Wormholes in Quantum Ergodicity

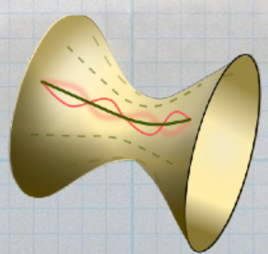
Altland, Bagrets, PN, Sonner, Vielma
arXiv: 2105.12129

Belin, de Boer, PN, Sonner
arXiv: 2012.07875

Belin, de Boer, PN, Sonner
arXiv: 2111.xxxx

& some work in progress with J. Sonner and R. Baumgartner

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research



Ascona conference

UNIVERSITÉ DE GENÈVE

Good Old Dream of Holography

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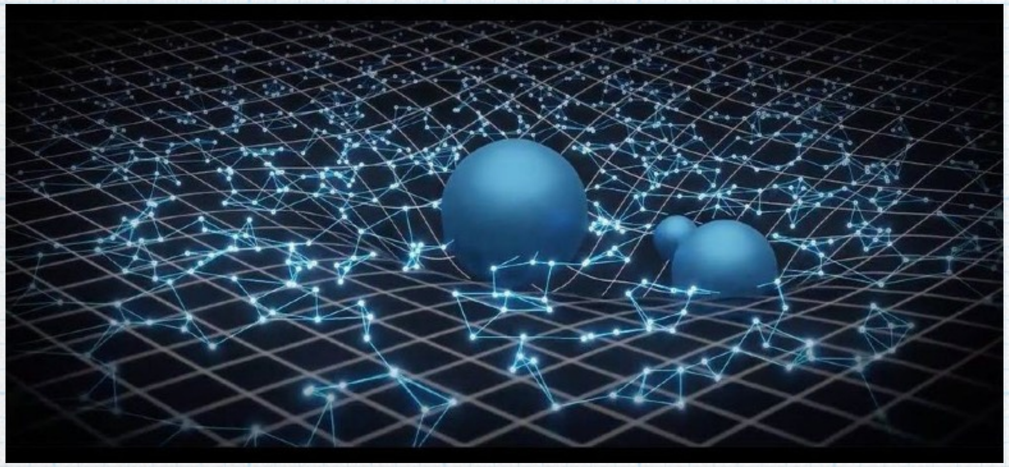

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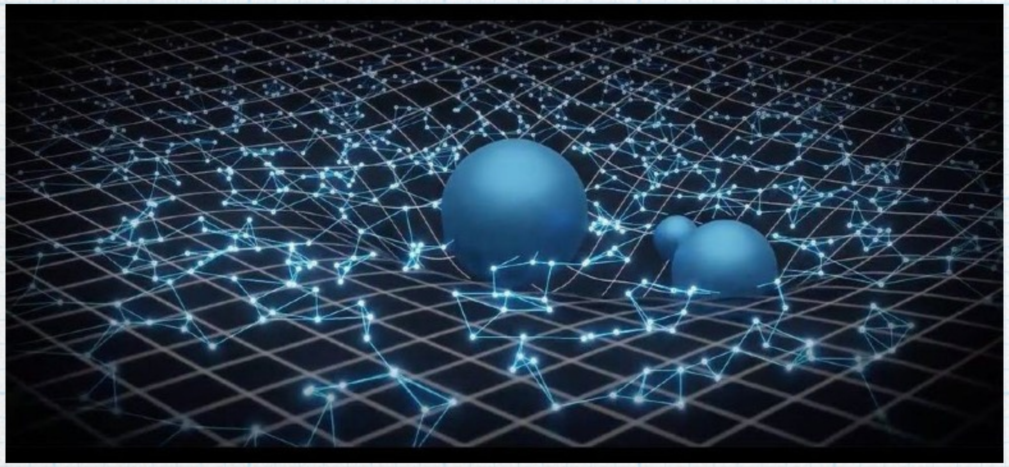

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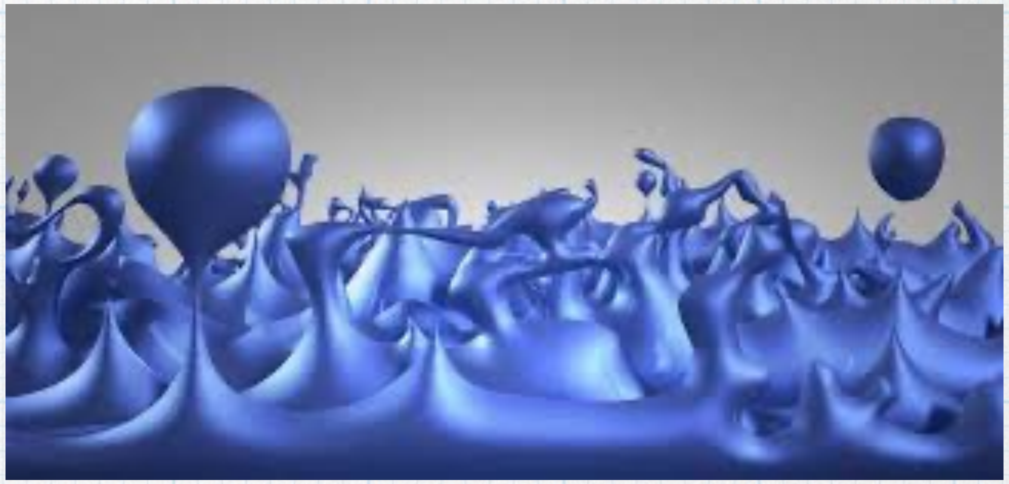

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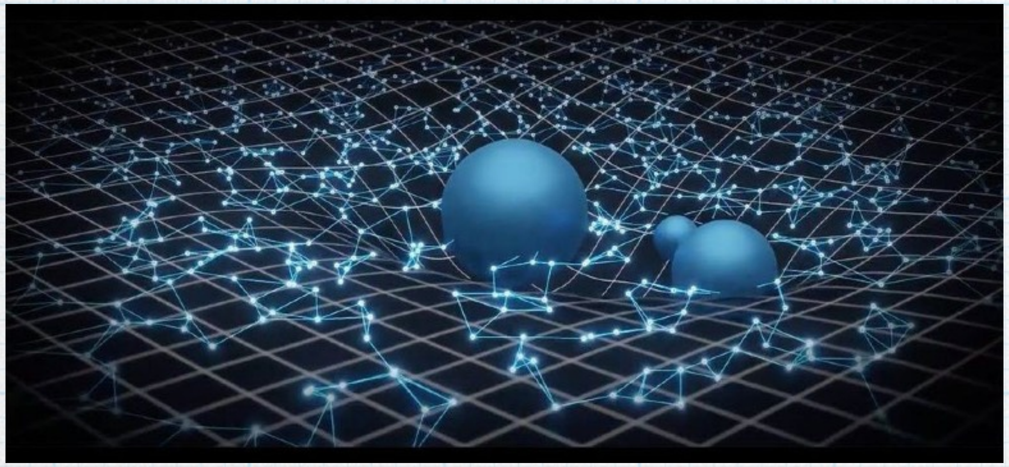

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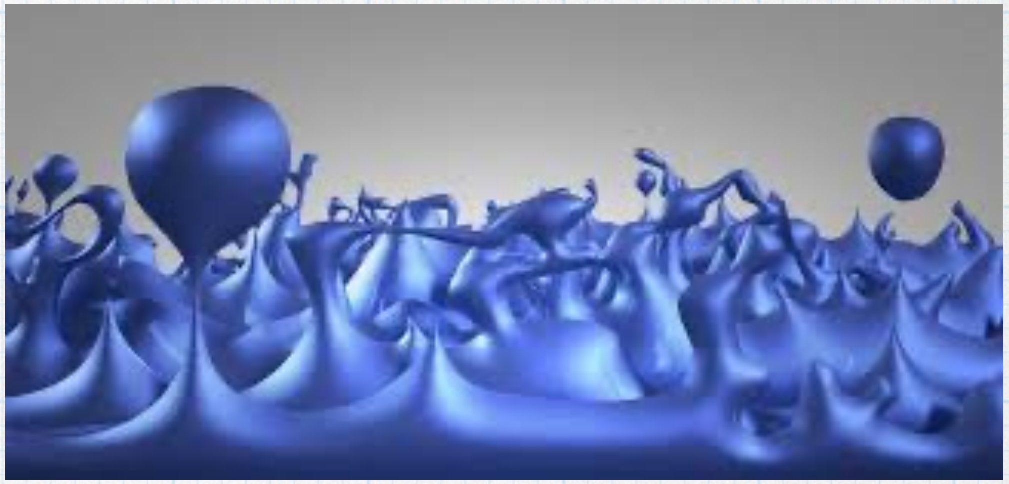
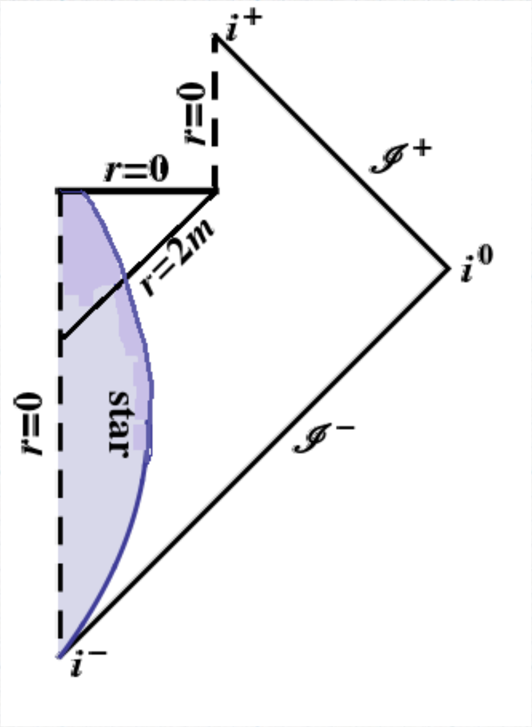
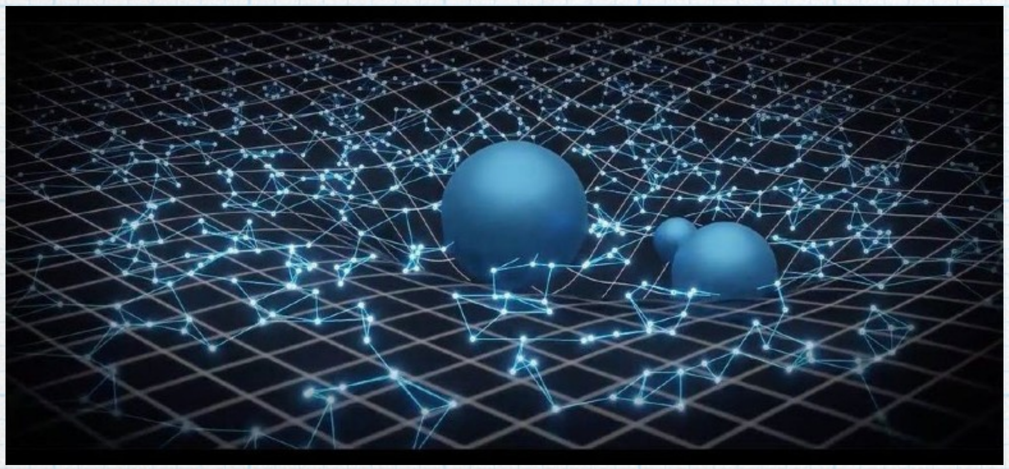




Information Loss Paradox

Good Old Dream of Holography



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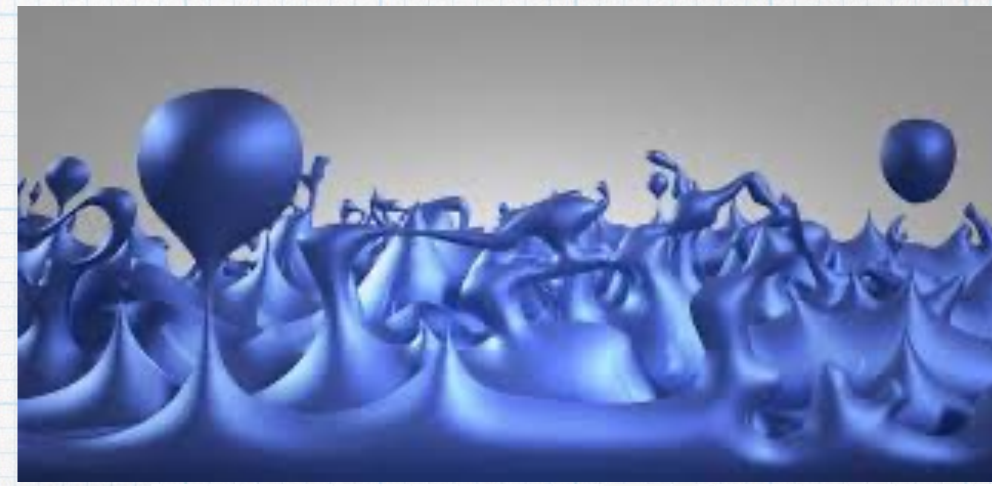
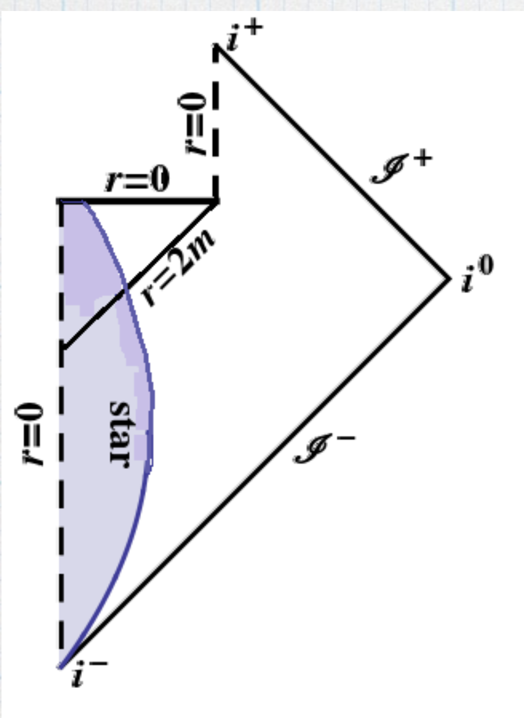
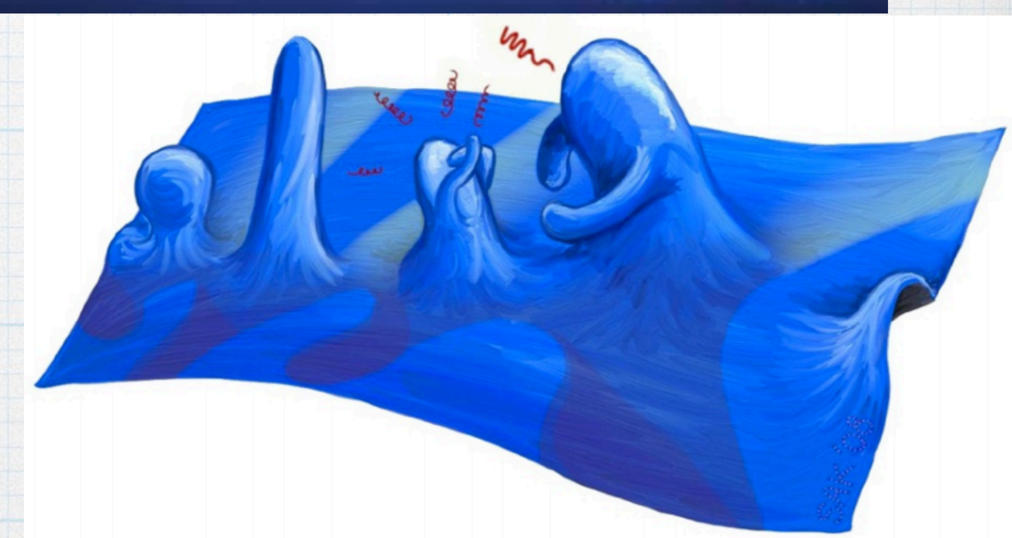
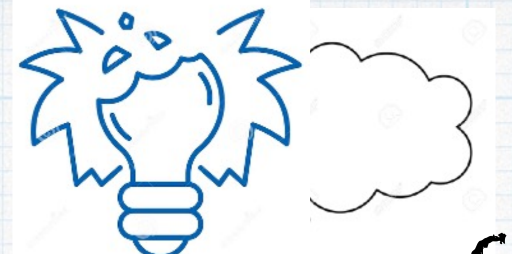






Information Loss Paradox

Strongly-coupled QFT



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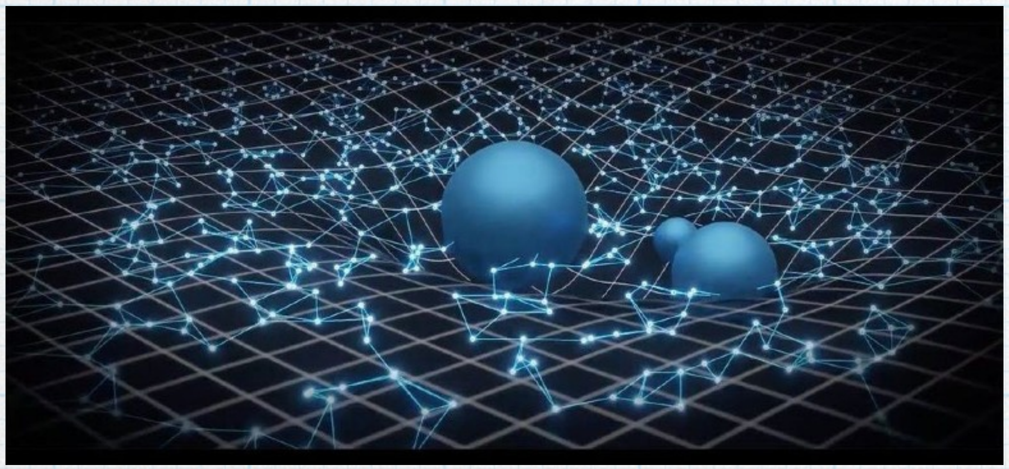

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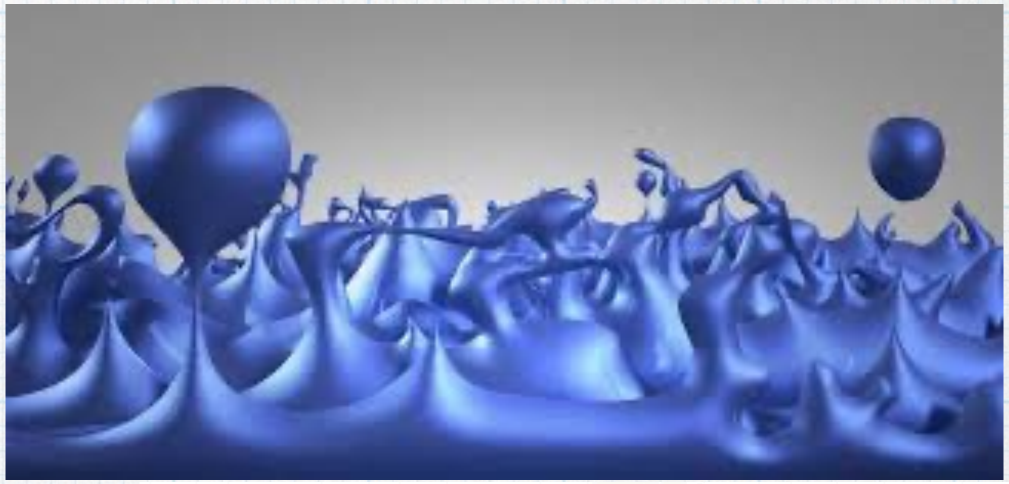

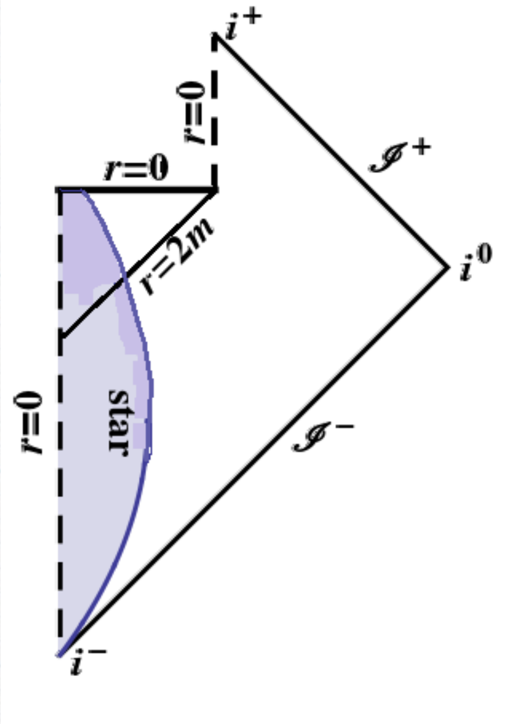

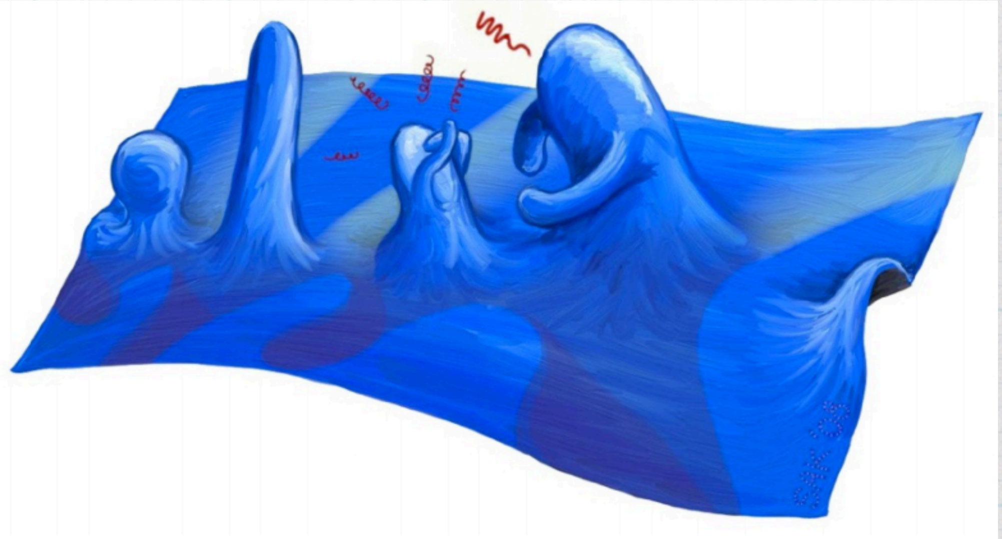

Information Loss Paradox

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A Partial Resolution?!

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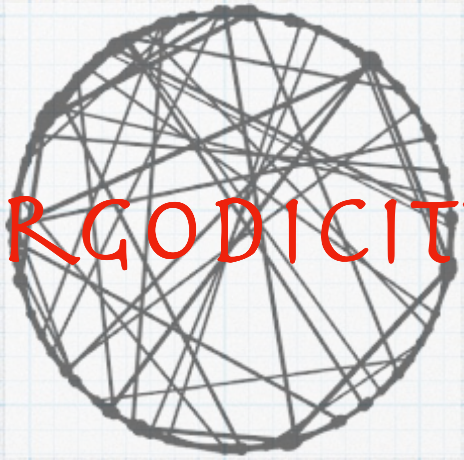

Altland, Bagrets '17;
Altland, Sonner '20;
Altland, Bagrets, *PN*, Sonner, Vielma *'21*;
Belin, de Boer, *PN*, Sonner *'21*;
more…

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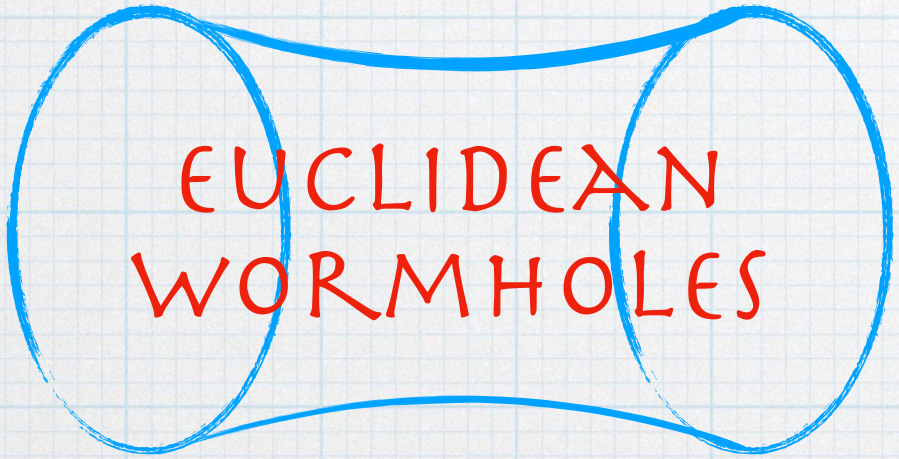

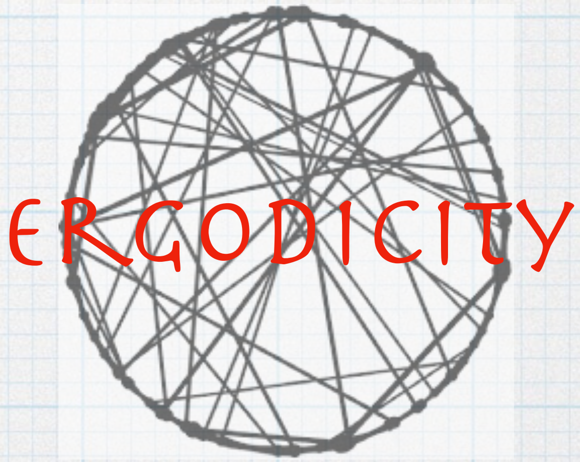

Saad, Shenker, Stanford '18 & '19;
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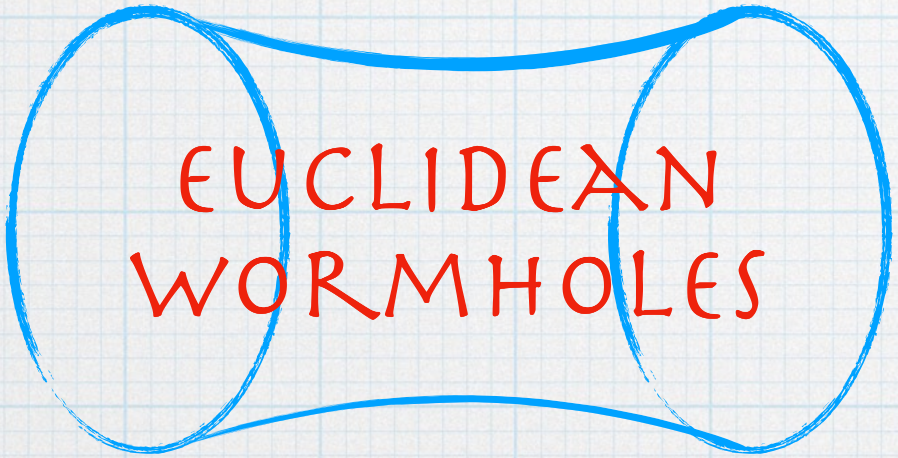
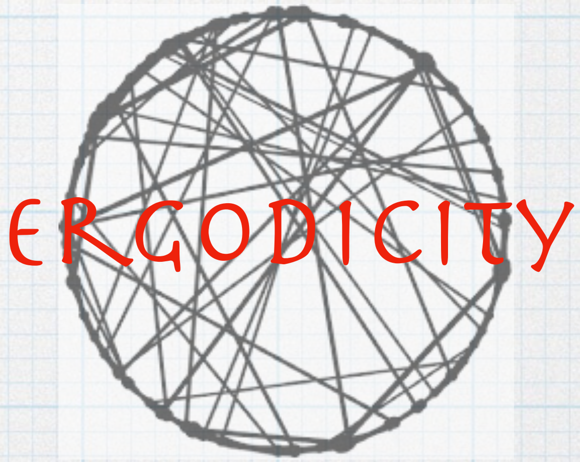
A Partial Resolution?!

Semi-classical Gravity ⟷ **Ergodic limit of Quantum theories**





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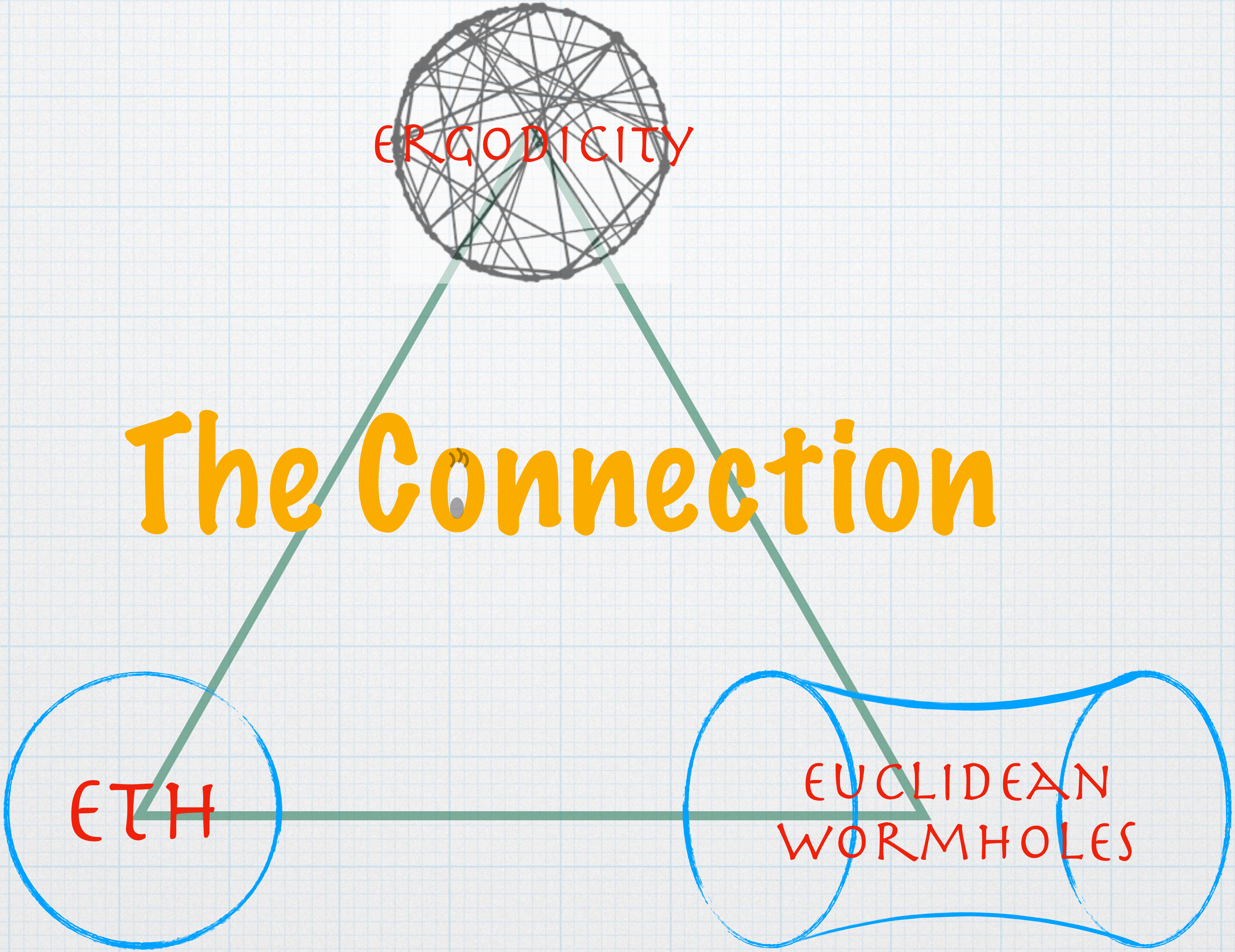
The Connection



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$$\xrightarrow{t\to\infty} \sum_i e^{-\beta E_i}|O_{ii}|^2 > 0$$

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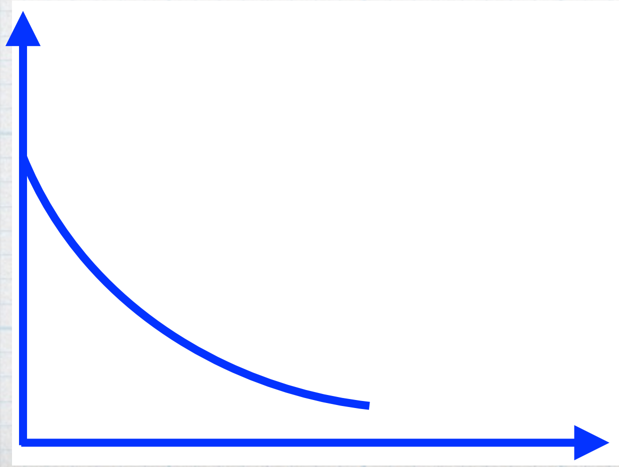
* The gravitational computation, however, gives a decaying answer



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Quantum Ergodicity

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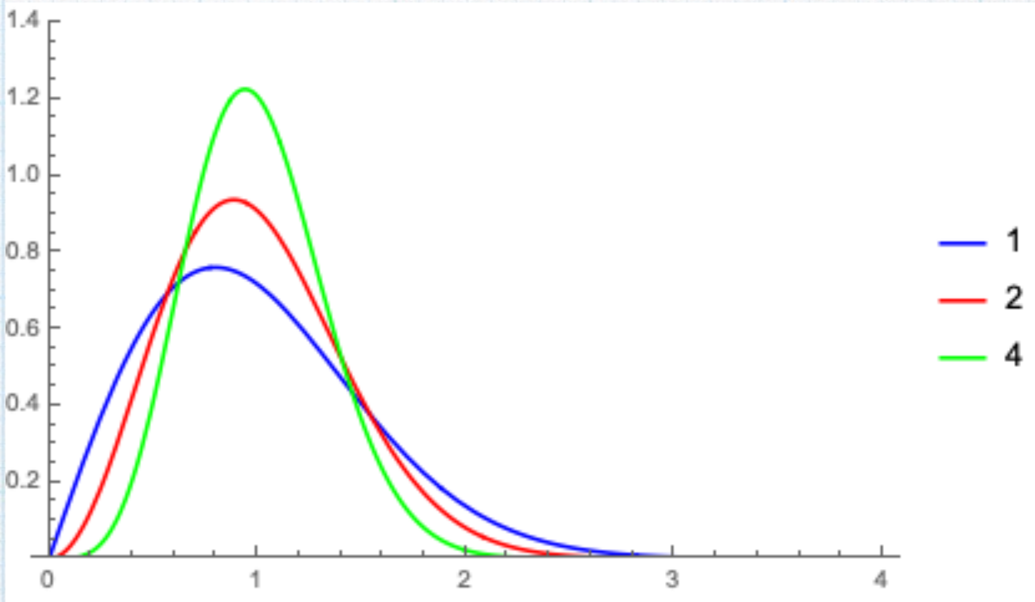
$$P^n[\omega] = A_n \omega^n \exp\left[-B_n \omega^2\right]$$



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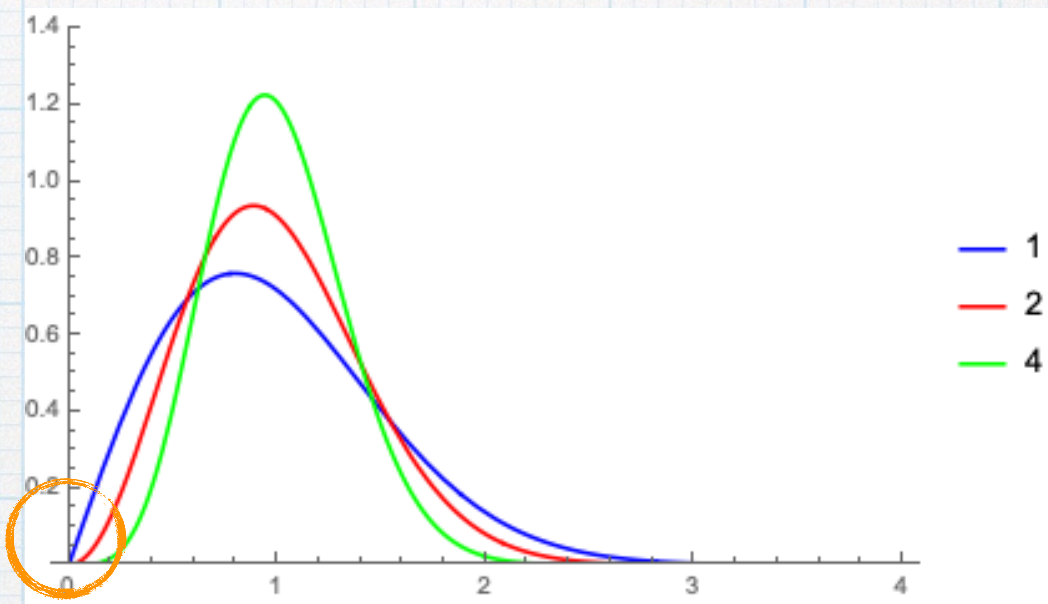

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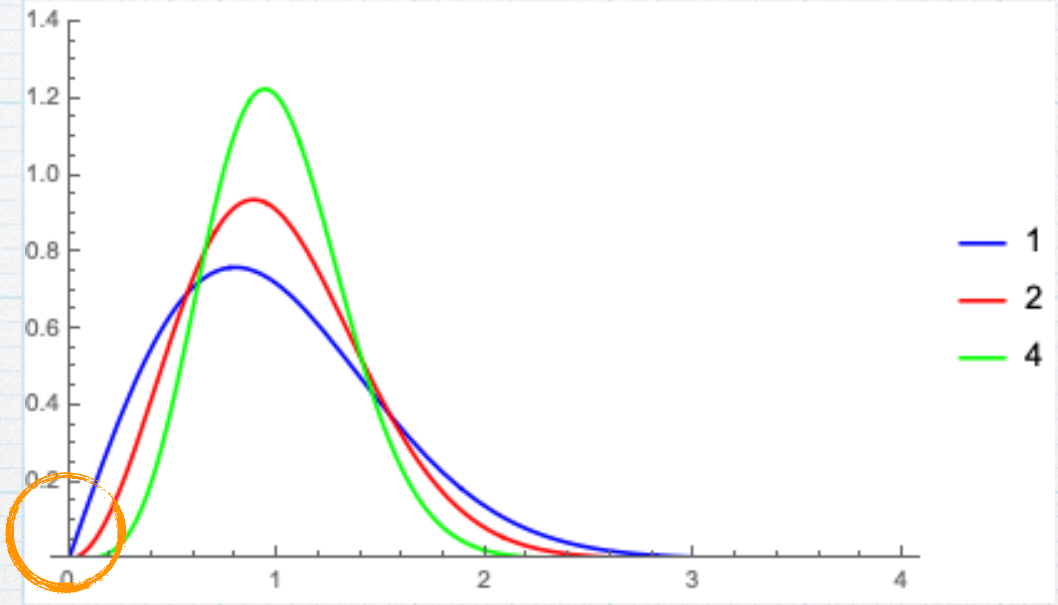

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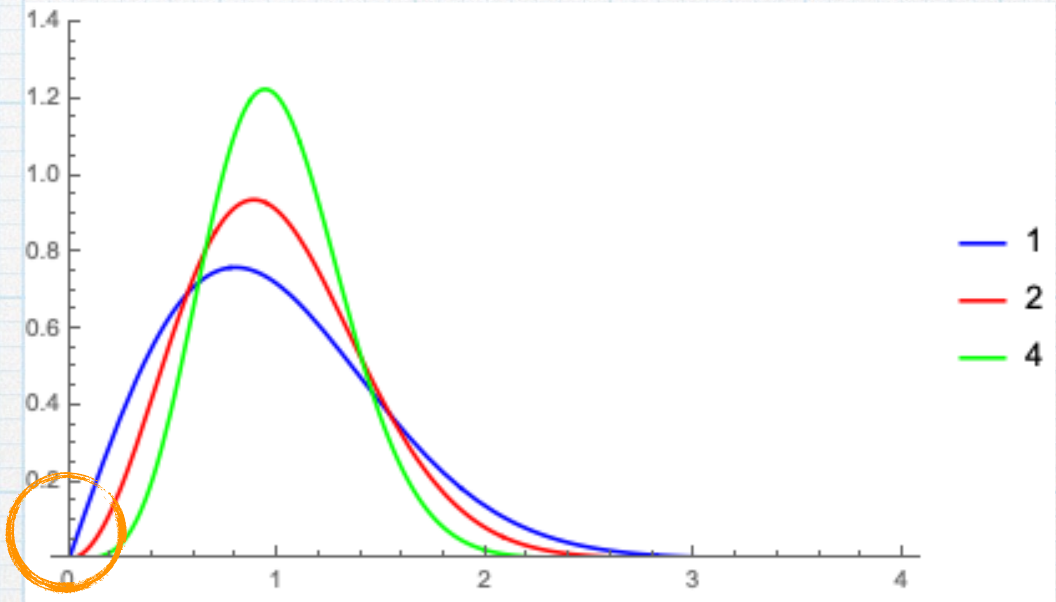

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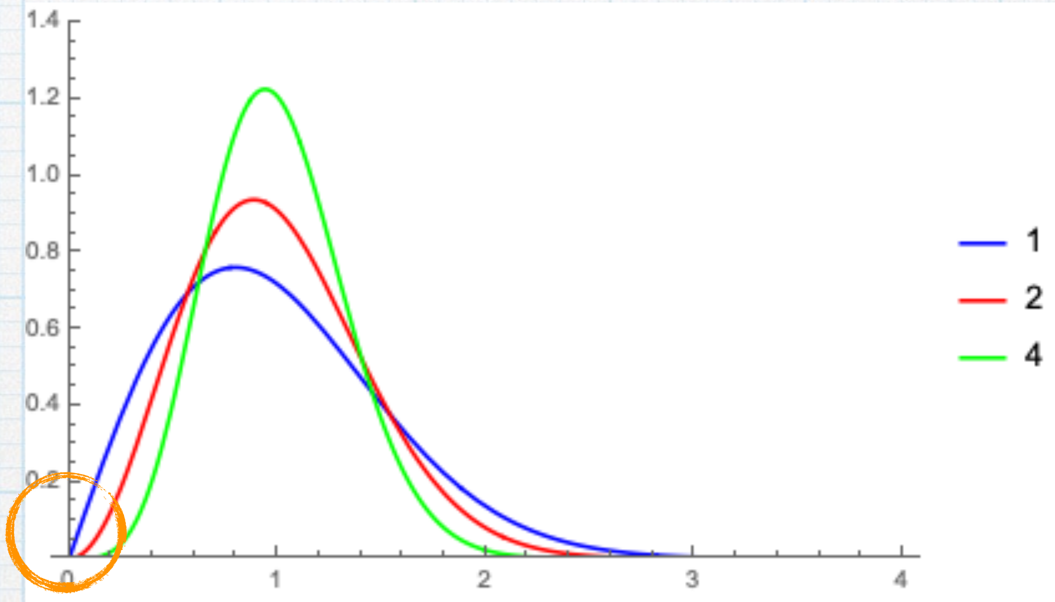

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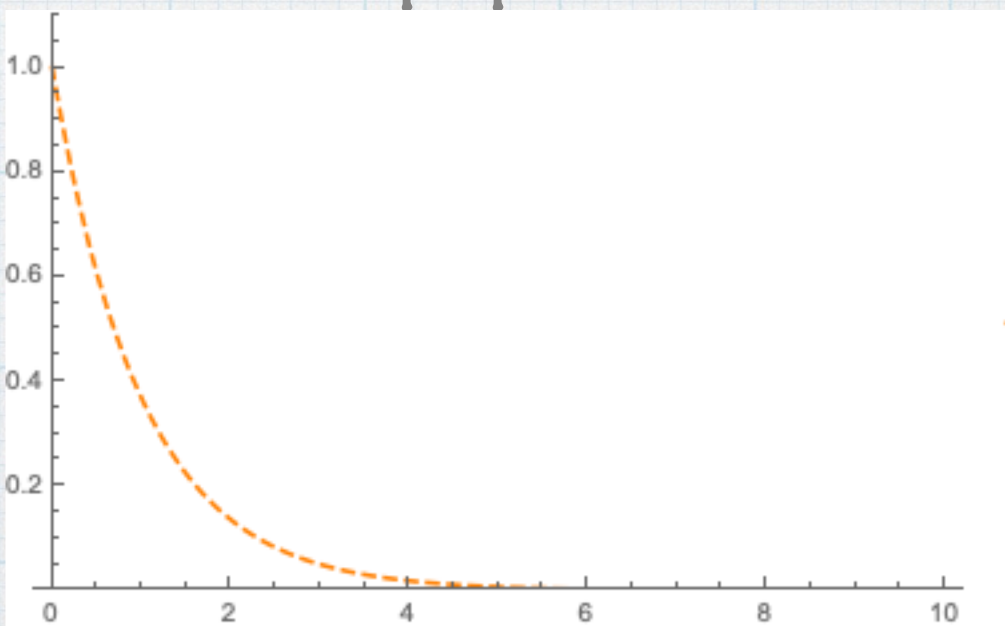
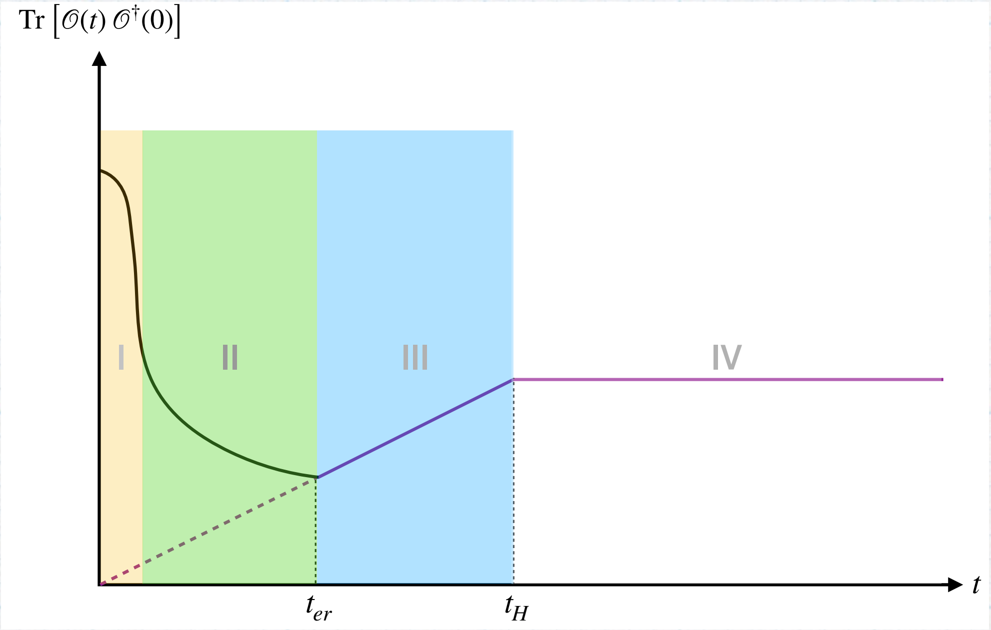
$$P^n[\omega] = \exp\left[-\omega\right]$$



* ω is the level spacing between eigenstates

Quantum ergodicity... manifestations

* Level repulsion and Spectral Rigidity in energy eigenstates of RMT leads to a characteristic **slope-ramp-plateau** behavior of observables like Spectral Form factor and correlation functions in RMT.



* Ergodic limit is defined as the energy domain in which RMT statistics persists for a many-body (chaotic) system. Corresponding time is called t_{er}

Quantum ergodicity... a history

* Quantum ergodicity has been studied numerically for a large class of models
* Numerical computations of such observables in the SYK model demonstrate a similar behavior.
* However, the microscopic description of this behavior in the SYK model (or any other system, for that matter) was incomplete.

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[Efetov '97 and references therein]

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[Cotler et. al. '16; Gharibyan, Hanada, Shenker, Tezuka '18
Saad, Shenker, Stanford '18]

* However, the **microscopic description** of this behavior in the SYK model (or <u>any</u> other system, for that matter) was **incomplete**.

Ergodic limit with σ-model

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- Let us look at the resolvent of an operator which is defined as,

$$\tilde{R}(E,\omega) = \sum_{\beta} |\langle \alpha | \mathcal{O} | \beta \rangle|^2 \, \delta\left(E_\alpha - E_\beta - \omega\right)$$

$$= \int dE' \, \rho(E)\rho(E') \, \left|\langle E | \mathcal{O} | E' \rangle\right|^2 \delta\left(E - E' - \omega\right)$$

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$|\langle E | \mathcal{O} | E' \rangle|^2 = \mathcal{O}_i \mathcal{O}_j^* U_{E,i} U^\dagger_{i,E'} U_{E',j} U^\dagger_{j,E}$

[Pollack, Rozali, Sully, Wakeham '20]

- Note that the averaging $\bar{\cdot}$ can be over of various kinds: disorder averaging, coarse graining, microcanonical averaging, etc.

The Resolvent

- The resolvent is related to the Fourier transform of the thermal 2-point function,

$$\mathrm{Tr}[e^{-\beta H}\mathcal{O}(t)\,\mathcal{O}^\dagger(0)] = \int dE\, d\omega\, e^{-\beta E + i\omega t}\, \tilde{R}(E,\omega)$$

ω is conjugate to the time, t

Late time $\longleftrightarrow$ small ω

- Recall the identity,

$$\rho(E) = \pm\, \mathrm{Im}\ \mathrm{Tr}\left[\frac{1}{E \mp i\varepsilon - H}\right]_{\lim \varepsilon\to 0}$$

$$\rho(E_1)\rho(E_2) \sim -\,\mathrm{Re}\ \left(\mathrm{Tr}\left[\frac{1}{E + i\varepsilon - H}\right]\mathrm{Tr}\left[\frac{1}{E - i\varepsilon - H}\right]\right)_{\lim \varepsilon\to 0}$$

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$$\frac{\mathrm{Det}(z_2^+ - H)\,\mathrm{Det}(z_4^- - H)}{\mathrm{Det}(z_1^+ - H)\,\mathrm{Det}(z_3^- - H)}$$

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Rotations in the 4d graded space

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Rotations in the 4d graded space

$$U(2|2) \to U(1|1)\times U(1|1)$$

Symmetry broken b/w advanced & retarded section

Also spontaneously broken by the saddle point in the limit $\dim(H) \gg 1$

Ergodic limit with σ-model

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[Altland, Bagrets, *PN*, Sonner, Vielma '21]

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$$\tilde{R}(E,\omega) = \sum_{\beta} |\langle \alpha | \mathcal{O} | \beta \rangle|^2 \, \delta\left(E_\alpha - E_\beta - \omega\right)$$

$$\approx \mathrm{Tr}[\mathcal{O}]\mathrm{Tr}\left[\mathcal{O}^\dagger\right] \pi\,\delta(s) + \left(\mathrm{Tr}\left[\mathcal{O}\mathcal{O}^\dagger\right] - \frac{1}{D}\mathrm{Tr}\mathcal{O}\mathrm{Tr}\mathcal{O}^\dagger\right)\left(\pi\delta(s) + 1 - \frac{\sin^2(s)}{s^2}\right)$$

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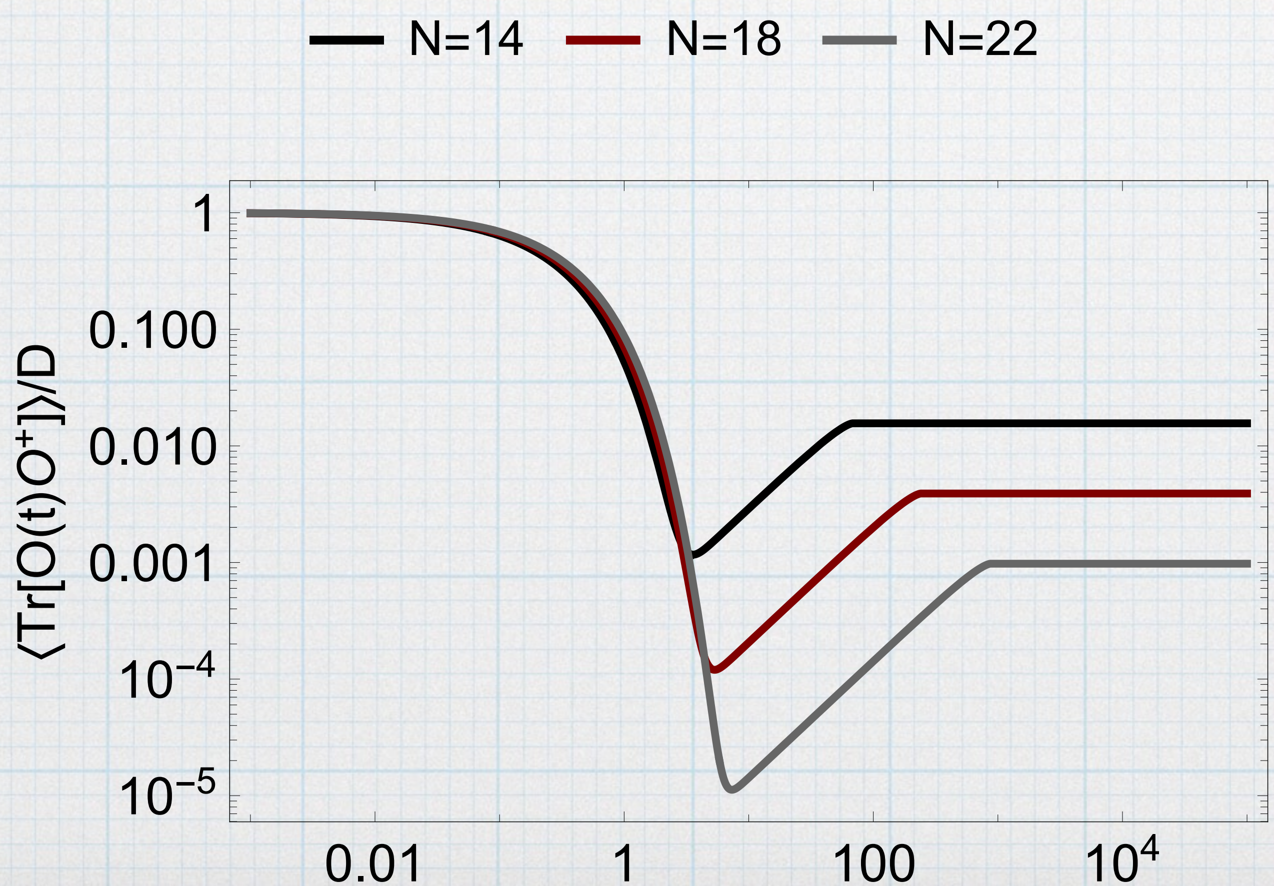

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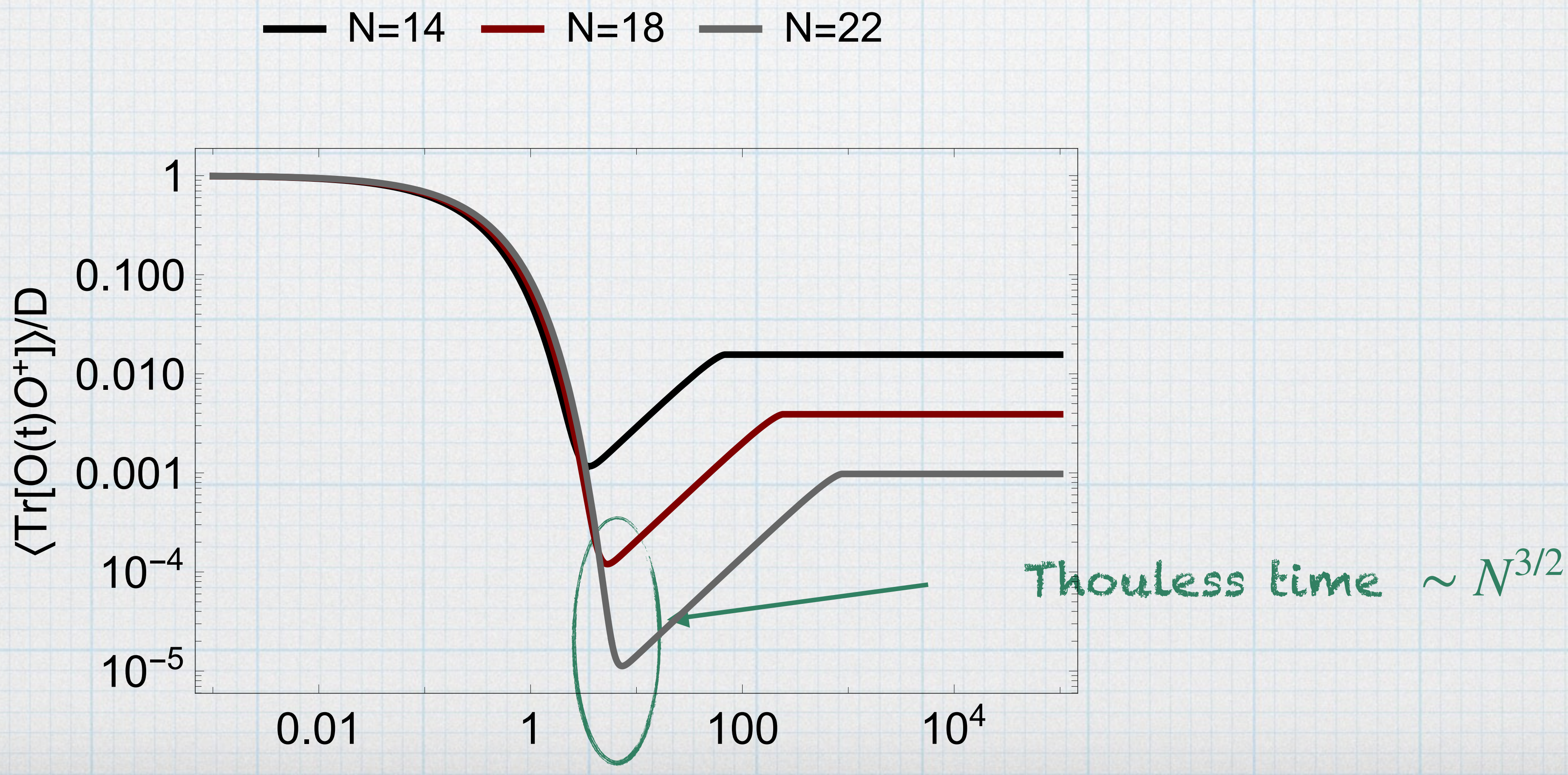
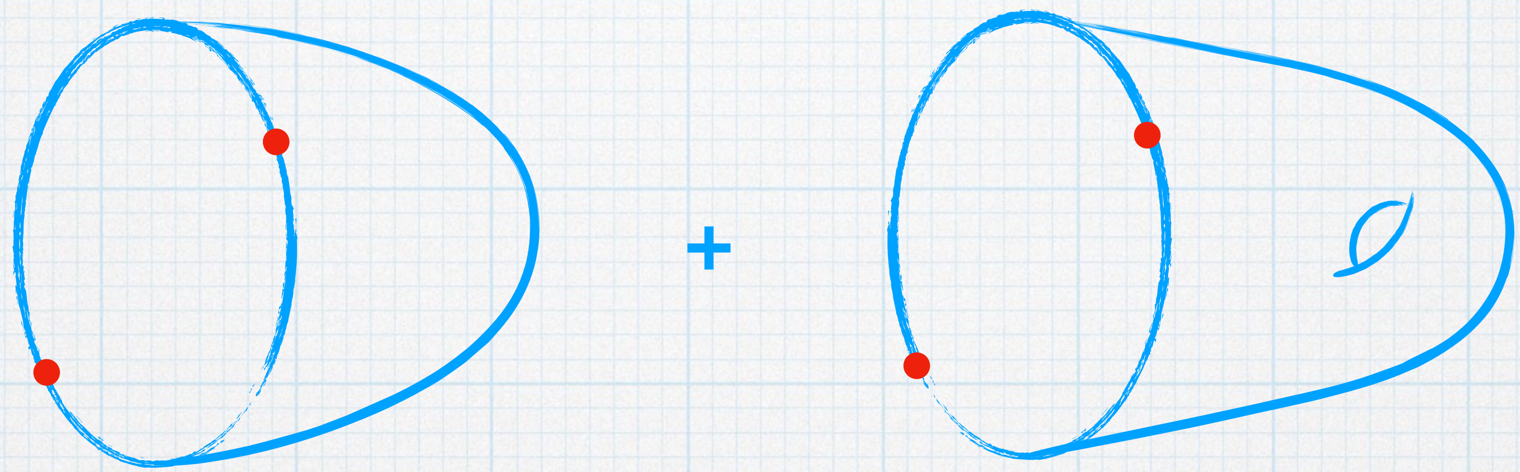
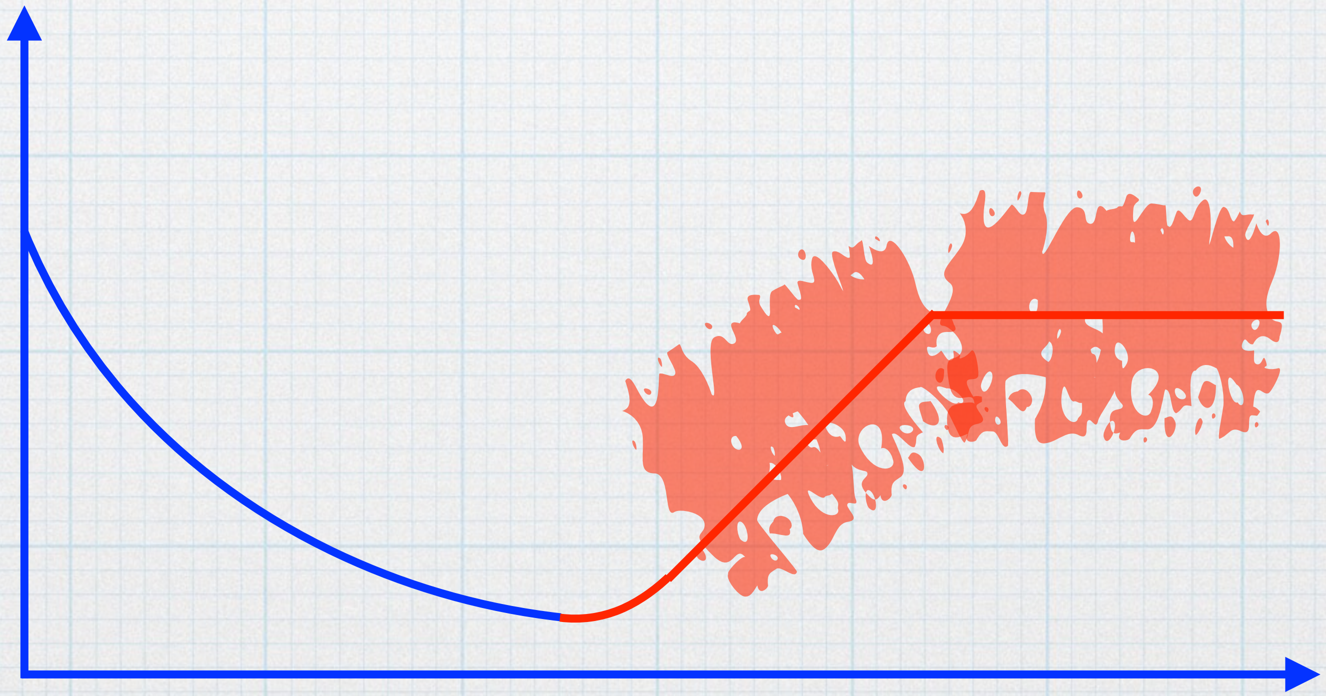


Euclidean Wormholes

[Saad '19; Altland, Bagrets, *PN*, Sonner, Vielma '21]





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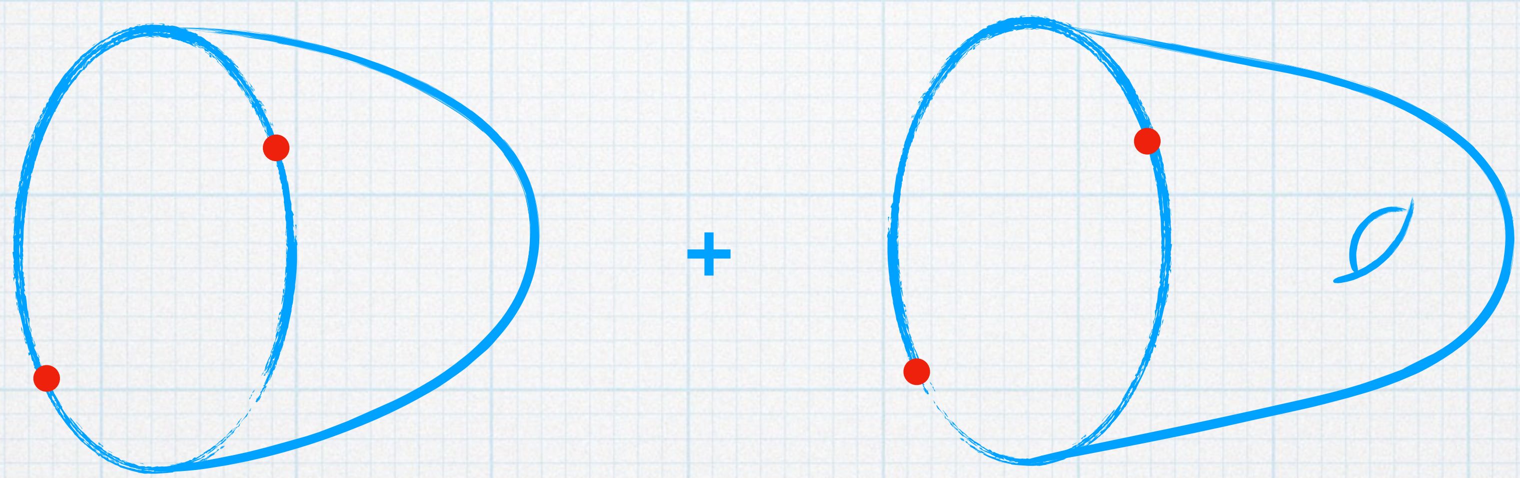

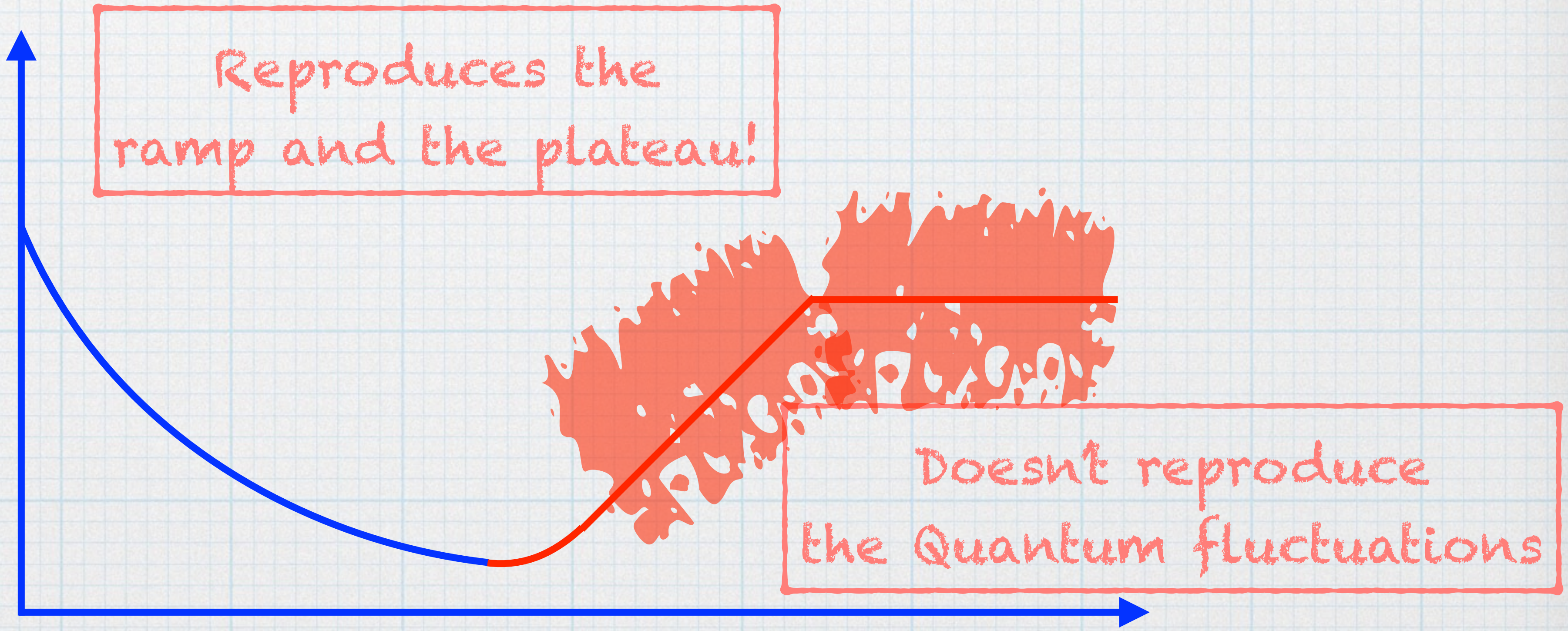

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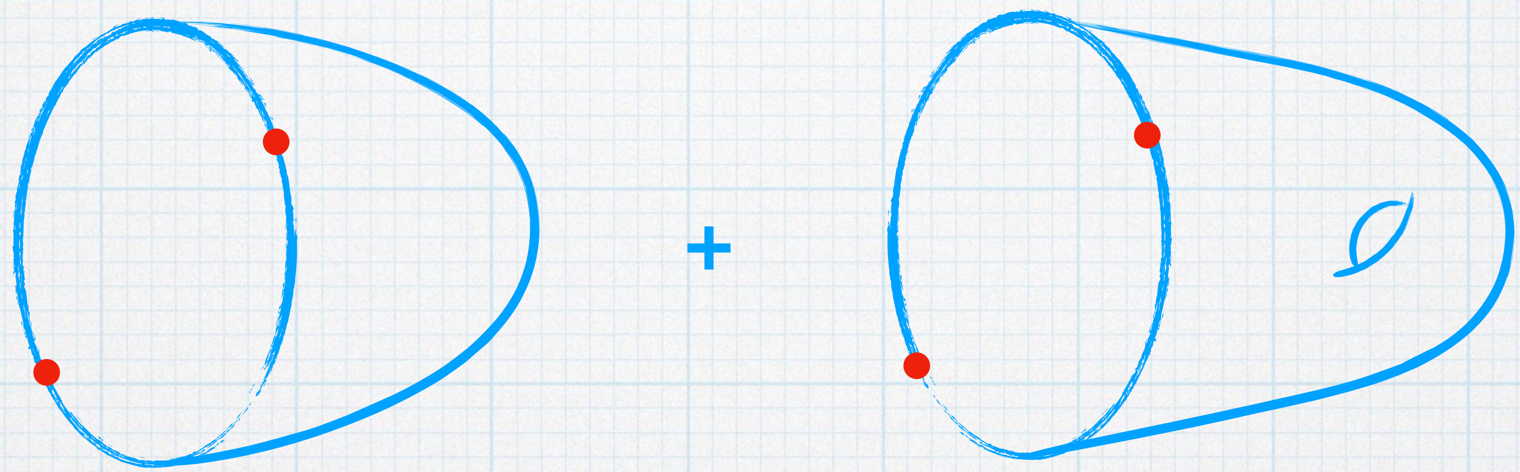

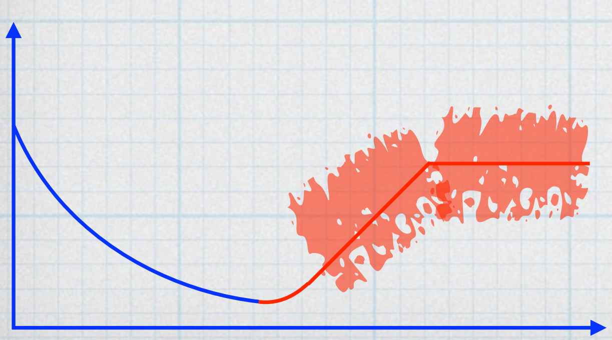

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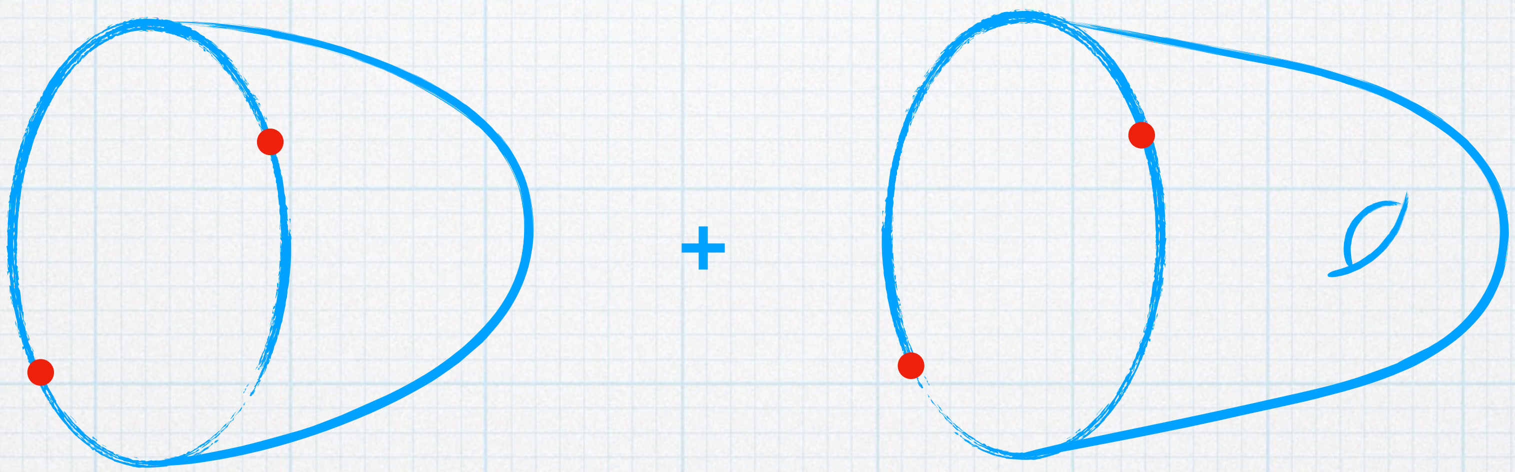

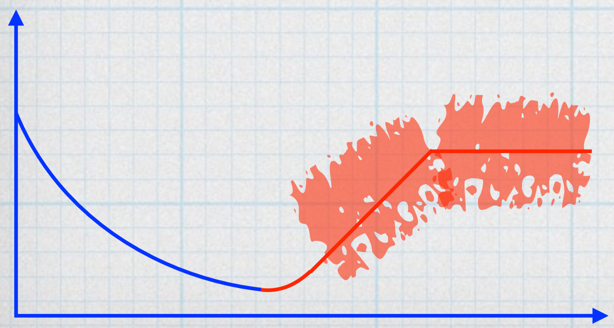

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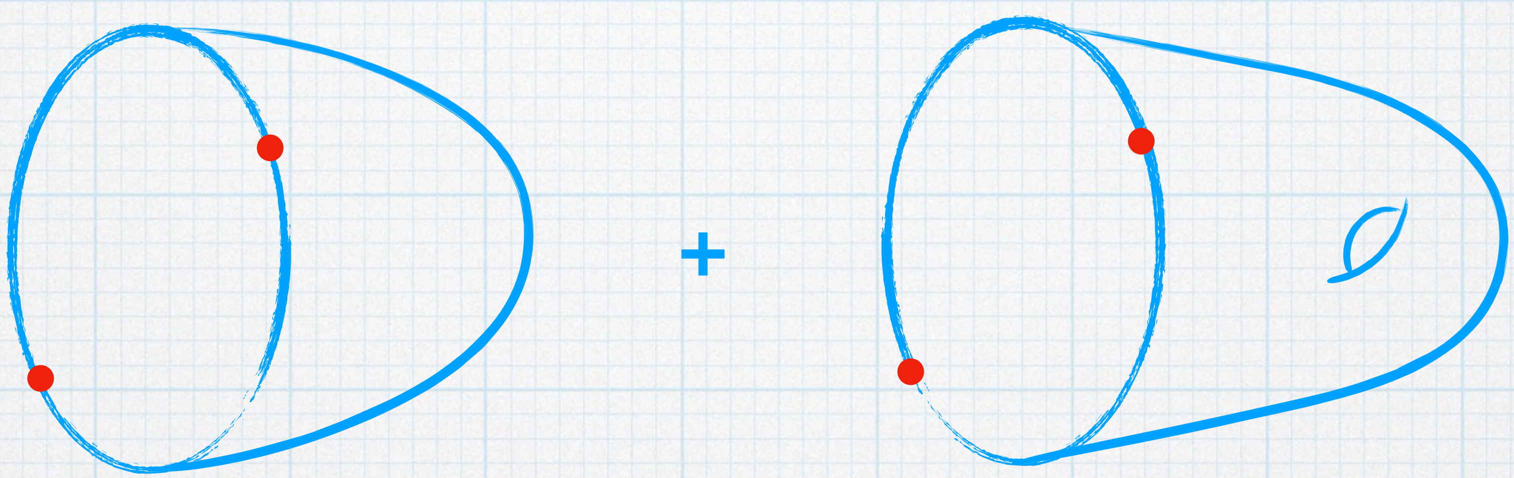

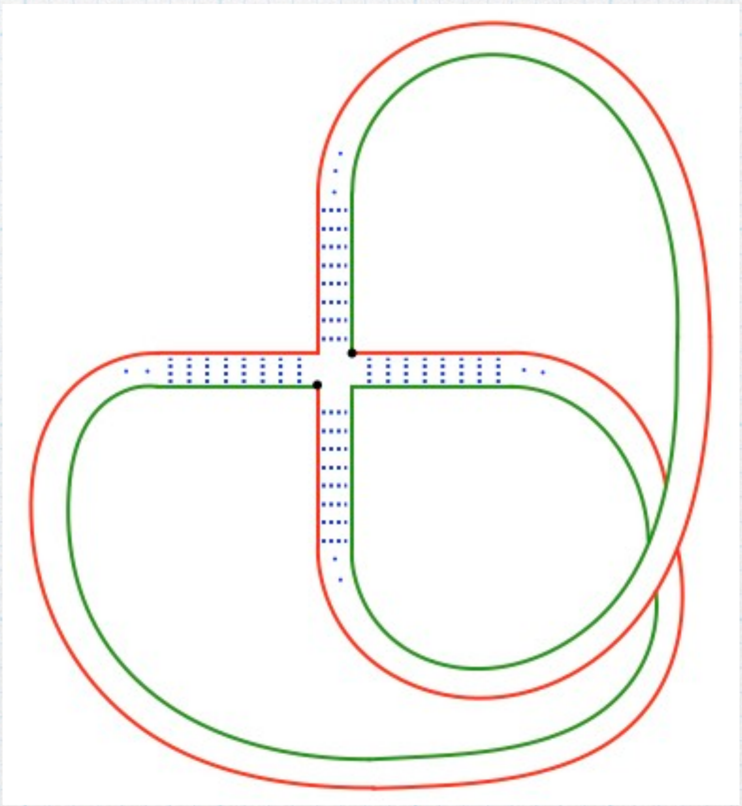

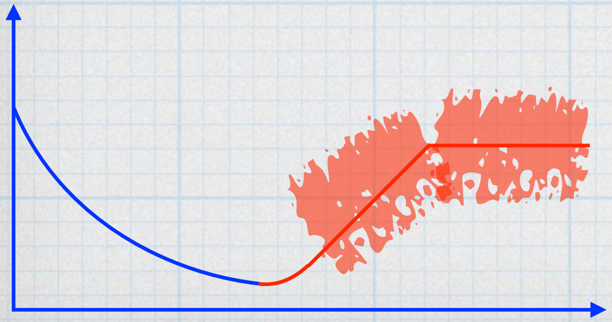

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Fewer random parameters in H $\Rightarrow$ corrections to mean field approximation, $\Psi\bar{\Psi} \propto \mathbb{1}_{\text{Fock}}$

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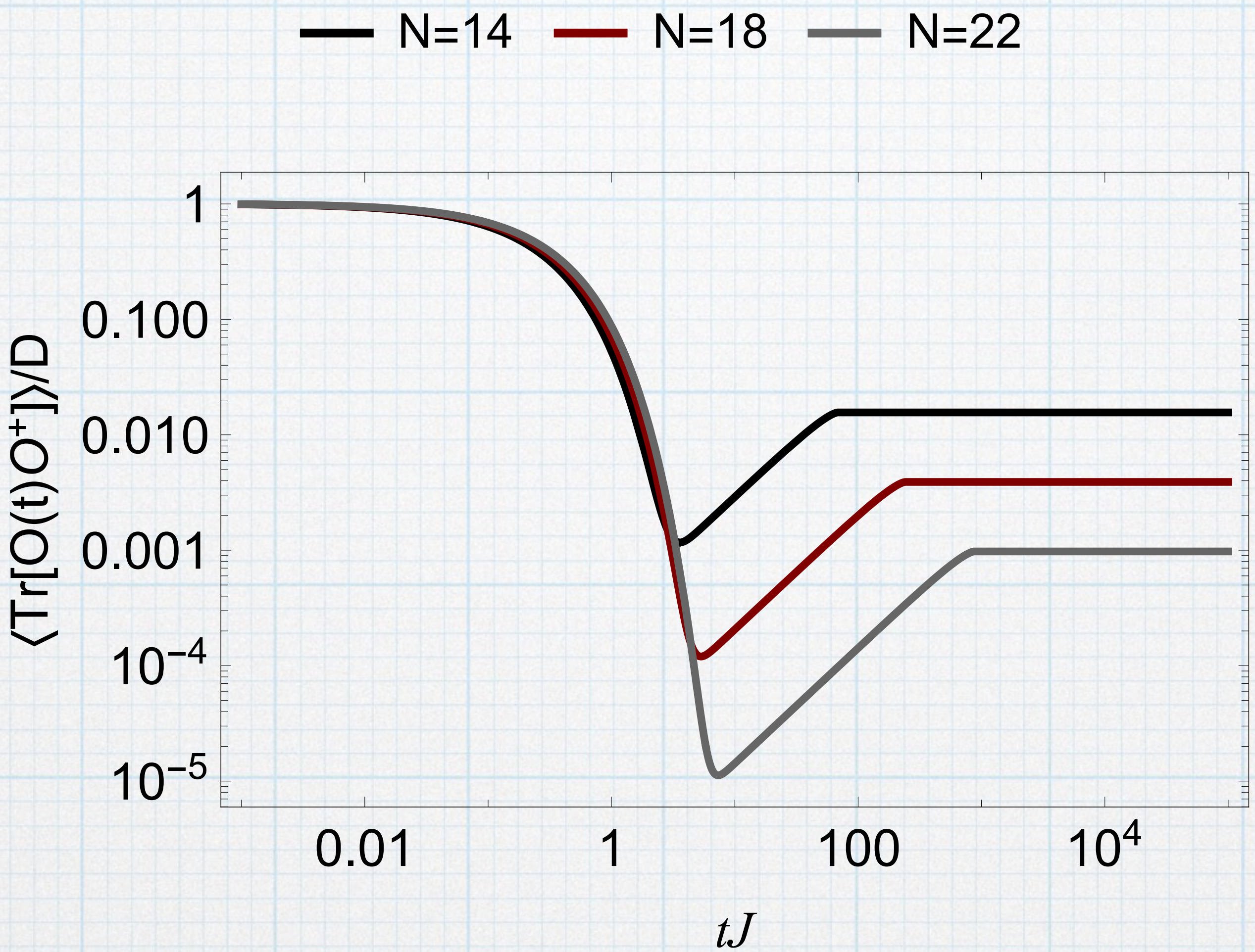

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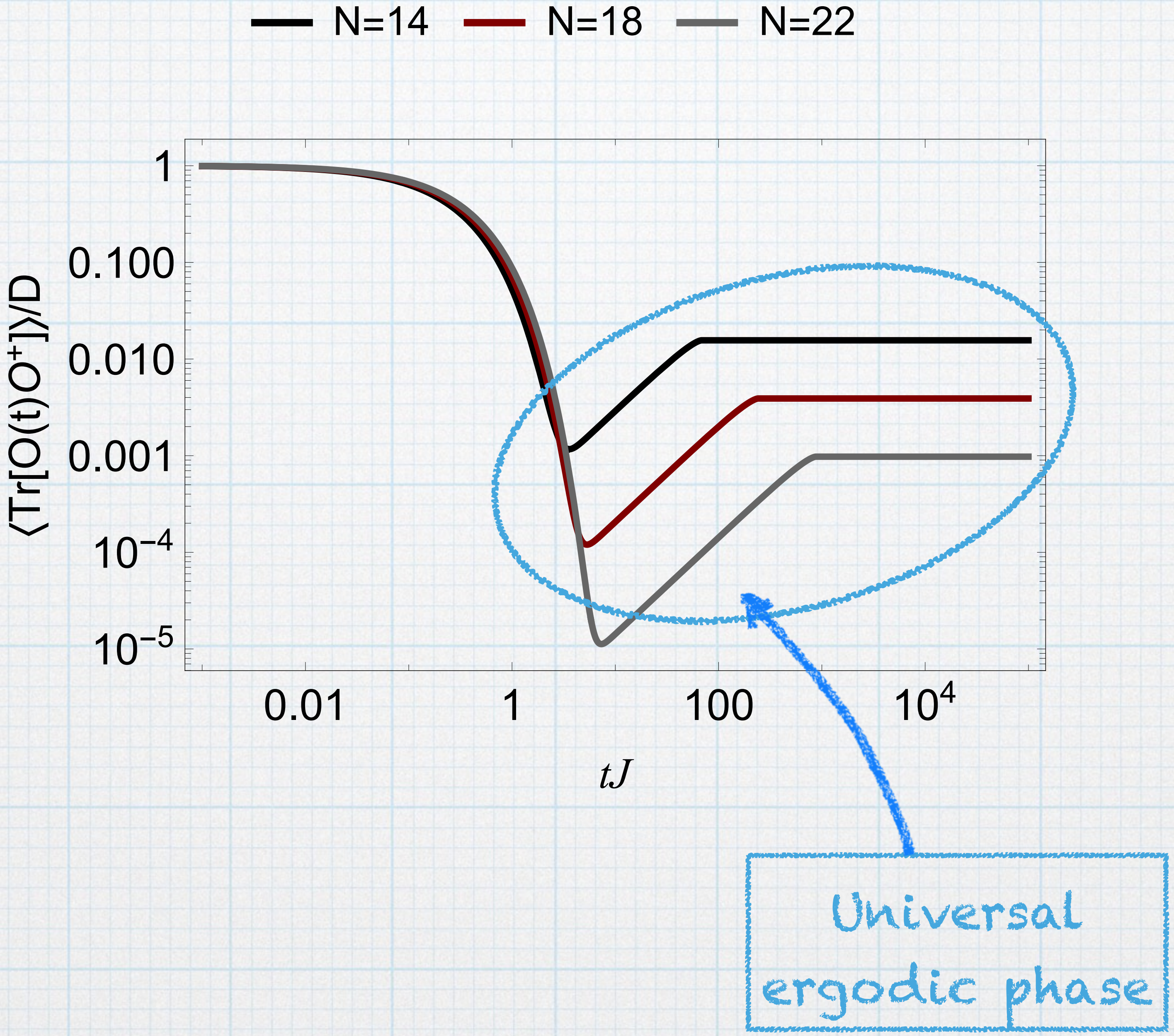

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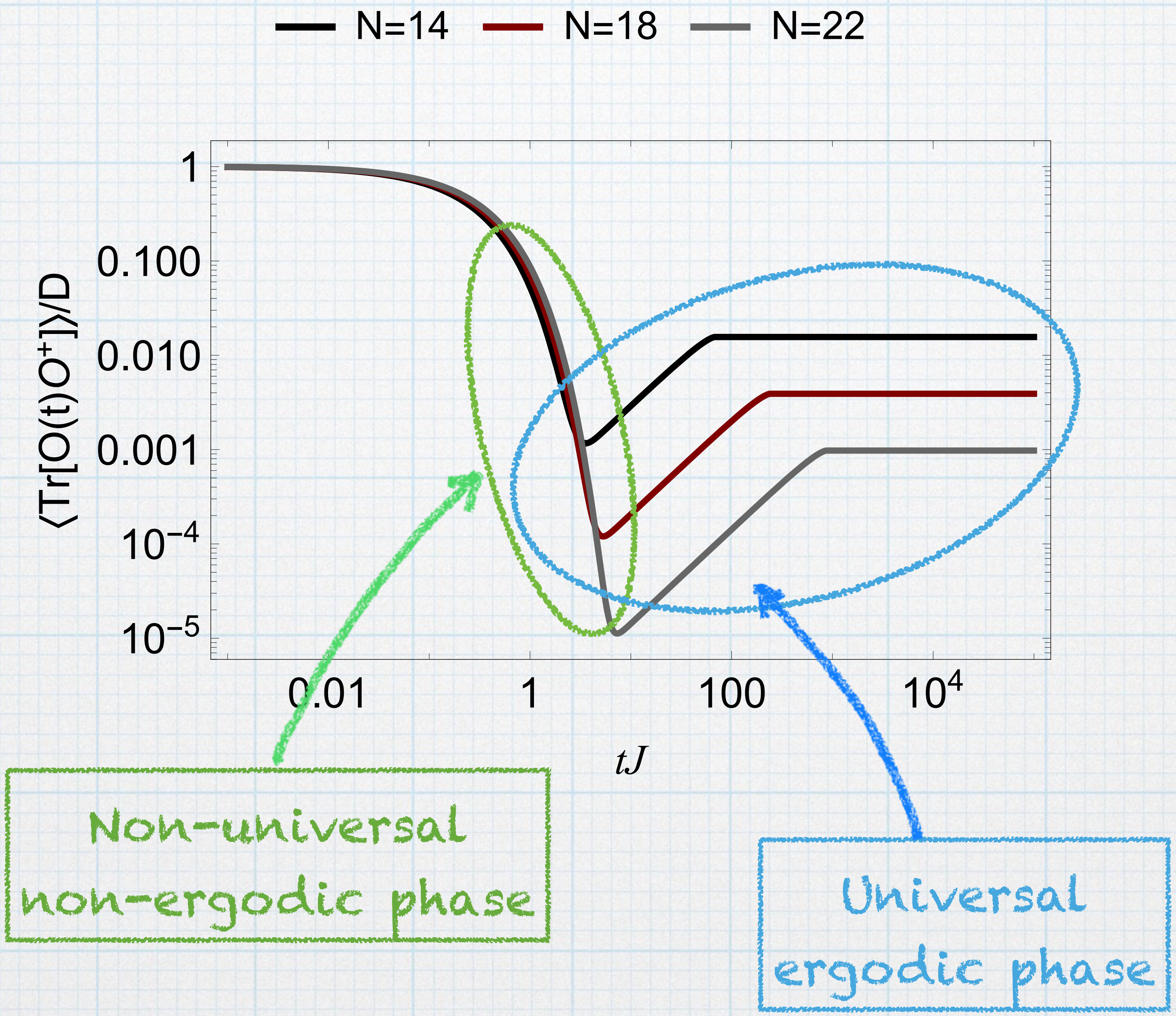

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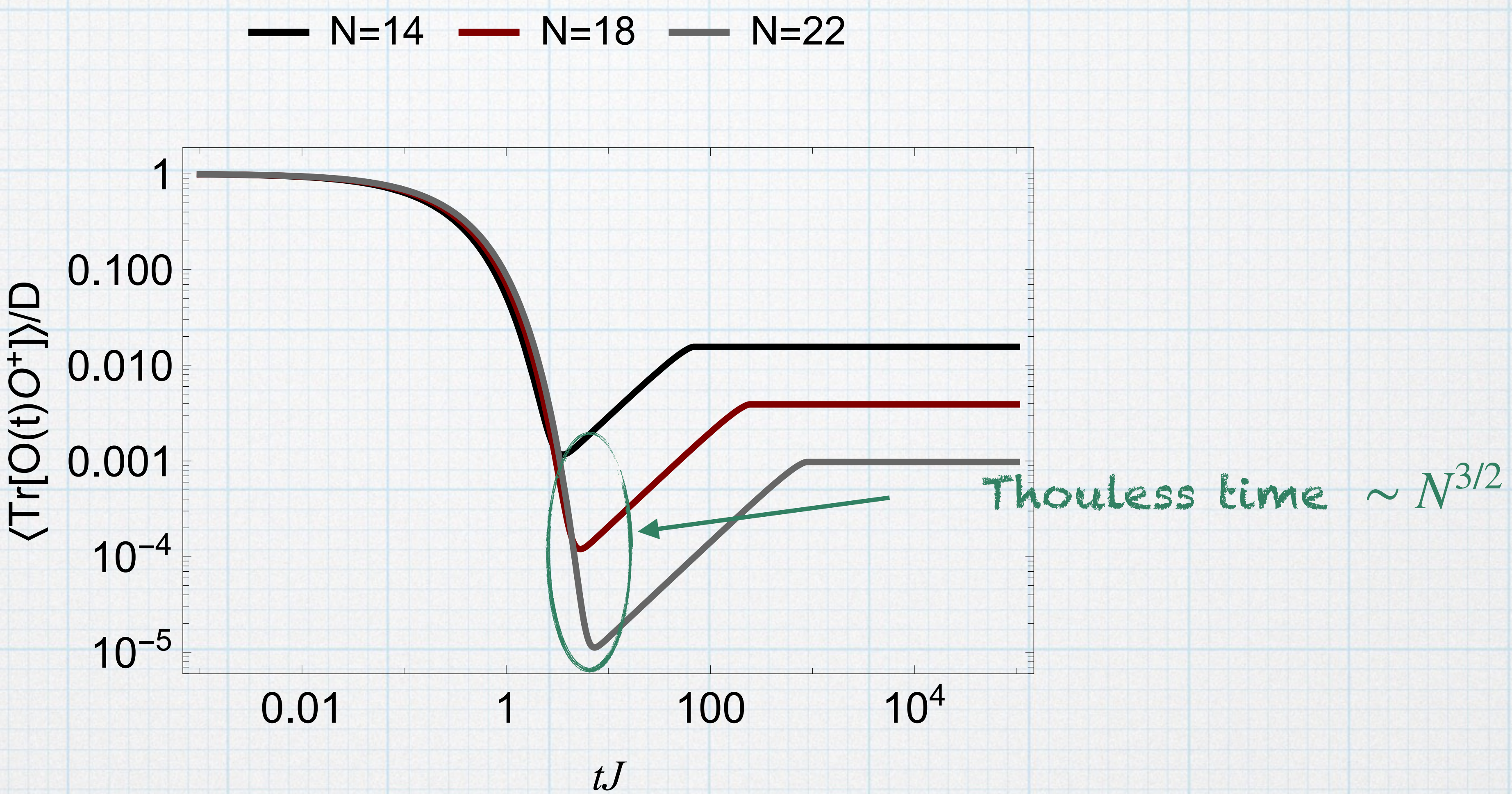

New Conjecture

Semi-classical Gravity $\Leftrightarrow$ Ergodic limit of Quantum theories

[Altland, Sonner '20; Altland, Bagrets, *PN*, Sonner, Vielma '21]

New Conjecture

Semi-classical gravity has more information about the fine-grained structure of the spectrum of the Quantum theory!

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Ergodic Predictions in CFT_2

Belin, de Boer '20;

Belin, de Boer, *PN*, Sonner

2111.xxxxx

Ergodic Predictions in CFT$_2$

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* Observables:

  Statistics of OPE coefficients $\left\langle\!\left\langle c_{ijk} c^*_{lmn} \right\rangle\!\right\rangle$ in ergodic regime

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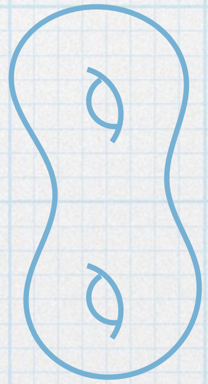
Genus-2 partition function

$$Z_{g=2} = \sum_{ijk} c_{ijk} c^*_{ijk} \; q_1^{\Delta_i} q_2^{\Delta_j} q_3^{\Delta_k}$$



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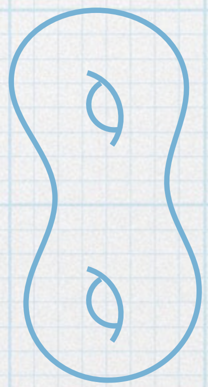
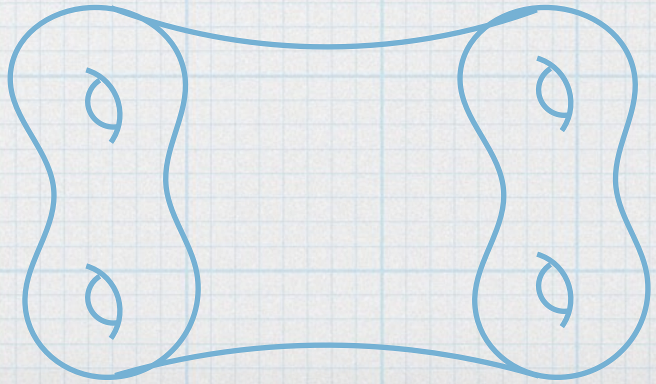


Genus-2 spectral form factor

$$\left| Z_{g=2} \right|^2$$



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Different cycles of genus-2 surface

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$$\mathbb{O} = c^*_{def} c_{abc} |d\rangle |e\rangle |f\rangle \langle a| \langle b| \langle c|$$

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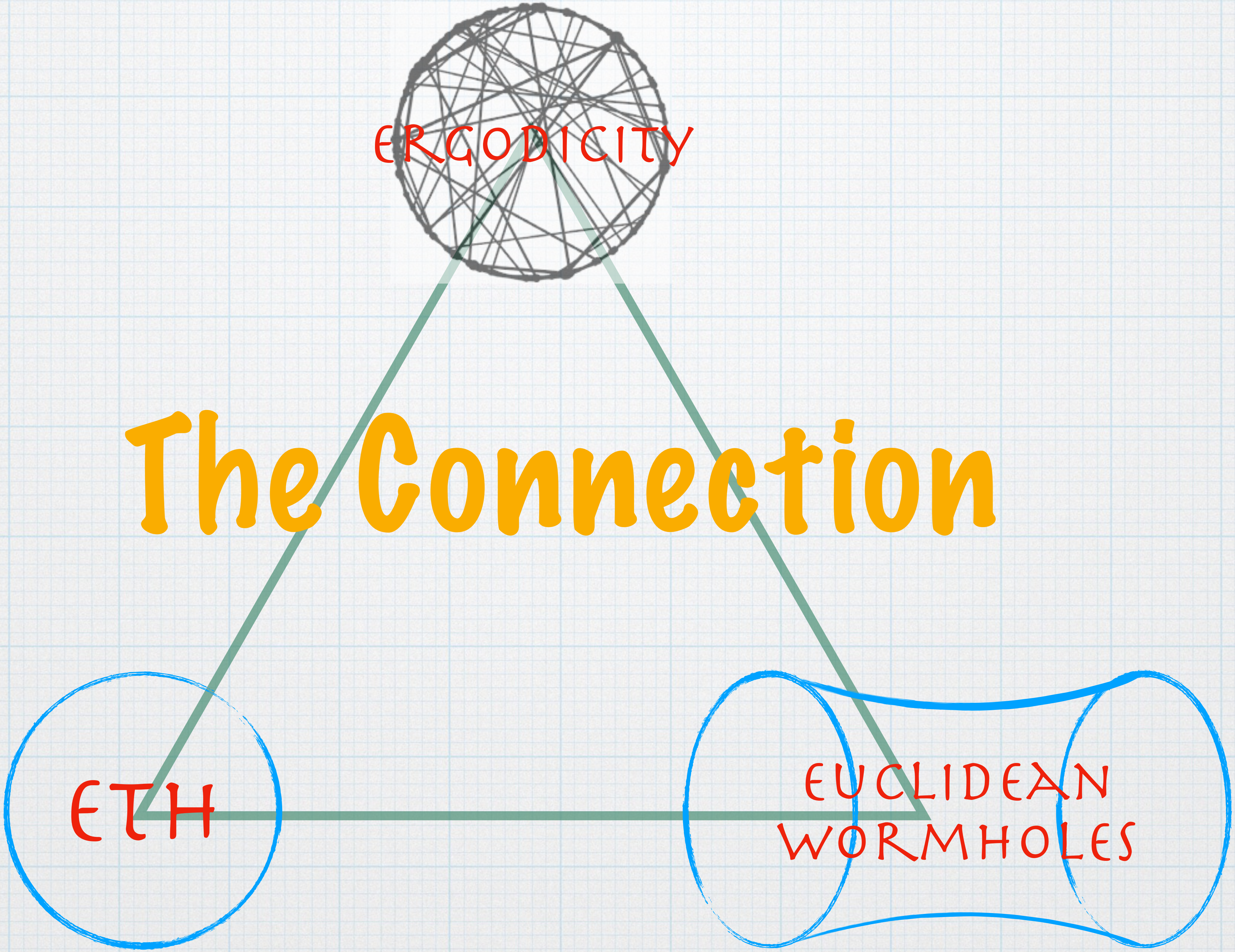
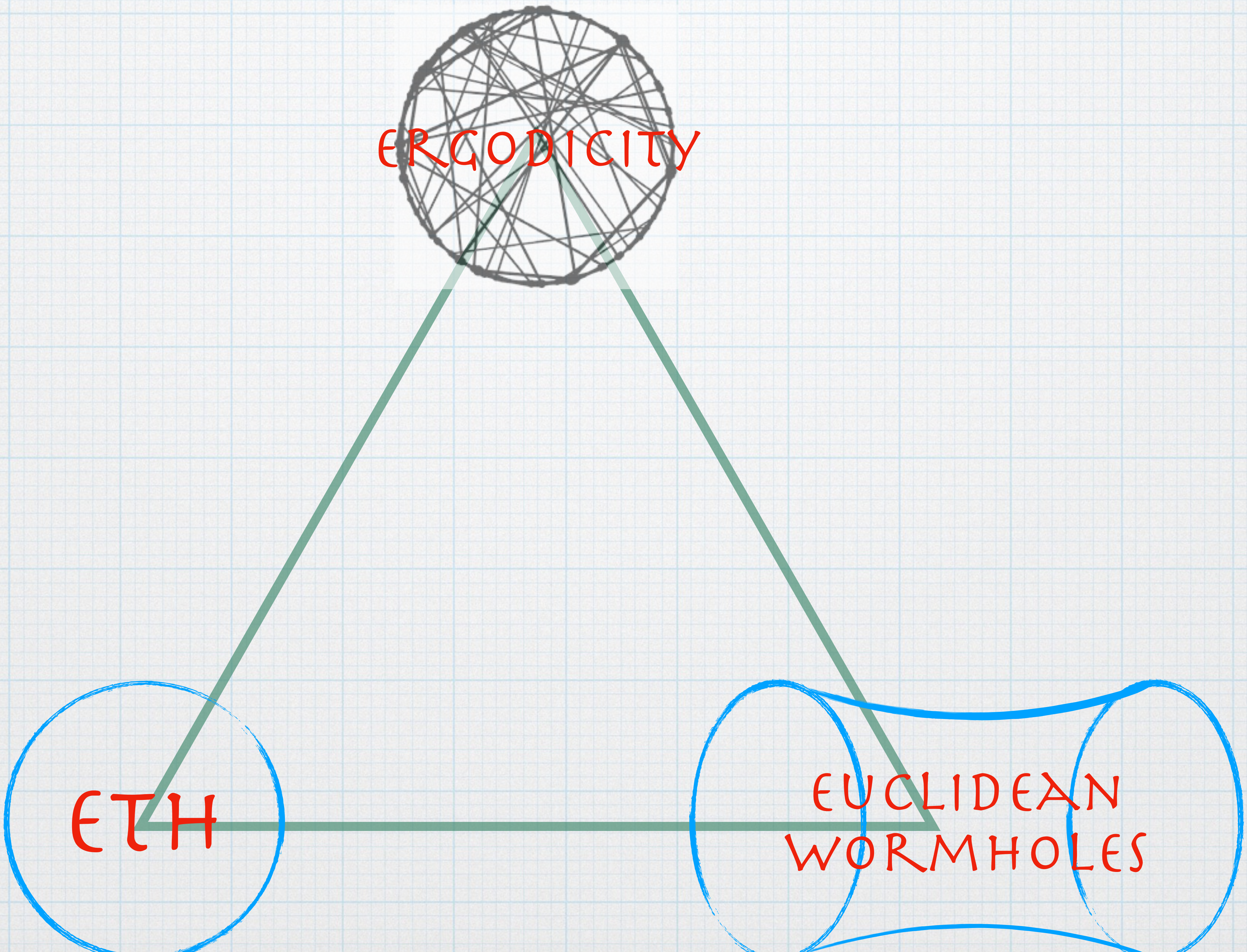
☑ Reproduce the correct entropic suppressions as predicted

The Connection



How are the three things related?



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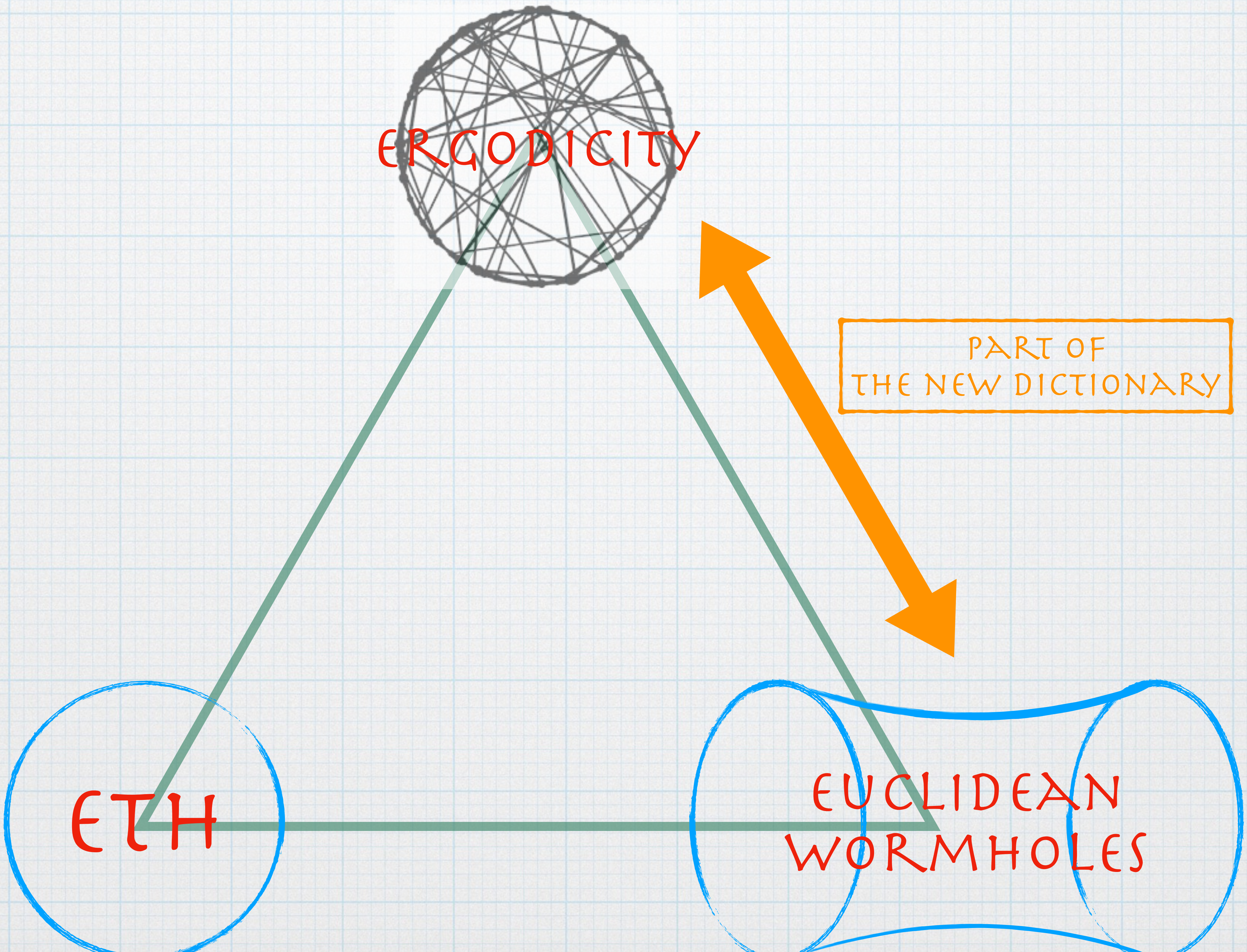

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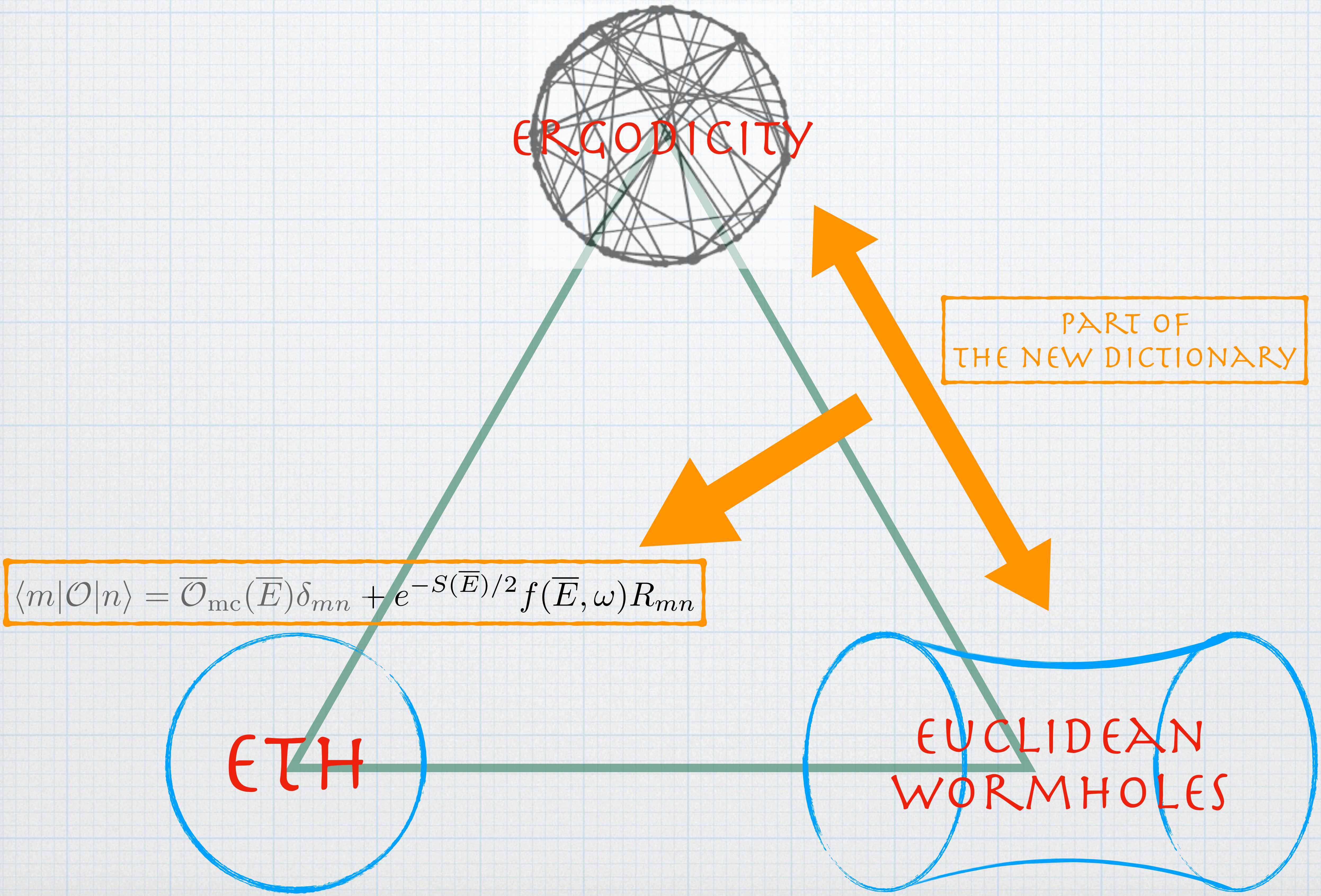

ETH for charged operators

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* A Tale of Two Conjectures:

ETH for charged operators

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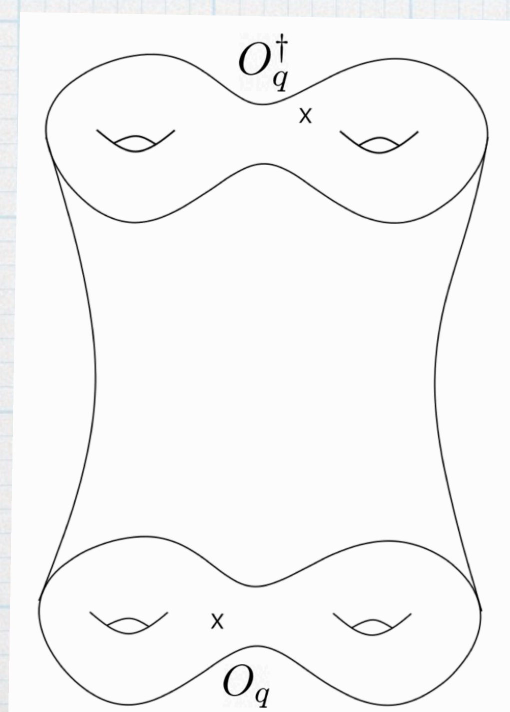
- Wormholes or no wormholes



Summary

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- We developed a EFT description of the observables in terms of the Goldstones of the causal symmetry breaking
- We studied the connection between the Euclidean wormholes geometries and the ergodic limit of the physical theories

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THANK YOU!

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Arriving at sigma model

$$Z[h] = \left\langle \int \mathscr{D}\bar{\Psi}\mathscr{D}\Psi \exp\left[i\bar{\Psi}\cdot(z-H+h)\cdot\Psi\right] \right\rangle_{\text{dis}}$$

$$Z[h]=\int \mathscr{D}\bar{\Psi}\mathscr{D}\Psi\mathscr{D}A \exp\Bigg[i\bar{\Psi}\cdot\left(z+h\right)\cdot\Psi - \frac{1}{2n}\sum_a \mathrm{STr}\left[X^a A^a X^a A^a\right] + \frac{i\gamma}{n}\sum_a \mathrm{STr}\left[\Psi\bar{\Psi}A^a\right]\Bigg]$$

$$Z_0[h] = \int \mathscr{D}y \;\exp\left[-\frac{D}{2}\mathrm{STr}(y^2) - \mathrm{STr}\ln(K_0)\right], \qquad K_0 = \left(z+h+\gamma\, y\otimes \mathbb{I}^{\mathrm{Fock}}\right)$$

Sigma model

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$$\Lambda_{AA} = \sigma_3^{\mathrm{RA}} \otimes \sigma_3^{\mathrm{bf}}$$

$$Z[h] = \int \mathscr{D}Q \exp\left[\frac{i\pi\rho(E)}{2}\mathrm{Str}\left(\omega\Lambda Q\right) - S_{\mathrm{src}}(h)\right], \qquad Q = T\Lambda T^{-1}$$

$$T = \exp(-W), \quad W = -\begin{pmatrix} 0 & B \\ \tilde{B} & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} x & \mu \\ \nu & iy \end{pmatrix}, \qquad \tilde{B} = \begin{pmatrix} x^* & \bar{\nu} \\ -\bar{\mu} & iy^* \end{pmatrix}.$$

$$\partial_h^2 Z[h]\Big|_{h=0} = \int \mathscr{D}Q \exp\left[2i\pi\rho(E)\omega\,\mathrm{Str}\left[B\tilde{B}\right] + \ldots\right] \times \mathrm{STr}\left[\tilde{B}P_b\tilde{B}BP_bB\right]$$

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$$\mathrm{STr}[B \cdot X]\,\mathrm{STr}[\tilde{B} \cdot Y] = \frac{i}{2\pi\rho(E)\,\omega}\mathrm{STr}[XY]$$

$$\mathrm{STr}[B \cdot X \cdot \tilde{B} \cdot Y] = \frac{i}{2\pi\rho(E)\,\omega}\mathrm{STr}[X]\mathrm{STr}[Y]$$

Sources for various correlation functions

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* Sources for the operator correlation function that we have been looking at,

* Source for 2-point function of d.o.s.

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$$h = \begin{pmatrix} 0 & h_+ P_b \otimes \mathcal{O} \\ -h_- P_b \otimes \mathcal{O}^\dagger & 0 \end{pmatrix}$$

* Source for 2-point function of d.o.s.

$$h = \begin{pmatrix} h_+ P_b \otimes \mathbb{I} & 0 \\ 0 & h_- P_b \otimes \mathbb{I} \end{pmatrix}$$