{1}{n}\sum_{a\neq b} q_{ab}^2\right)\left[1 - \frac{J^2}{M}\chi_{\rm loc}^2\right] + \mathcal{O}(q_{ab}^4)$$

$$\chi_{\rm loc} = \frac{1}{J\sqrt{\pi}}\ln\left(\frac{J}{T}\right) + \ldots .$$

$$T_{\rm sg} \sim J \exp\left(-\sqrt{M\pi}\right) \quad , \quad e^{-\sqrt{2\pi}} = 0.0815\ldots$$

Georges, Parcollet, S.S. (2001)



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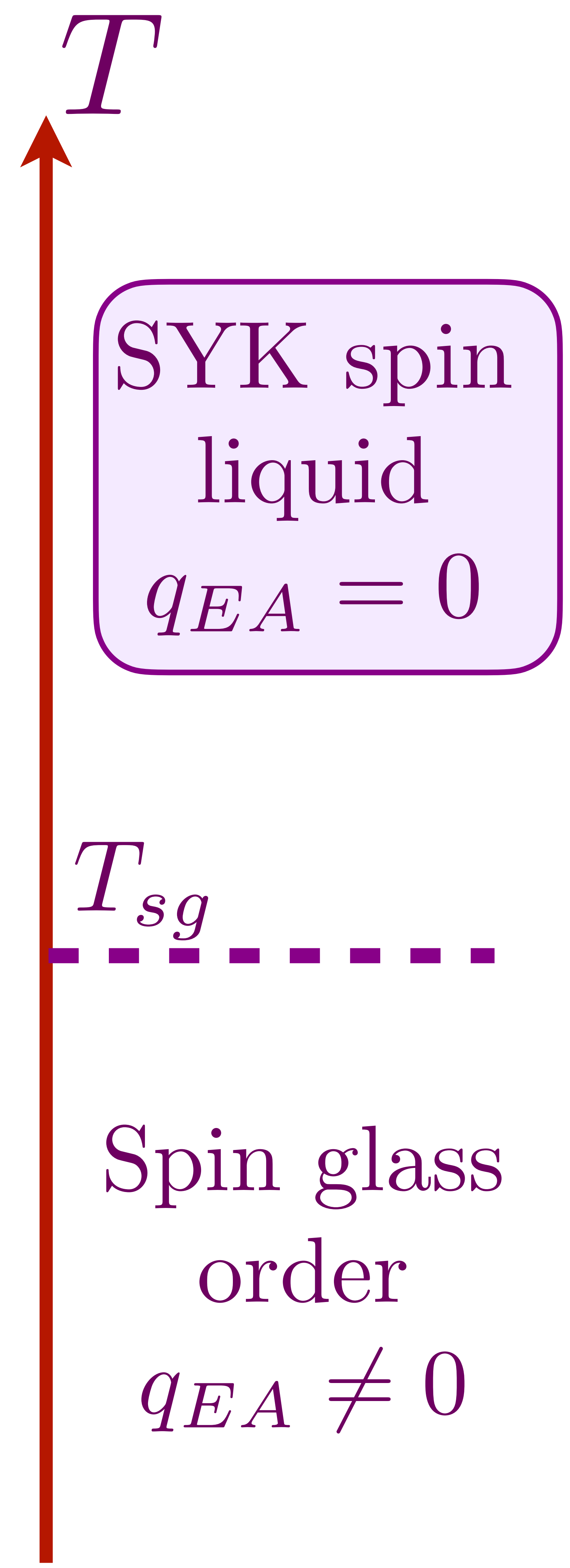


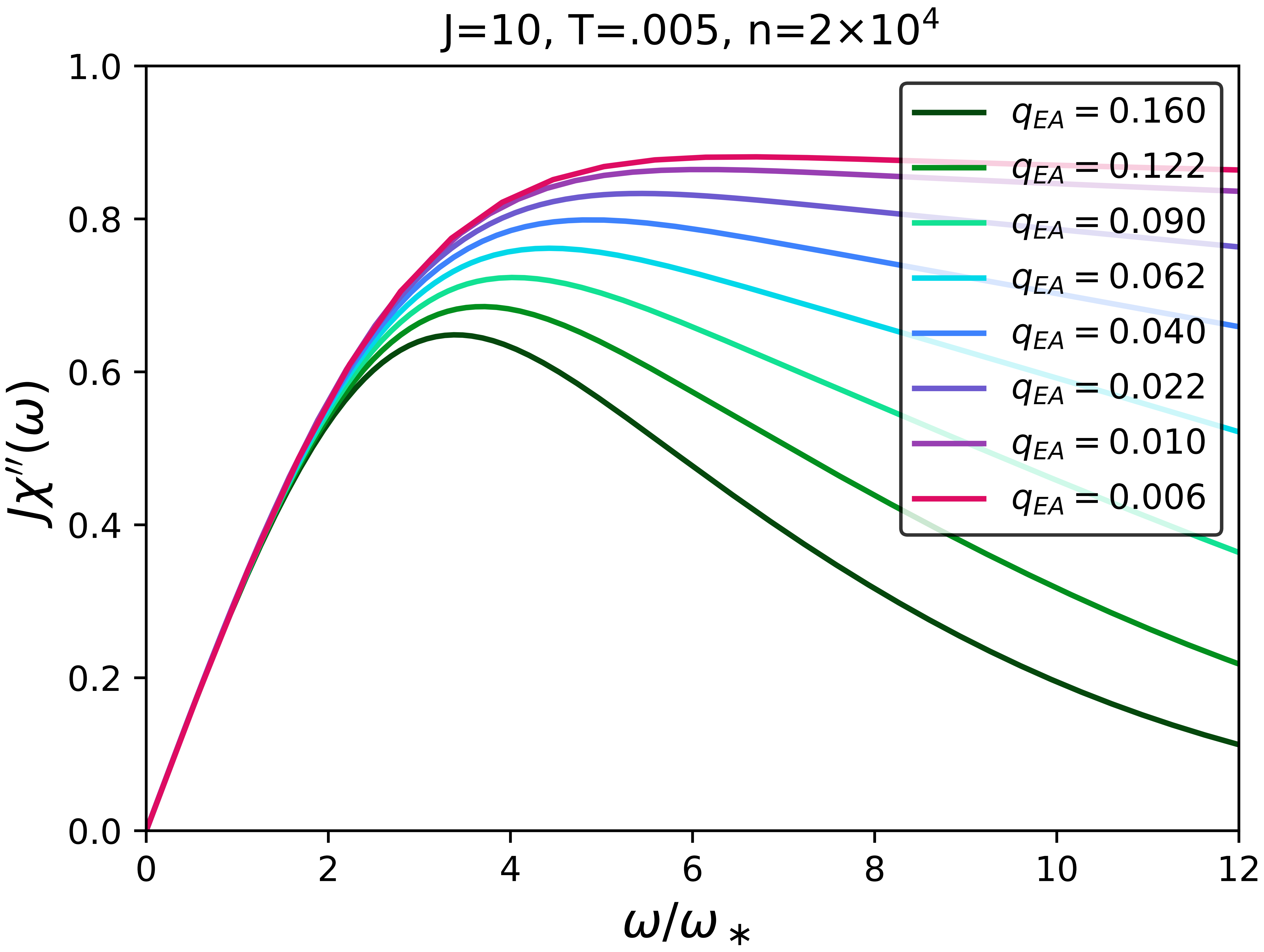

Adding spin glass order to the SU($M \to \infty$) equations:

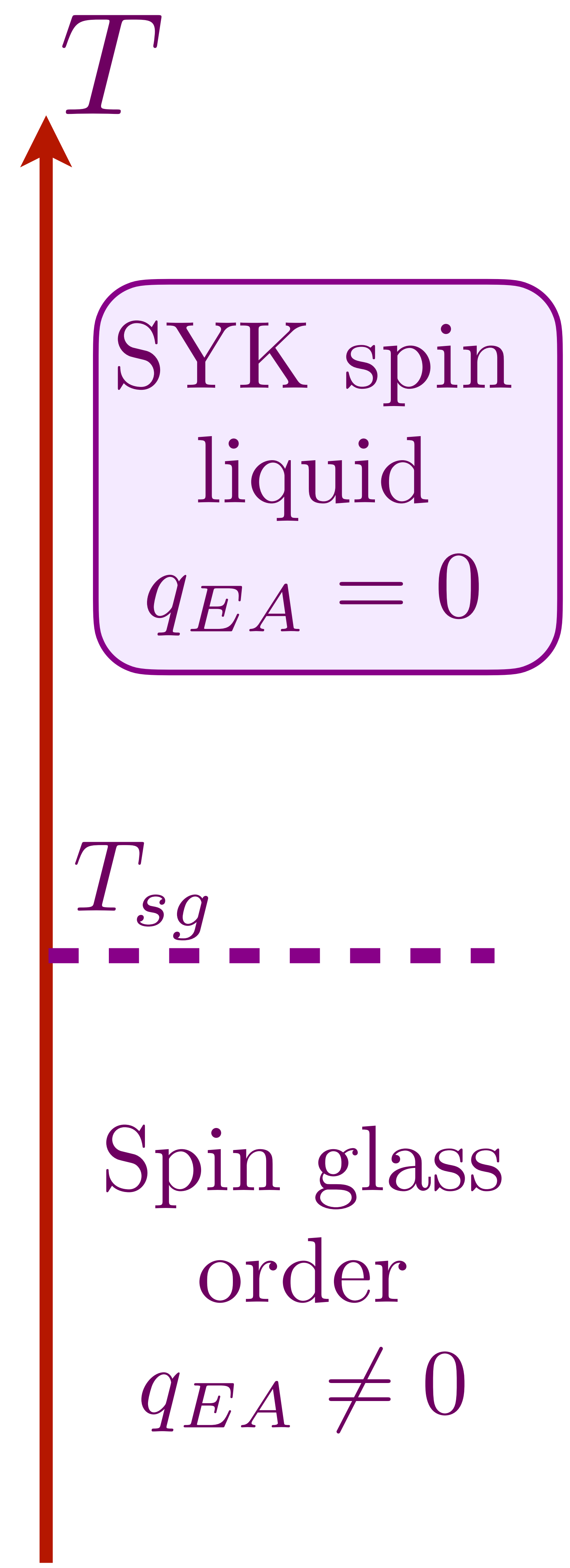
$$\Sigma(\tau) = J^2 Q_{aa}(\tau) G(\tau)$$
$$G(i\omega) = [i\omega - \Sigma(i\omega)]^{-1}$$
$$Q_{ab}(\tau) = -G(\tau)G(-\tau)\delta_{ab} + q_{ab}$$

Need only add the static spin glass order parameter $q_{ab}$, which is determined by the $1/M$ corrections.

$$T_{\mathrm{sg}} \sim J \exp\left(-\sqrt{M\pi}\right) \quad , \quad e^{-\sqrt{2\pi}} = 0.0815\ldots$$

$$\chi''(\omega) = \frac{\pi\omega}{T} q_{EA}\, \delta(\omega) + \frac{1}{J} \Phi_\chi\left(\frac{\omega}{J q_{EA}}\right) + \ldots, \quad T \to 0$$



# Dope the quantum Sherrington-Kirkpatrick model with mobile electrons

$$H = \sum_{i<j} \left[ -t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \text{H.c.} + J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \right]$$

$$\boldsymbol{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$

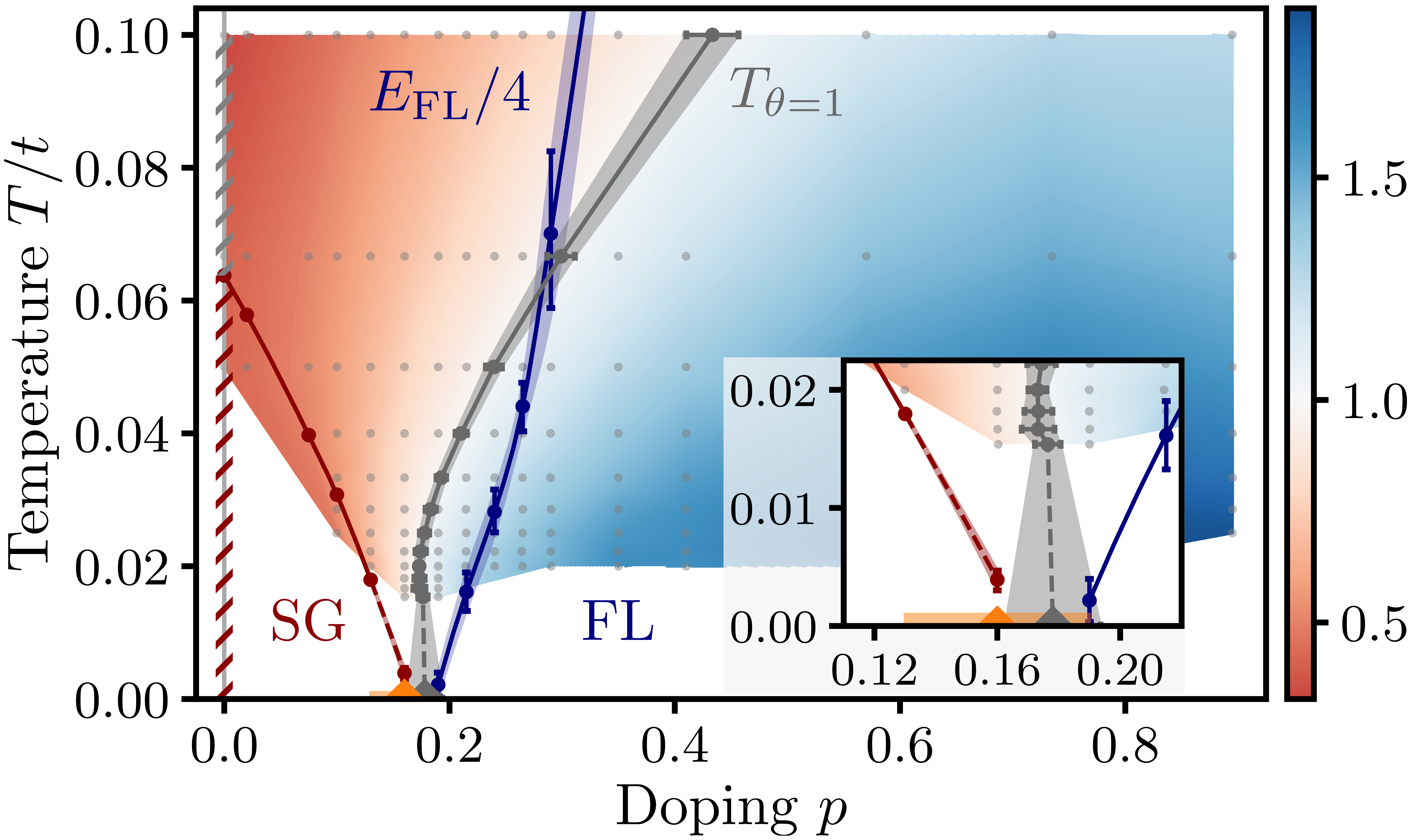
$$[c_{i\alpha}, c_{j\beta}^\dagger]_+ = \delta_{ij}\delta_{\alpha\beta} \quad , \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1$$

$$\frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

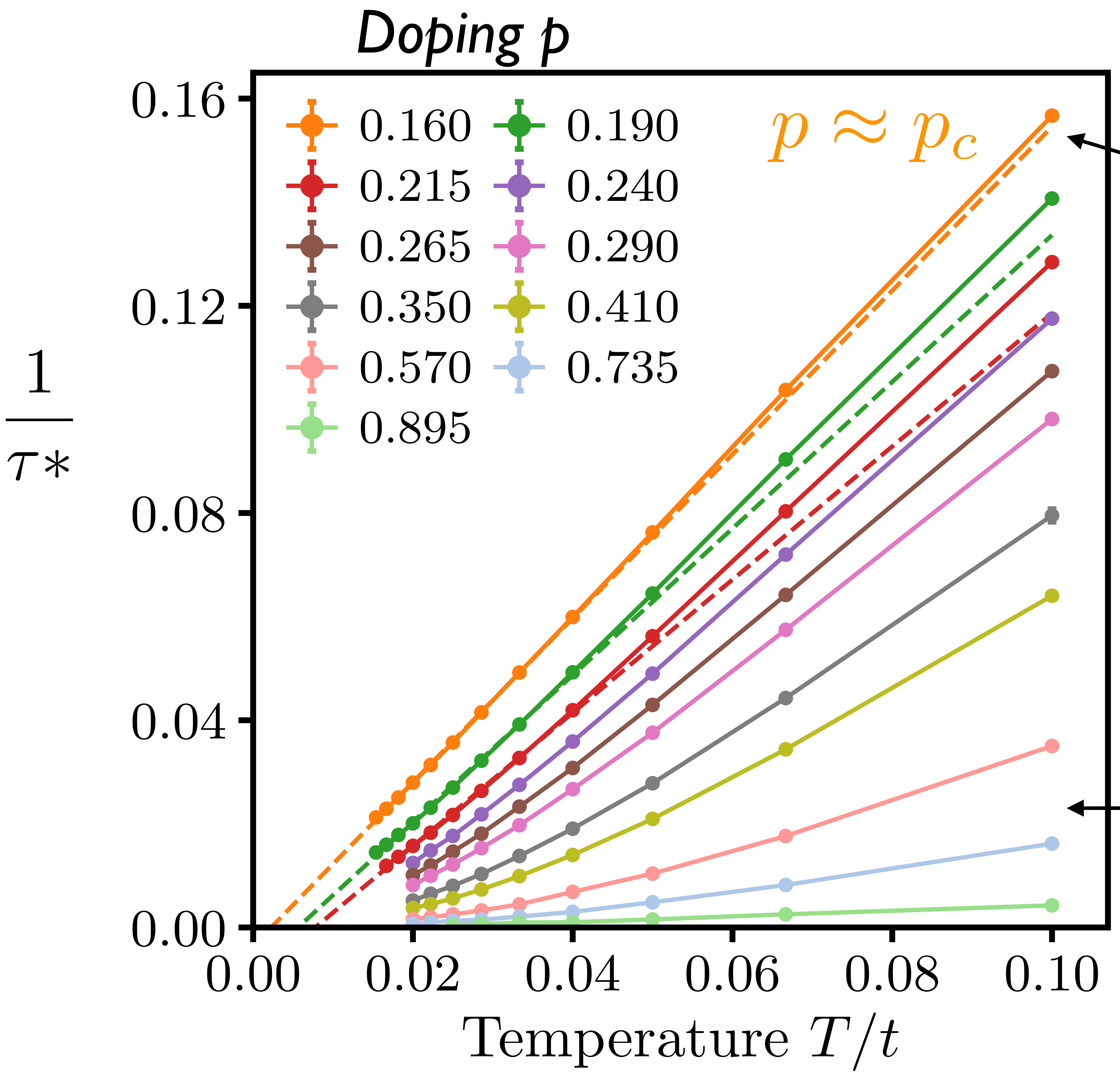
$$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad \text{Different } J_{ij} \text{ uncorrelated.}$$

$$\overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2, \quad \text{Different } t_{ij} \text{ uncorrelated.}$$

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv:2109.05037



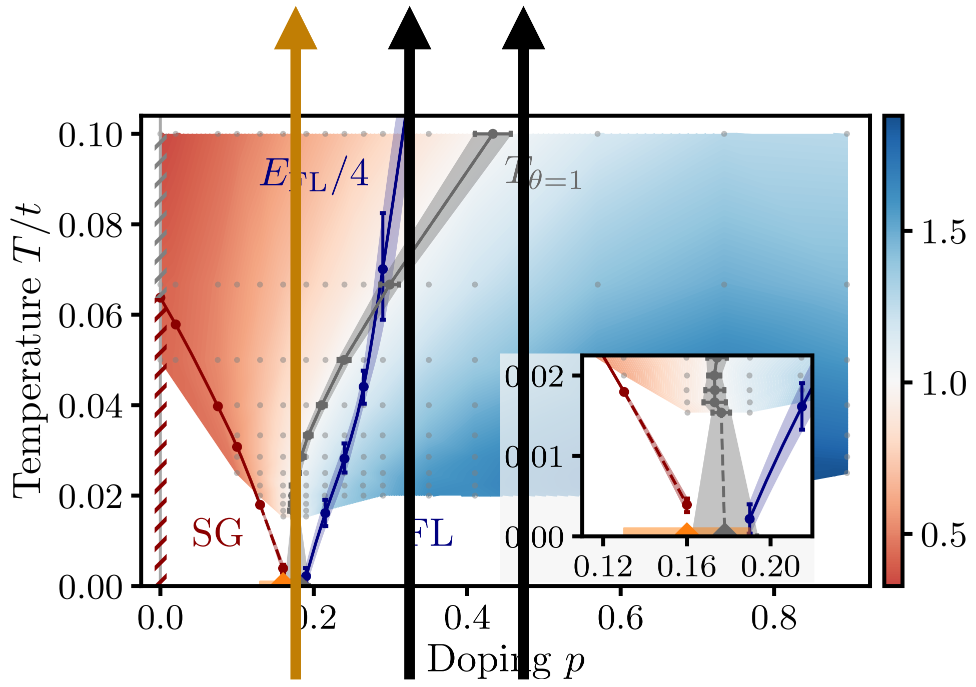

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607;
also H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)




$$\frac{1}{\tau*} \simeq c \, \frac{k_B T}{\hbar}$$

Planckian metal
for $p \approx p_c$

$$\frac{1}{\tau*} \propto T^2$$

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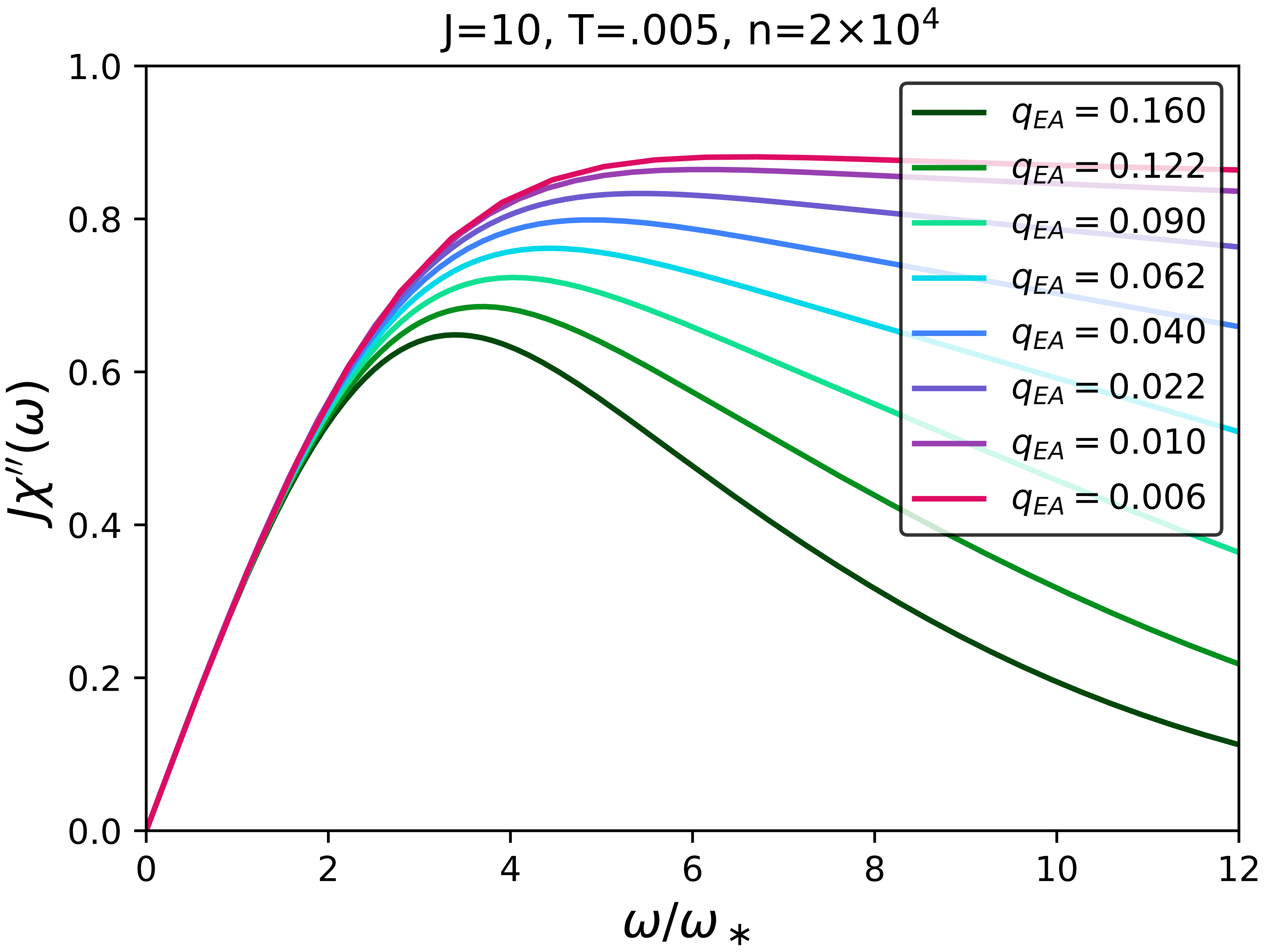
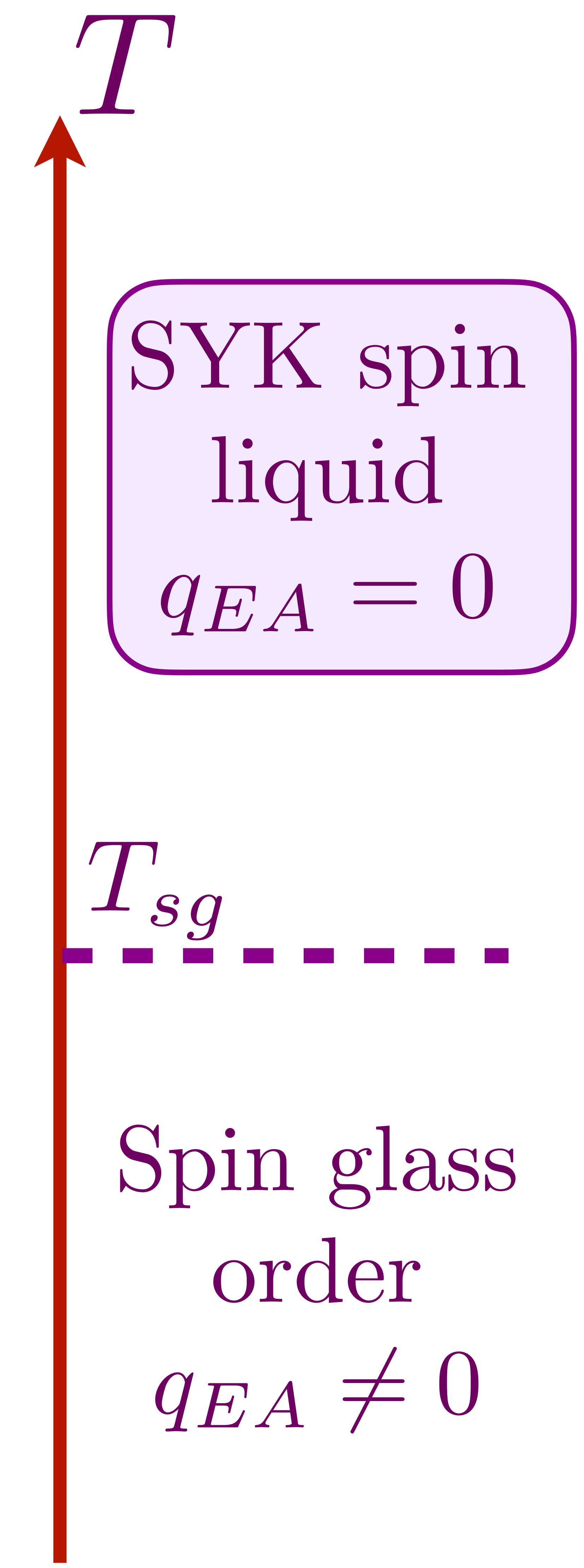
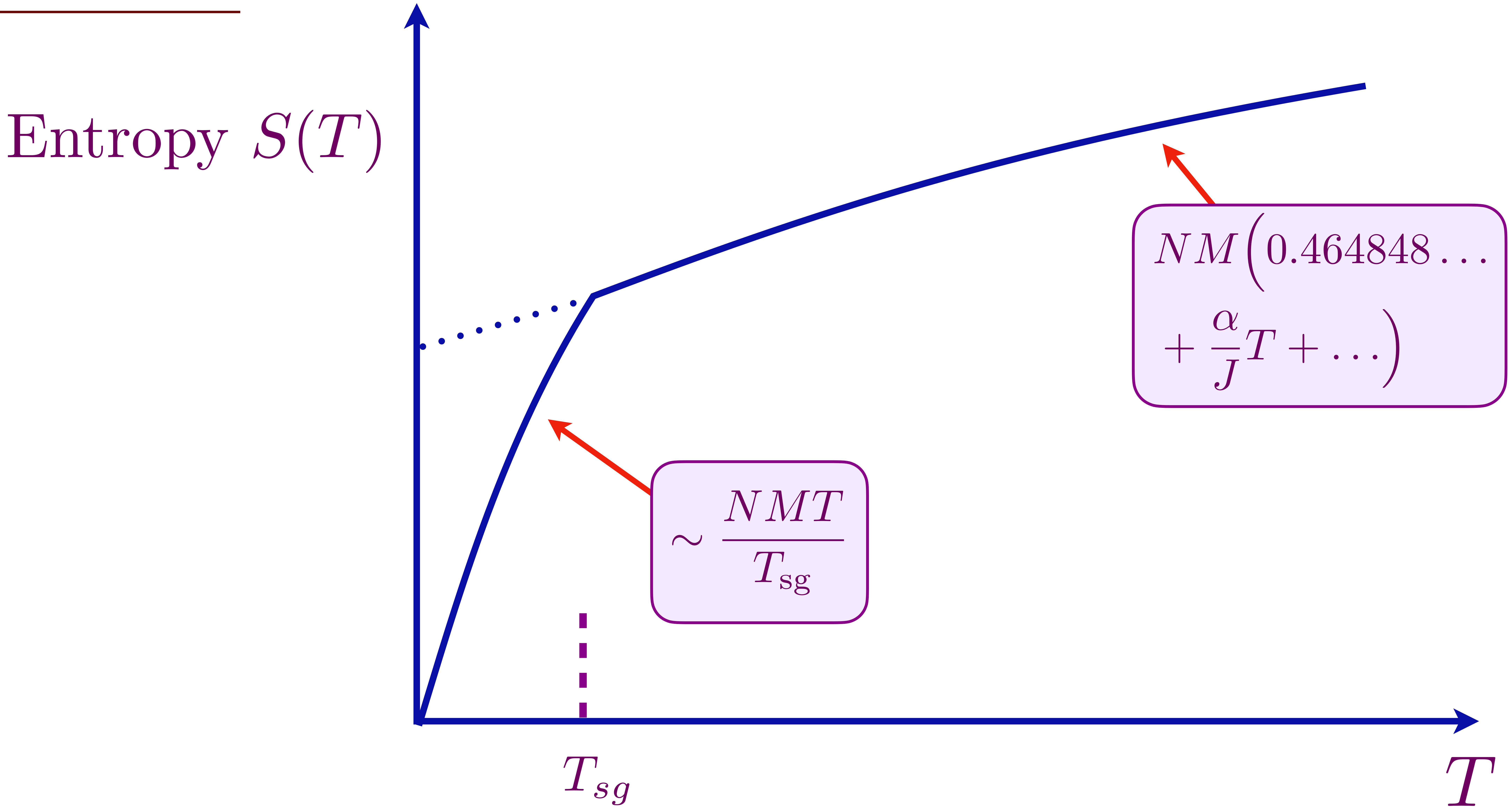
$$T_{\rm sg} \sim J \exp\left(-\sqrt{M\pi}\right) \quad , \quad e^{-\sqrt{2\pi}} = 0.0815\ldots$$

$$\chi''(\omega) = \frac{\pi\omega}{T} q_{EA}\, \delta(\omega) + \frac{1}{J}\Phi_\chi\left(\frac{\omega}{J q_{EA}}\right) + \ldots, \quad T \to 0$$

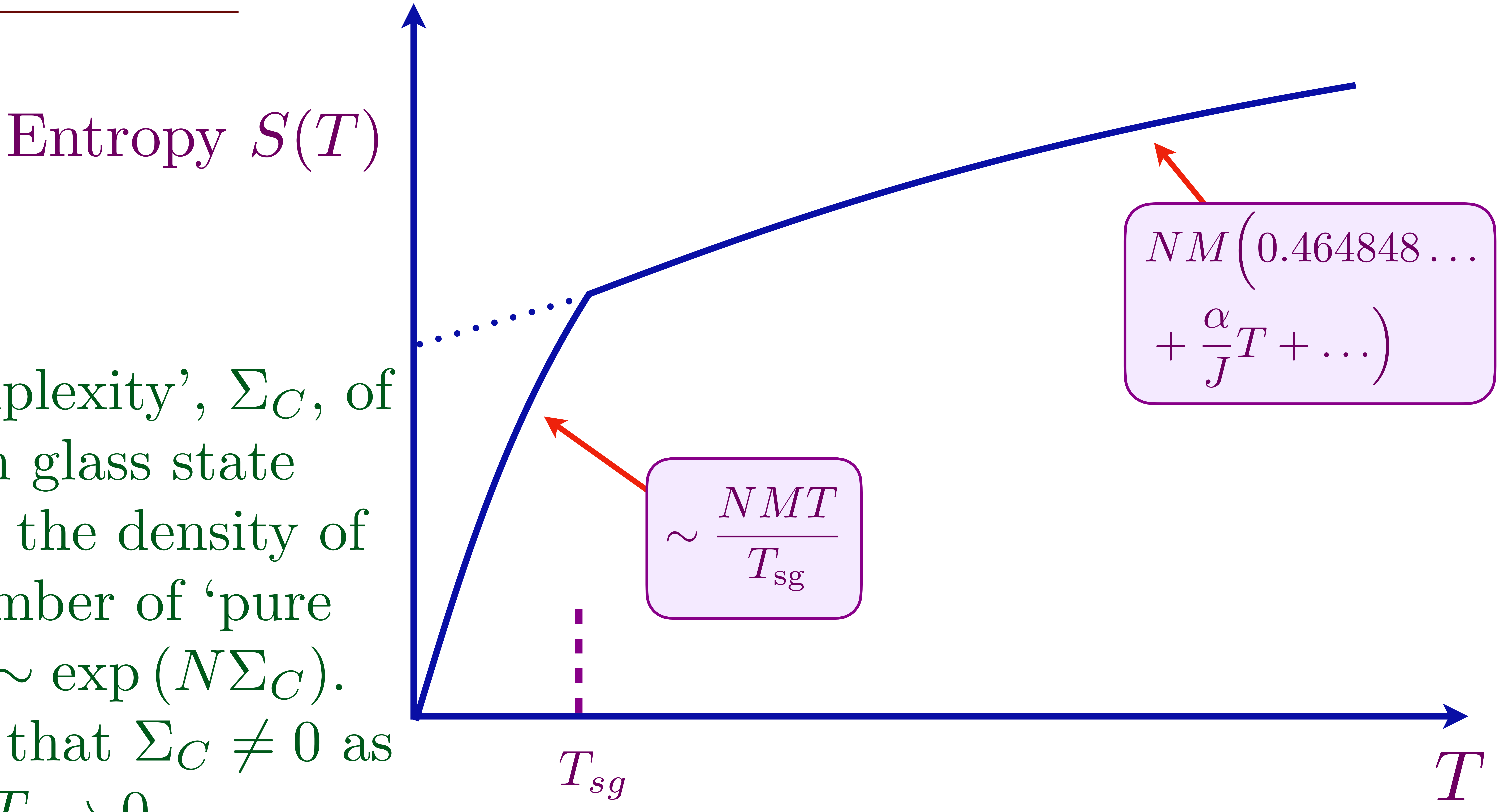


## SU($M$) spin model



## SU($M$) spin model




The ‘complexity’, $\Sigma_C$, of a spin glass state measure the density of the number of ‘pure states’ $\sim \exp(N\Sigma_C)$. We show that $\Sigma_C \neq 0$ as $T \to 0$.

1. Classical and quantum Ising spin glass
2. $S=1/2$ SU(2) spins with random exchange
   A. SYK spin liquid
   B. Numerical results
   C. Spin glass: crossover from fractionalization to confinement
3. Entropy and Complexity
4. Holography: $AdS_2$ fragmentation ?

J. M. Maldacena, J. Michelson, and A. Strominger, “Anti-de Sitter fragmentation,” JHEP **02**, 011 (1999), arXiv:hep-th/9812073 .

D. Anninos, T. Anous, J. Barandes, F. Denef, and B. Gaasbeek, “Hot Halos and Galactic Glasses,” JHEP **01**, 003 (2012), arXiv:1108.5821 [hep-th] .

D. Anninos, T. Anous, F. Denef, and L. Peeters, “Holographic Vitrification,” JHEP **04**, 027 (2015), arXiv:1309.0146 [hep-th] .

D. Anninos, T. Anous, and F. Denef, “Disordered Quivers and Cold Horizons,” JHEP **12**, 071 (2016), arXiv:1603.00453 [hep-th] .

T. Anous and F. M. Haehl, “The quantum $p$-spin glass model: A user manual for holographers,” (2021), arXiv:2106.03838 [hep-th] .

arXiv.org > hep-th > arXiv:hep-th/9812073v2

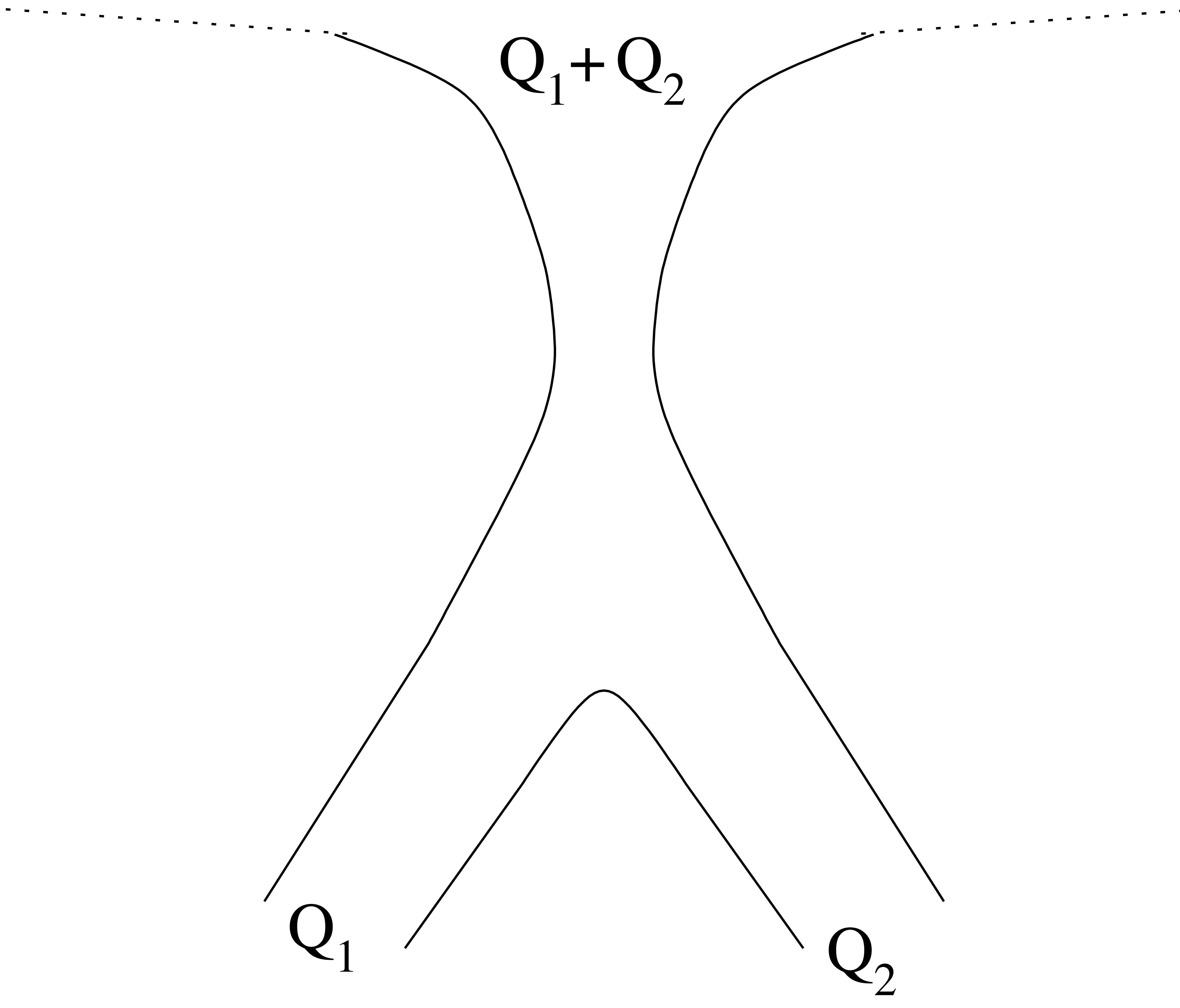
High Energy Physics – Theory

[Submitted on 8 Dec 1998 (v1), last revised 9 Dec 1998 (this version, v2)]

# Anti-de Sitter Fragmentation

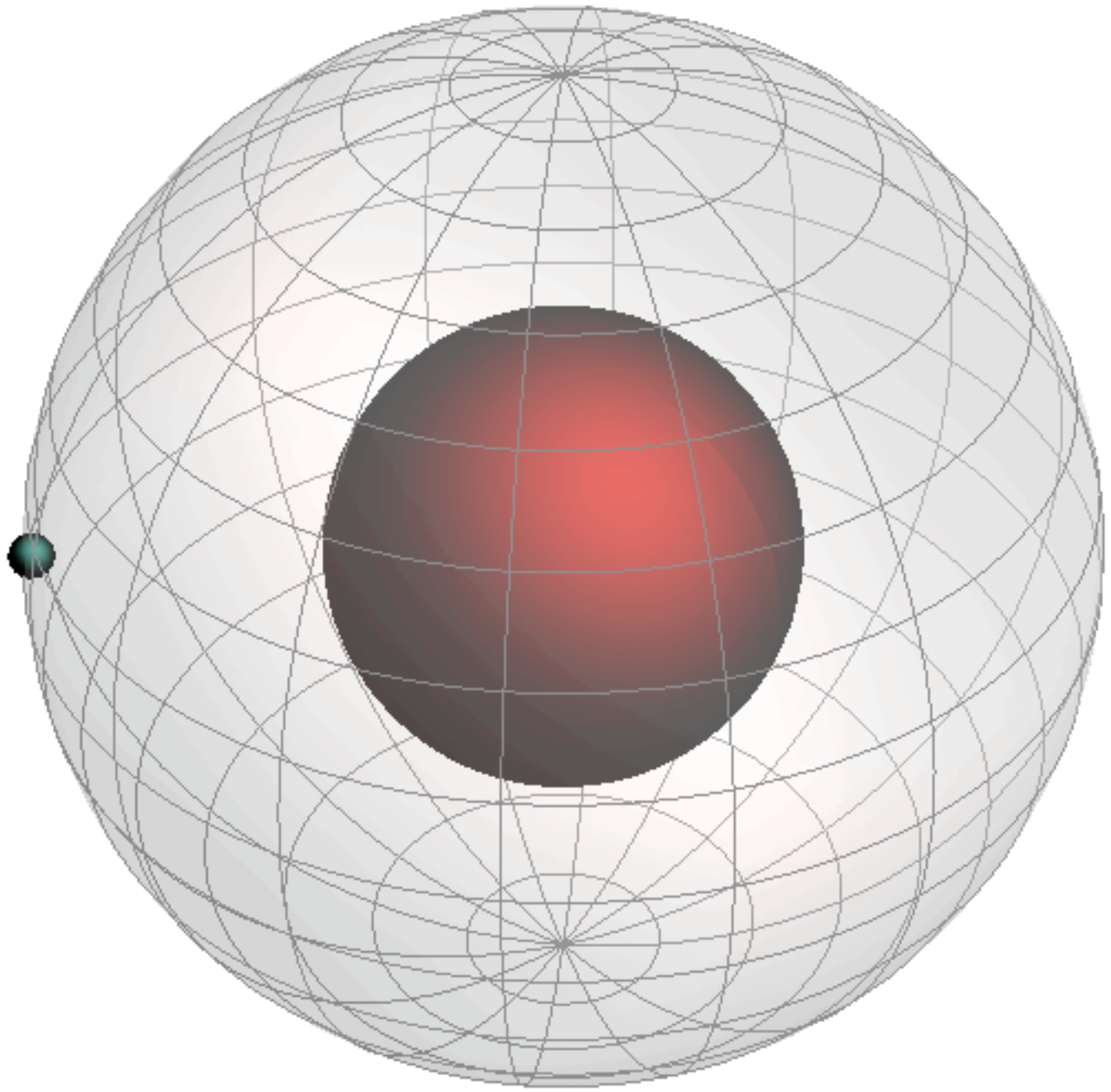
Juan Maldacena, Jeremy Michelson, Andrew Strominger

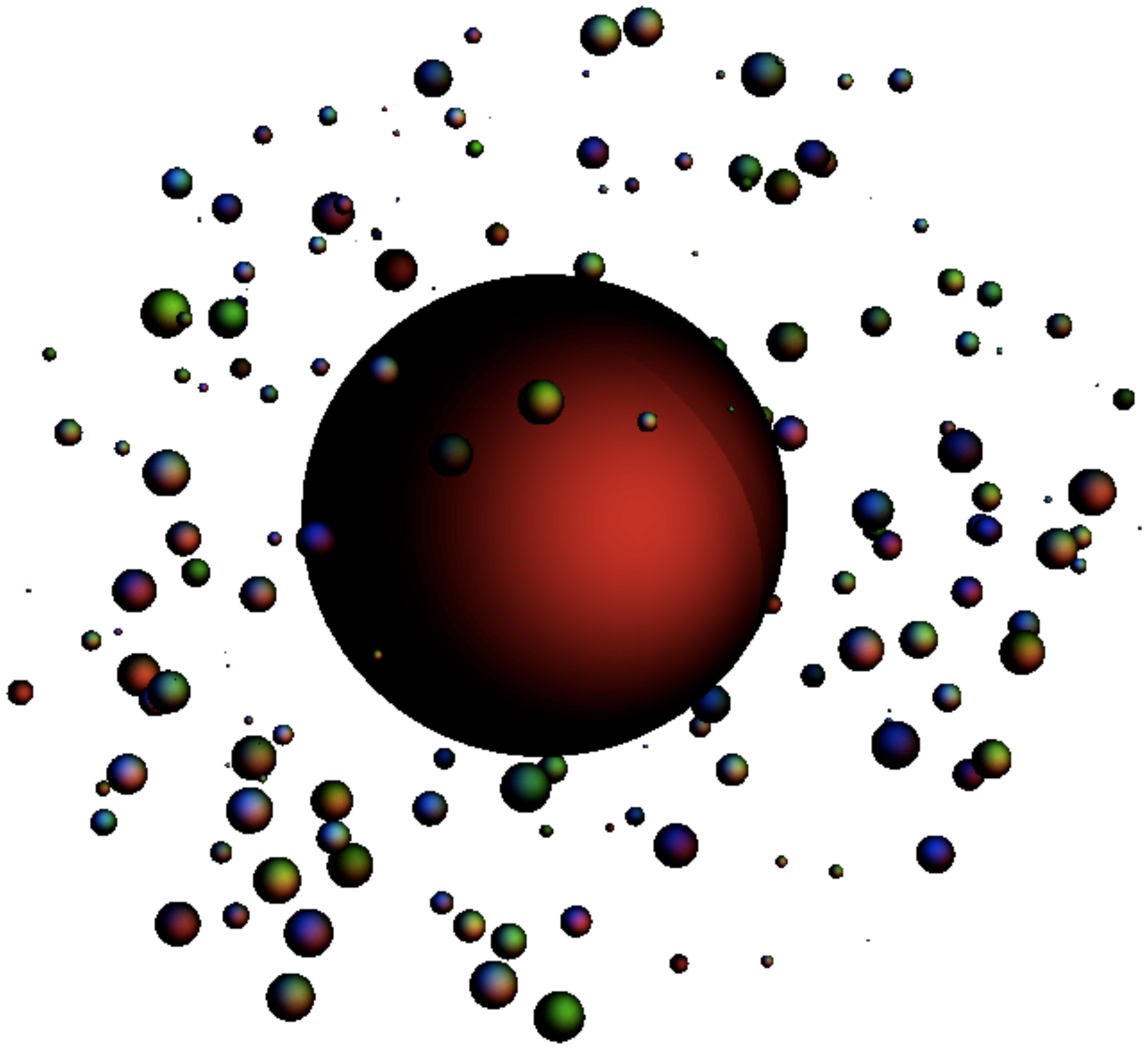
Low-energy, near-horizon scaling limits of black holes which lead to string theory on $AdS_2 \times S_2$ are described. Unlike the higher-dimensional cases, in the simplest approach all finite-energy excitations of $AdS_2 \times S_2$ are suppressed. Surviving zero-energy configurations are described. These can include tree-like structures in which the $AdS_2 \times S_2$ throat branches as the horizon is approached, as well as disconnected $AdS_2 \times S_2$ universes.




There is an asymptotically Minkowskian region and a single charge $Q_1 + Q_2$ throat region which divides into two throats of charges $Q_1$ and $Q_2$.

D. Anninos, T. Anous, J. Barandes, F. Denef, and B. Gaasbeek, "Hot Halos and Galactic Glasses," JHEP **01**, 003 (2012), arXiv:1108.5821 [hep-th] .





We show that an exponential multitude of classically stable "halo" bound states can be formed in four-dimensional $N = 2$ supergravity between large finite temperature D4-D0 black hole cores and much smaller, arbitrarily charged black holes at the same temperature. Several features of these systems, such as a macroscopic configurational entropy and exponential relaxation timescales, are similar to those of the extended family of glasses.

# Summary

- Theory of finite density gauge-charged matter: crossover from fractionalized SYK spin liquid to a confining spin glass state.

- Holography: SYK spin liquid dual to black holes with $AdS_2$ horizons

- Confining spin glass: dual to $AdS_2$ fragmentation (Maldacena, Strominger, Anous, Denef, Haehl...)