Quantum Singularities

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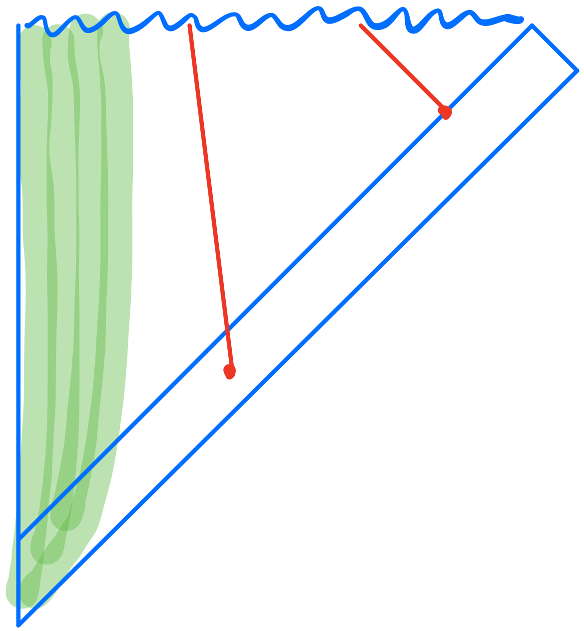
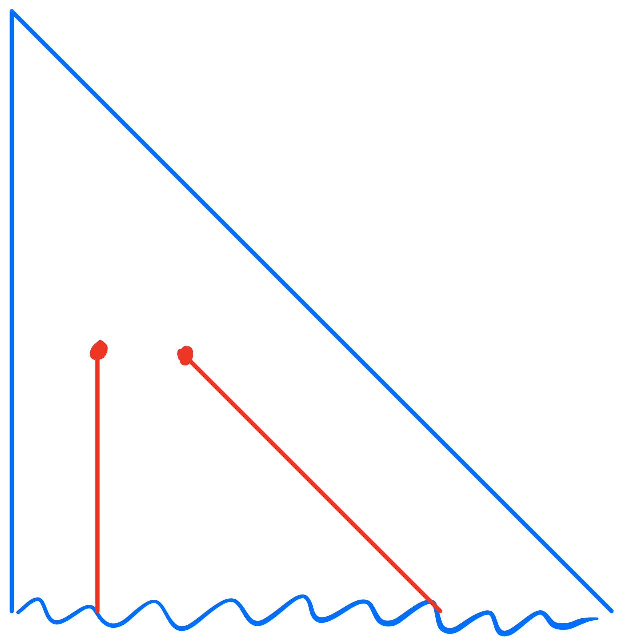
Singularity $\equiv$

incomplete timelike or null geodesic

in an inextendible spacetime M.

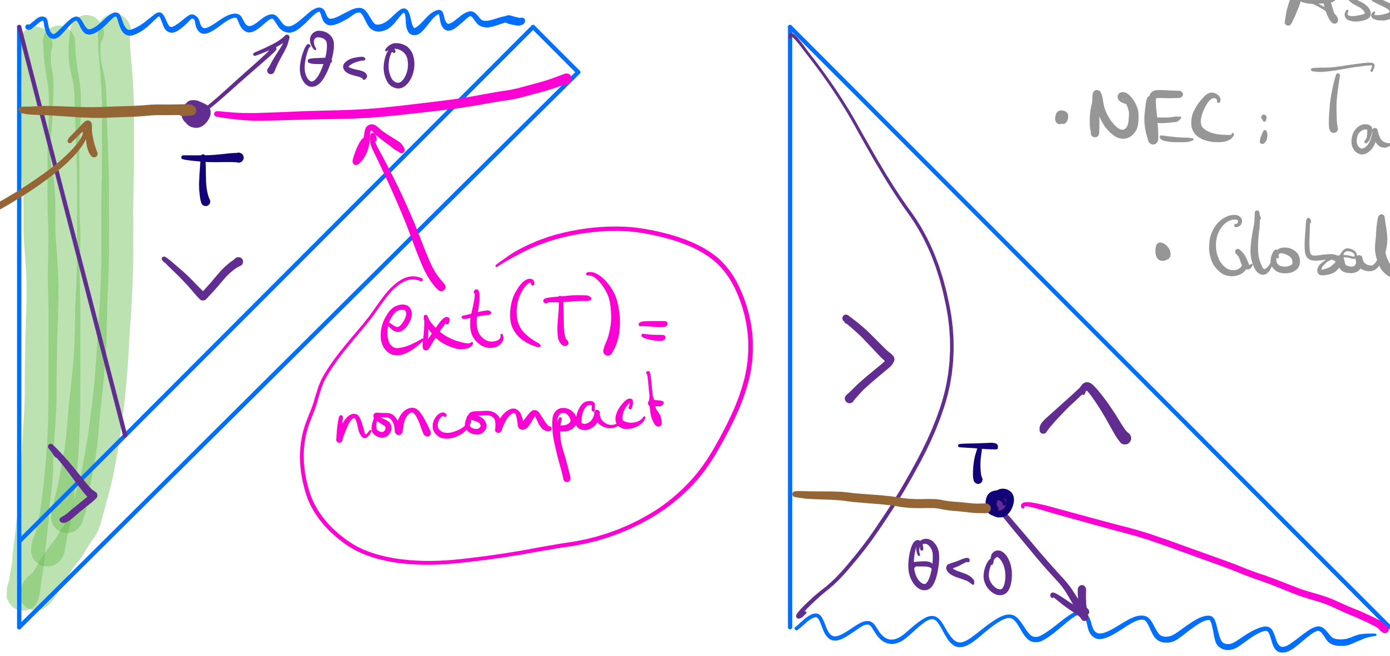




Penrose Singularity Theorem (1965)

Assume:

- NEC: $T_{ab}k^a k^b \geq 0$.
- Global hyperbolicity



T = outer-(anti)-trapped ← T = compact, ..., $\theta < 0$ in a spatially noncompact direction

$\Downarrow$

At least one orthogonal null geodesic = incomplete

Proof of Penrose Thm.: $L \equiv \dot{I}^+(\text{int}(T))$ is closed.

T = compact → rescale affine parameter λ so $\theta|_T = -1$.

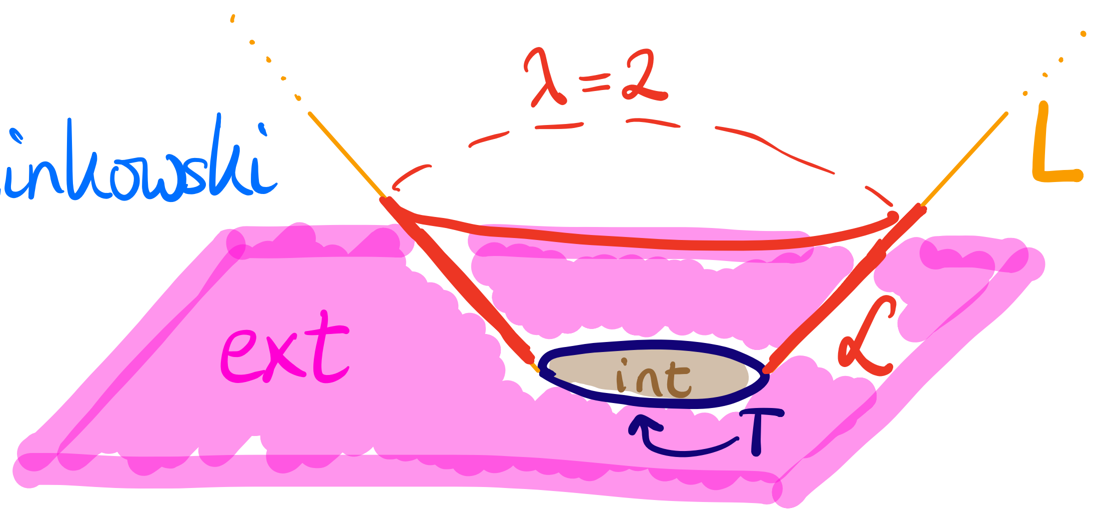
Assume all null geodesics complete → can define

$\mathcal{L}$ = {all points on geodesics with $\lambda \in [0,2]$}.

image of continuous map from compact set $T \times [0,2]$ into M, so $\mathcal{L}$ is compact.

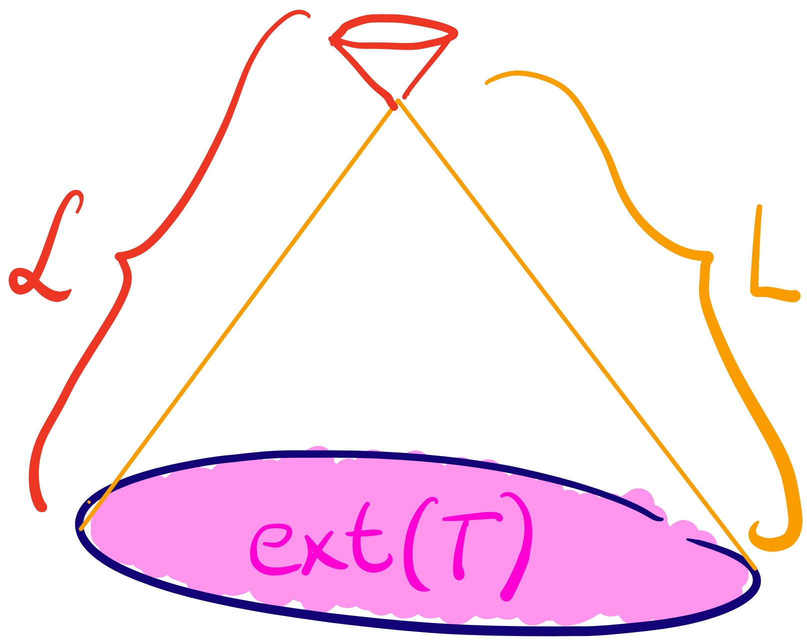
Example: T = sphere in Minkowski

But this has $\theta > 0$.



$\Theta|_T = -1$, so the NEC implies a conjugate point at or before $\lambda = 2$. Null geodesics leave L at or before such points. $\Rightarrow L \subset \mathcal{L} \Rightarrow L = \text{compact}$.

This implies*) that $\text{ext}(T)$ is compact ↯.



*) via a homeomorphism $L \leftrightarrow \text{ext}(T)$ defined by a smooth timelike congruence

Singularity Theorem for Hyperentropic Regions

Same as Penrose (1965), but replace

$$\mathrm{ext}(T) = \text{noncompact}$$

by the weaker*) assumption that

$$S(\mathrm{ext}(T)) > \frac{\mathrm{Area}(T)}{4G\hbar}$$

By $\theta|_T < 0$ and the NEC, L is a lightsheet

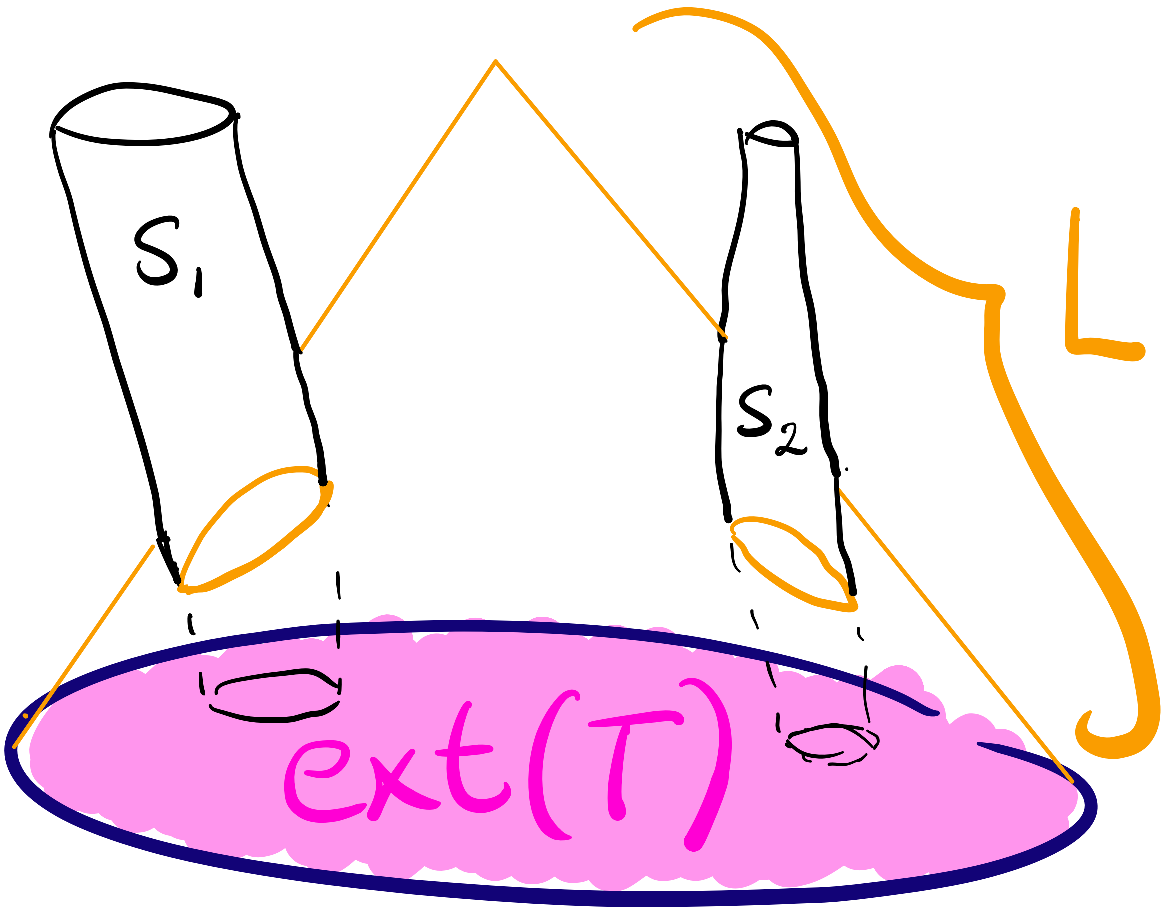
and the covariant entropy bound, $S(L) \leq \frac{A}{4G\hbar}$.

RB 1999

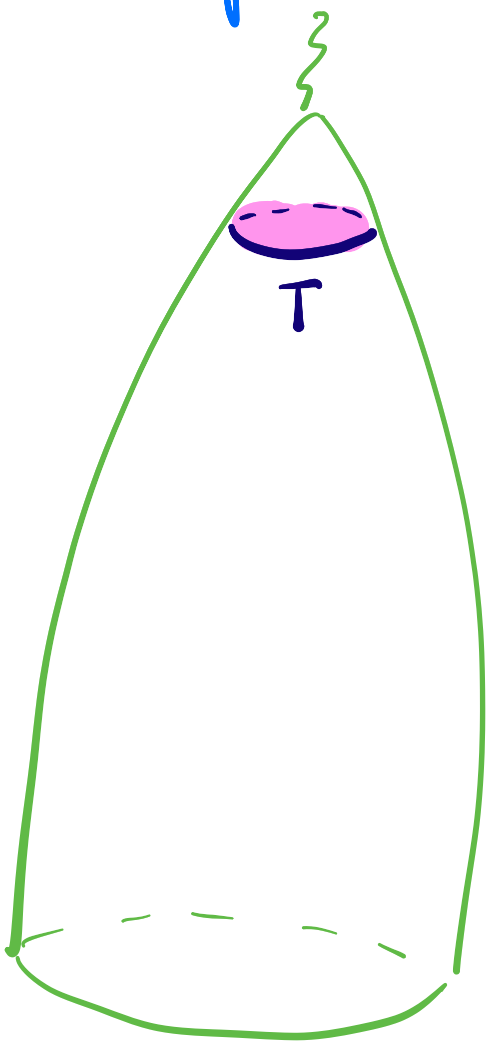
*) In a noncompact region, one can add arbitrarily large entropy with arbitrarily small backreaction.

Proof: by the previous proof, $D(L) = D(\mathrm{ext}(T))$.

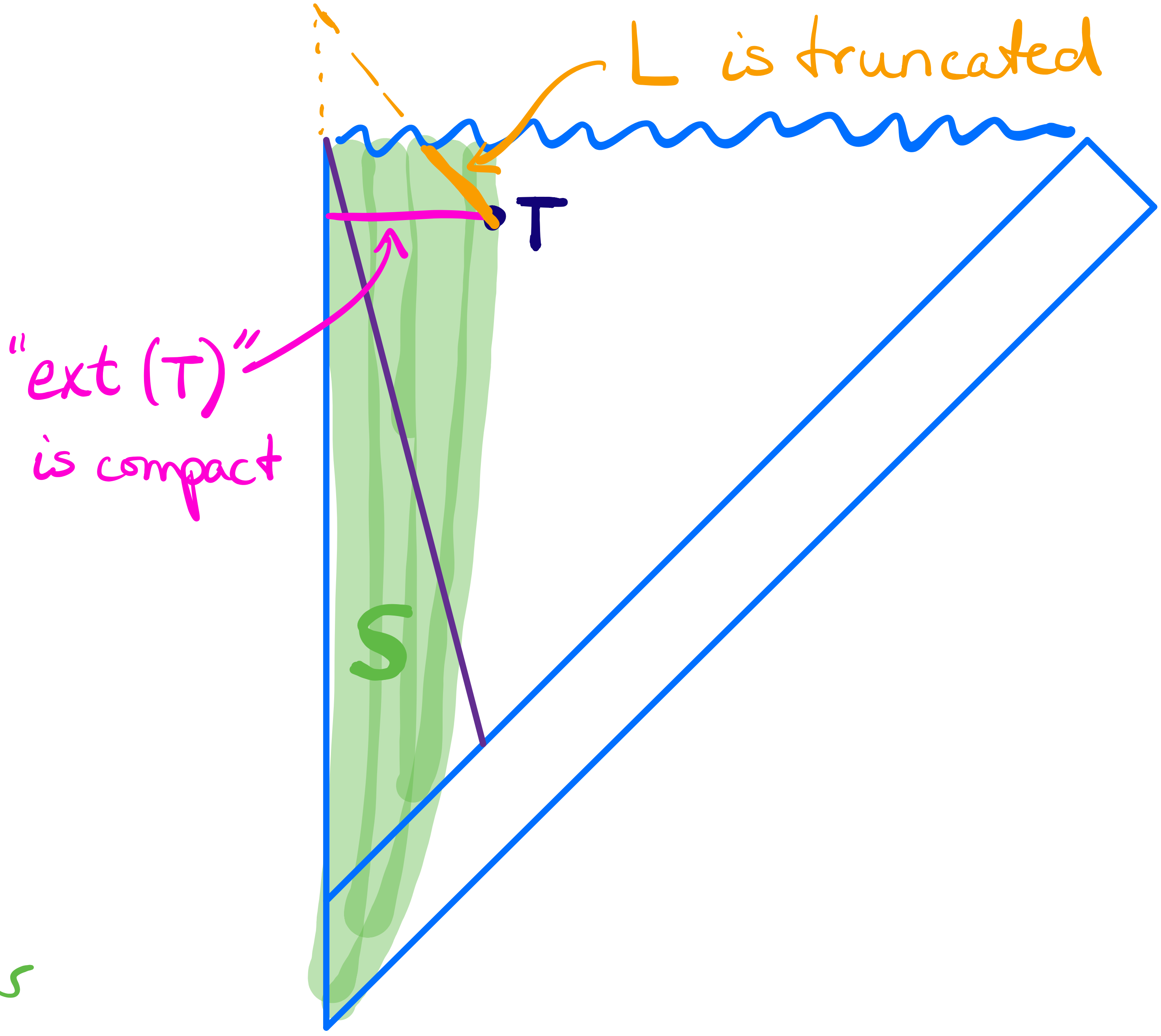
$$\Rightarrow S(L) = S(\mathrm{ext}(T)) > \frac{A}{4G\hbar} \quad \lightning \quad \square$$



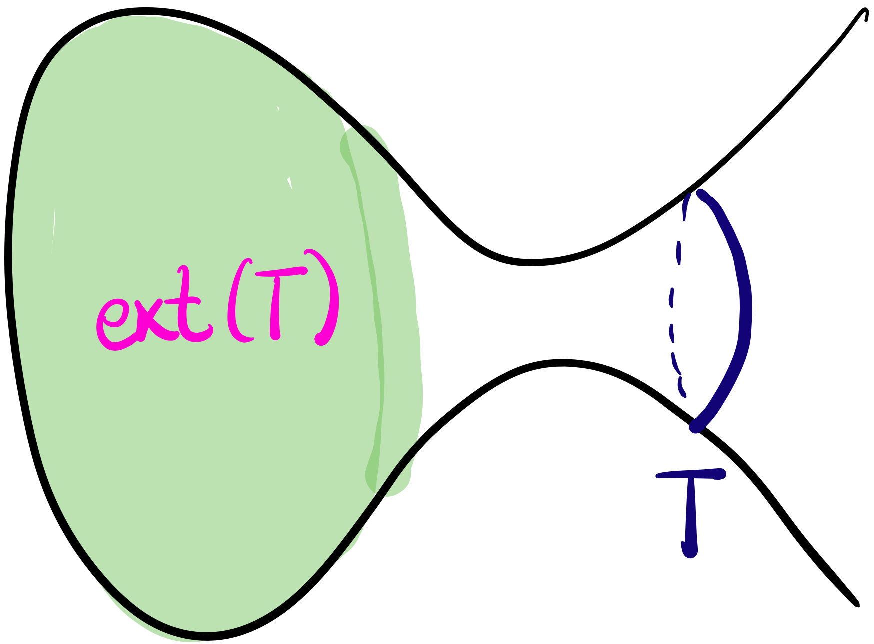
Examples:

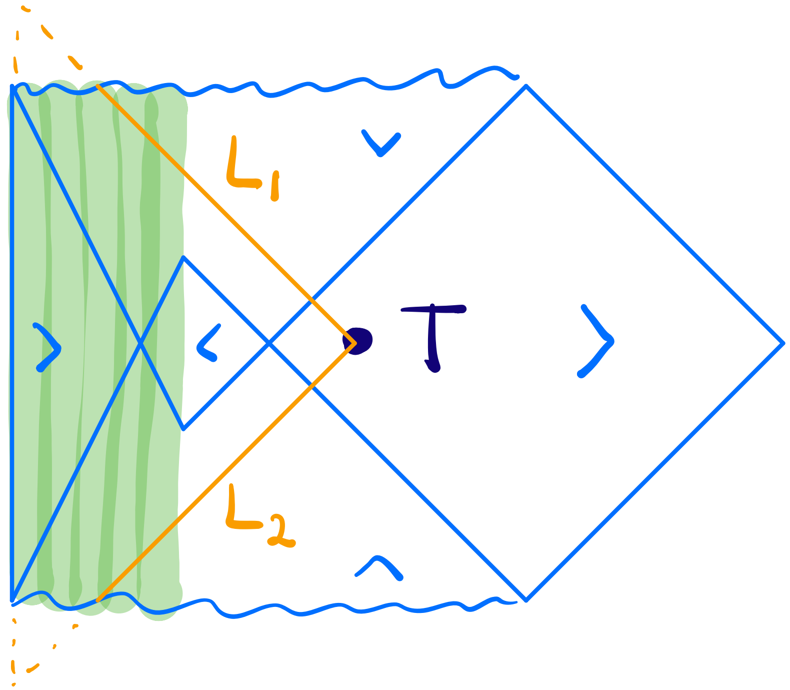


Collapsing star



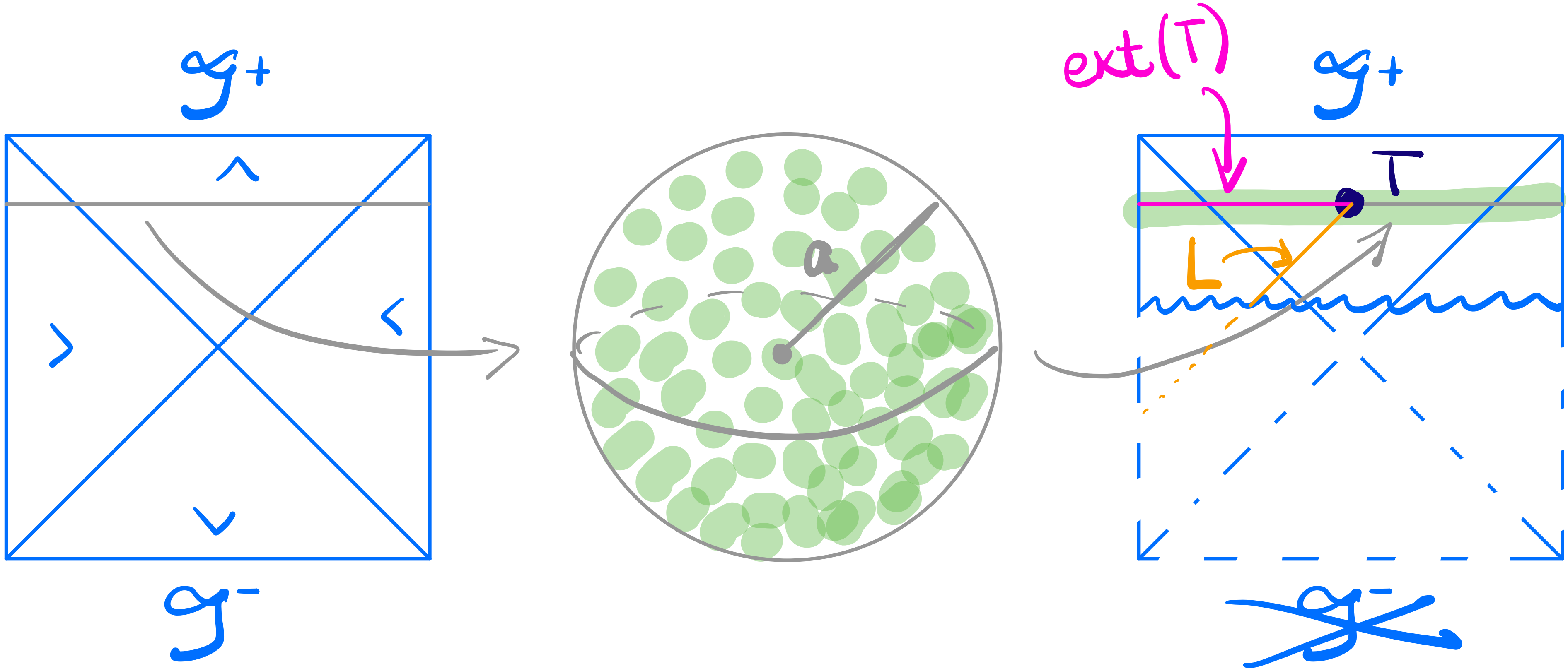
"Bag of Gold" / "Monster"





Note: T is a perfectly normal sphere → predict both past and future singularity.

de Sitter with almost no radiation



Add arbitrarily low-density radiation at late time

$\rightarrow S(\mathrm{ext}(T)) \sim \rho_{rad}^{3/4}\, a^3 > \mathrm{Area}(T) \sim a^2.$

Theorem implies a singularity in (distant!) past.

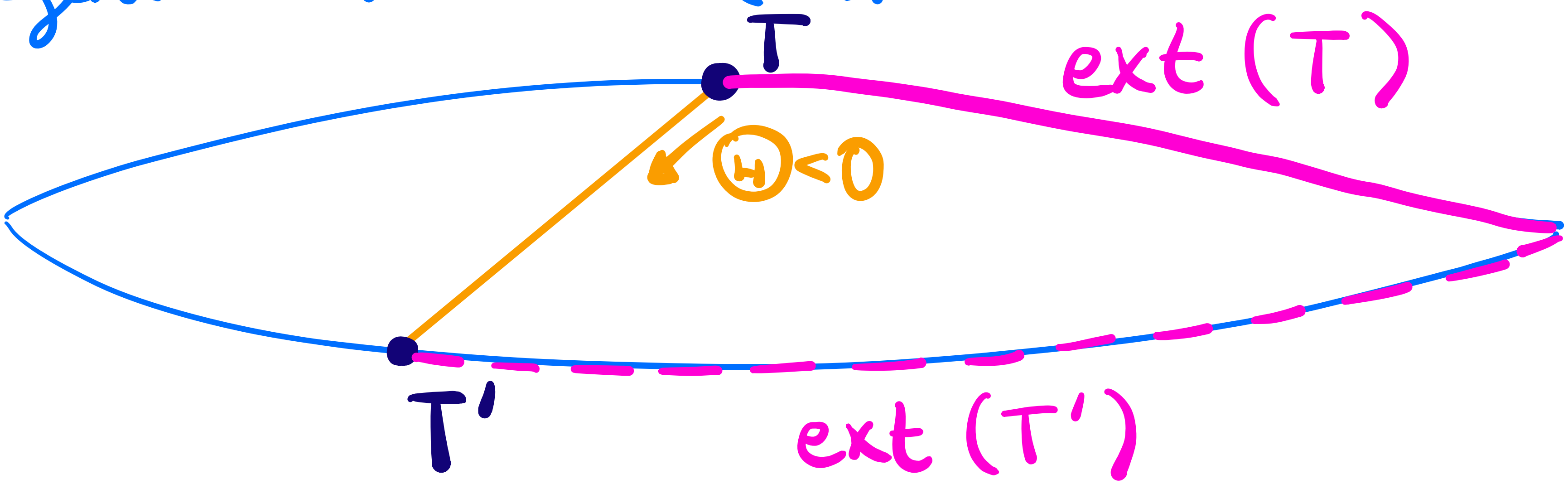
Quantum Covariant Entropy Bound

RB, Fisher, Leichenauer, Wall 2015

If $\Theta|_T \leq 0$ then $S_{gen}(\mathrm{ext}(T')) \leq S_{gen}(\mathrm{ext}(T))$.

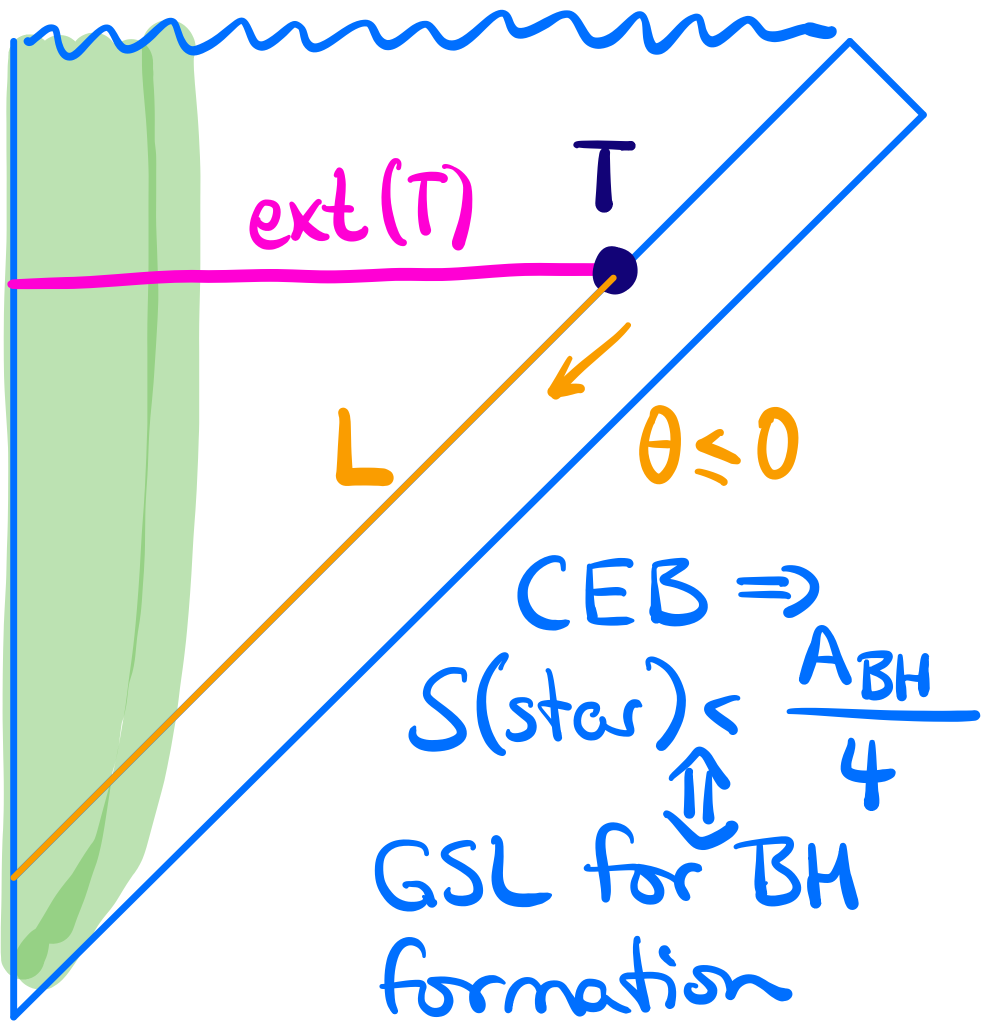
where

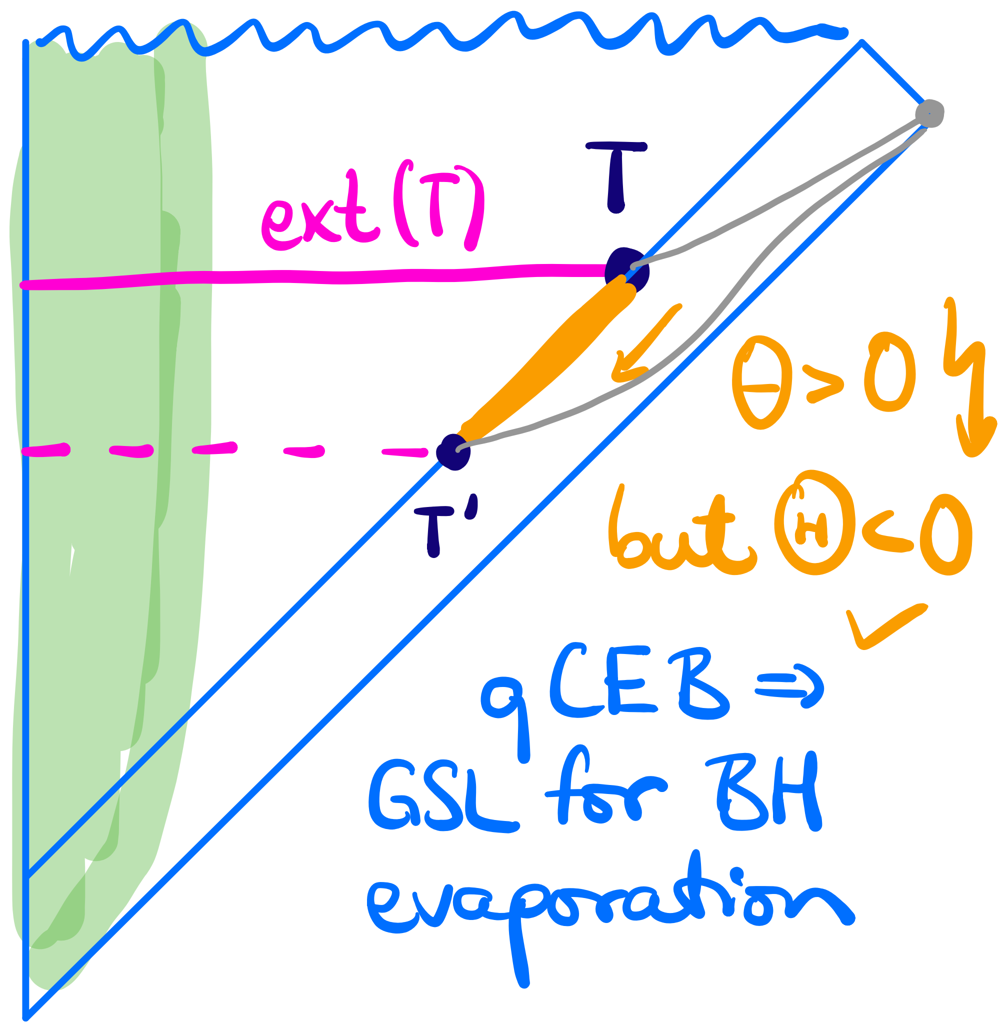
$$S_{gen}(\mathrm{ext}(T)) \equiv \mathrm{Area}(T)/4G\hbar + S(\mathrm{ext}(T))$$



and Θ is the shape derivative of $S_{gen}(\mathrm{ext}(T))$

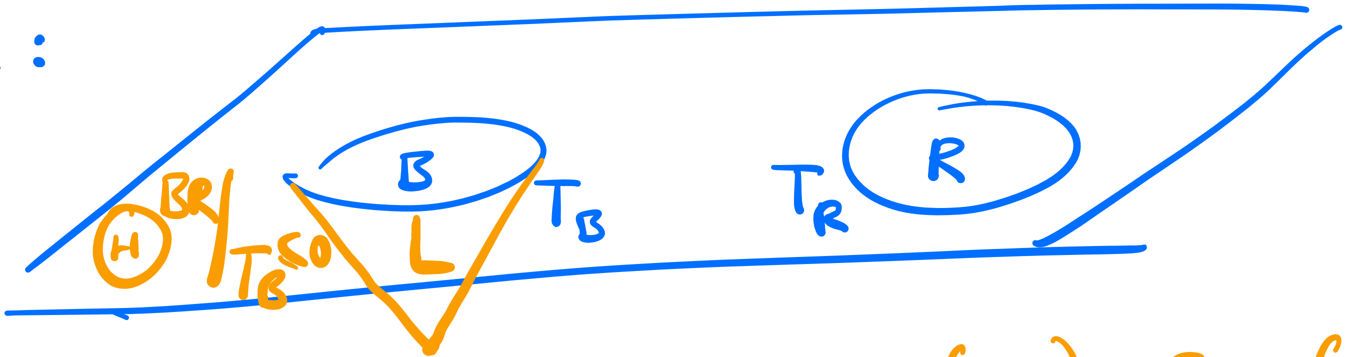
Why a quantum version? Applies when $\Theta \neq \theta$, for example:





For full generality, we can allow $\mathrm{ext}(T)$ to be disconnected:

$T = T_B \cup T_R$



If L is complete then qCEB $\Rightarrow S_{gen}(BR) \geq S_{gen}(R)$.

Conversely if $S_{gen}(BR) < S_{gen}(B) \Rightarrow$ singularity must cut off L. Proof: use an argument due to Wall (2011) to show that null geodesics cannot stay on L for infinite affine time (or else GSL is violated).

A puzzle:

After t_{Page}, $S_{gen}(BR) < S(R)$.

But there is no singularity!