The large N effective potential from large charge

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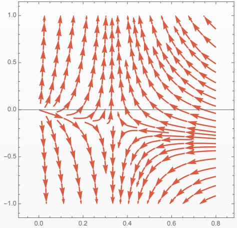
arXiv:1505.01537, arXiv:1610.04495, arXiv:1707.00711, arXiv:1804.01535, arXiv:1902.09542, arXiv:1905.00026,arXiv:1909.02571, arXiv:1909.08642, arXiv:2003.08396, arXiv:2005.03021, arXiv:2008.03308, arXiv:2010.07942, arXiv:2102.12488, arXiv:2103.05642 and more to come…

Who's who

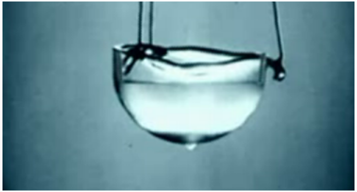
L. Álvarez Gaumé (SCGP and CERN);
D. Banerjee (Calcutta);
S. Chandrasekharan (Duke);
S. Hellerman (IPMU);
S. Reffert, N. Dondi, I. Kalogerakis , R. Moser, V. Pellizzani, T. Schmidt (AEC Bern);
F. Sannino (CP3-Origins and Napoli);
M. Watanabe (Weizmann).

Why are we here? Conformal field theories

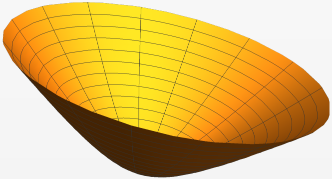
extrema of the RG flow



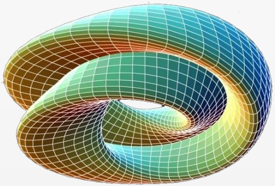
critical phenomena



quantum gravity



string theory



Why are we here? Conformal field theories are hard

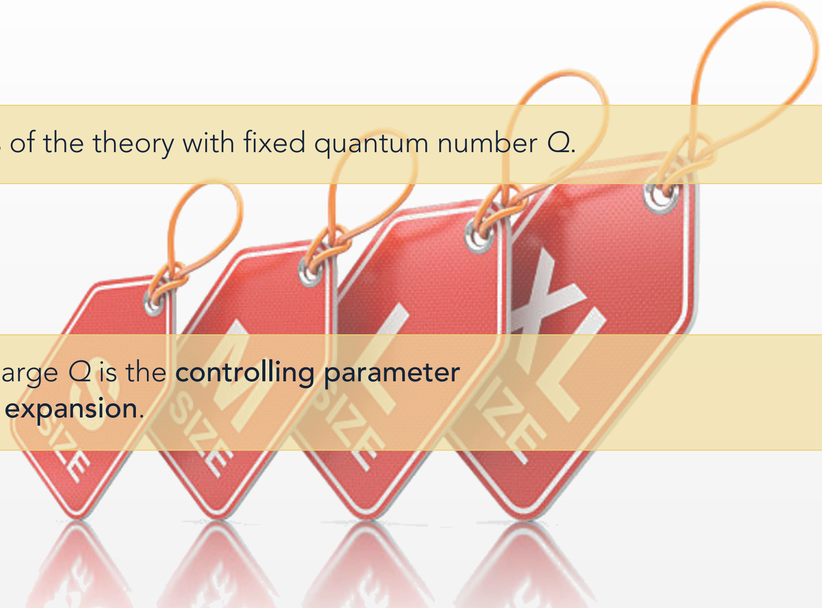
Most conformal field theories (CFTs) lack nice limits where they become simple and solvable.

No parameter of the theory can be dialed to a simplifying limit.

Why are we here? Conformal field theories are hard

In presence of a symmetry there can be sectors of the theory where anomalous dimension and OPE coefficients simplify.

The idea



Study **subsectors** of the theory with fixed quantum number Q.

In each sector, a large Q is the **controlling parameter** in a **perturbative expansion**.

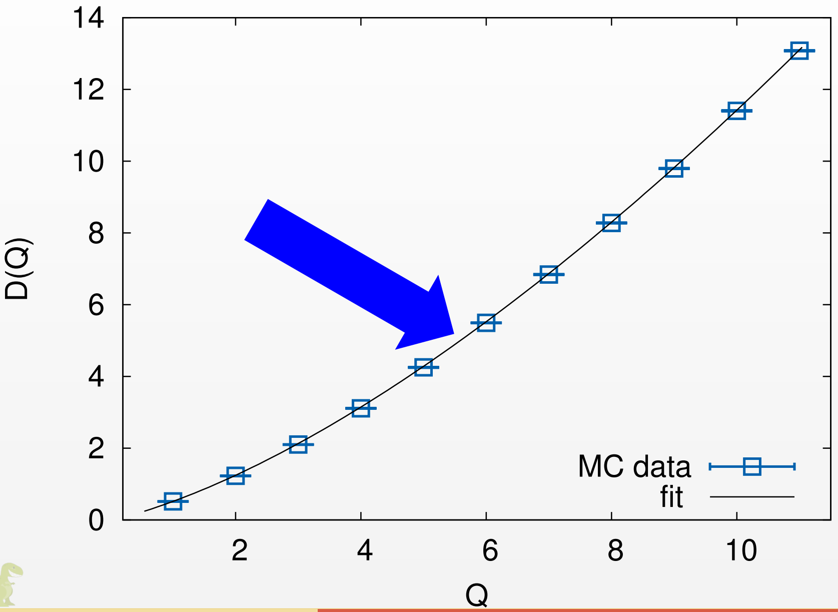
Concrete results

We consider the $O(N)$ vector model in three dimensions. In the IR it flows to a conformal fixed point [Wilson & Fisher].

We find an explicit formula for the dimension of the lowest primary at fixed charge:

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

Summary of the results: O(2)



Scales

We want to write a Wilsonian effective action.

Choose a cutoff Λ, separate the fields into high and low frequency ϕ_H, ϕ_L and do the path integral over the high-frequency part:

$$e^{iS_\Lambda(\phi_L)} = \int \mathscr{D}\phi_H \, e^{iS(\phi_H, \phi_L)}$$

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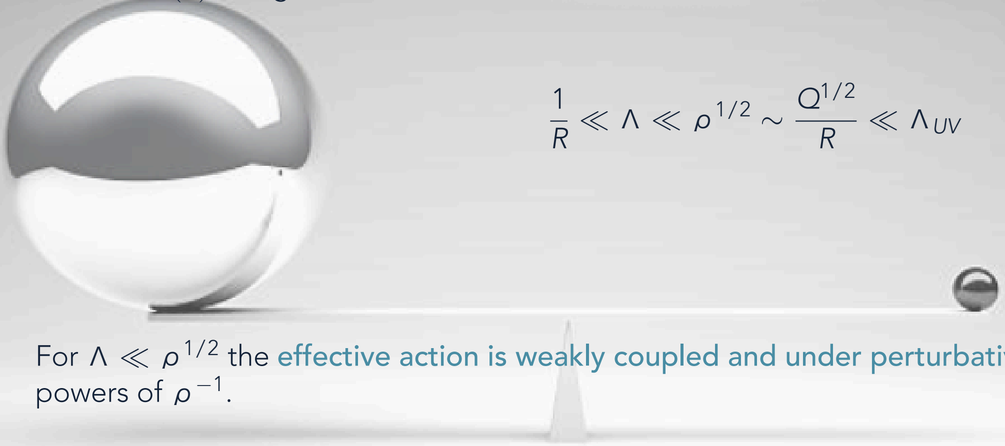
too hard

Scales

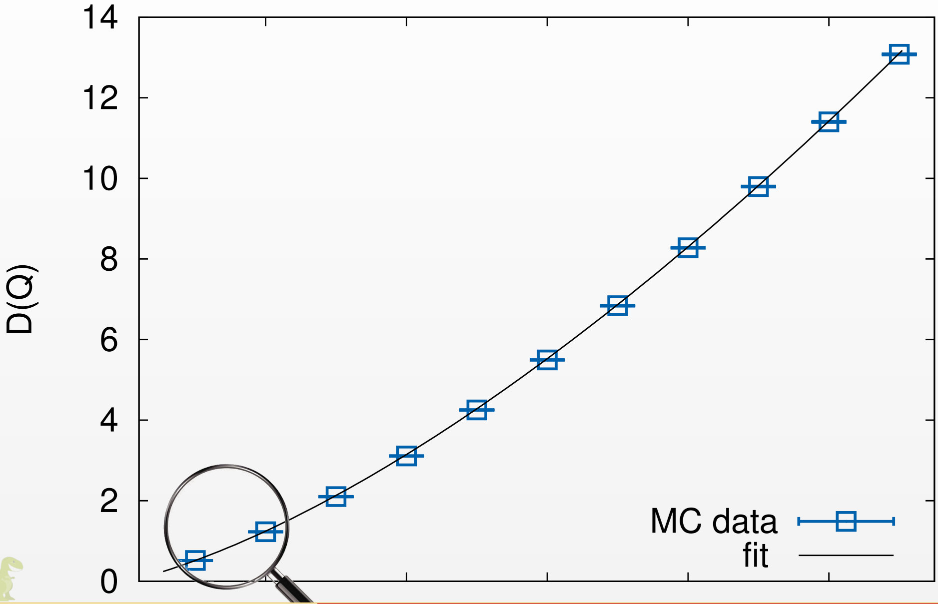
- We look at a finite box of typical length R
- The $U(1)$ charge Q fixes a second scale $\rho^{1/2} \sim Q^{1/2}/R$



$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

For $\Lambda \ll \rho^{1/2}$ the effective action is weakly coupled and under perturbative control in powers of ρ^{-1}.

Too good to be true?



Today's talk

The EFT for the $O(2)$ model in $2+1$ dimensions

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The EFT for the $O(2)$ model in $2+1$ dimensions

- An effective field theory (EFT) for a CFT.
- The physics at the saddle.
- State/operator correspondence for anomalous dimensions.

Today's talk

The EFT for the $O(2)$ model in $2+1$ dimensions

Justify and prove all my claims from first principles

- well-defined asymptotic expansion (in the technical sense)
- justify why the expansion works at small charge
- compute the coefficients in the effective action in large-N

Today's talk

The EFT for the $O(2)$ model in $2+1$ dimensions

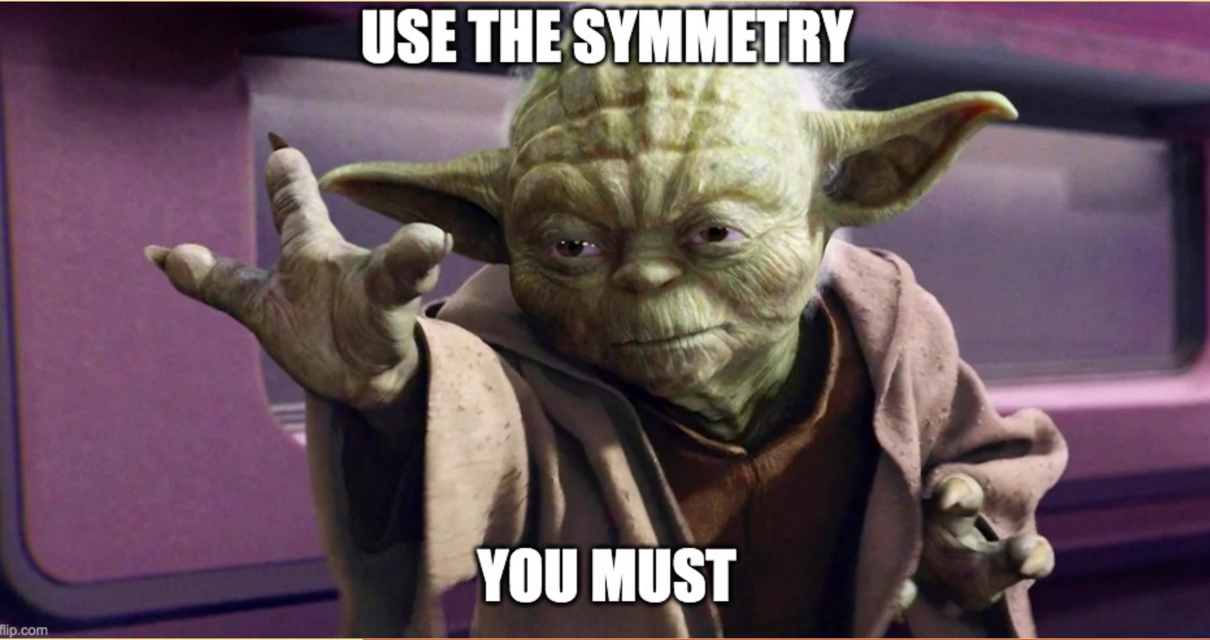
Justify and prove all my claims from first principles

Use large charge to write the effective potential

- Legendre transform, convexity and unitarity
- Complex CFT in $4 < D < 6$

PARENTAL
ADVISORY
EXPLICIT CONTENT

An EFT for a CFT



The $O(2)$ model

The simplest example is the Wilson–Fisher (WF) point of the $O(2)$ model in three dimensions.

- Non-trivial fixed point of the ϕ^4 action

$$L_{UV} = \partial_\mu \phi^* \partial_\mu \phi - u(\phi^* \phi)^2$$

- Strongly coupled
- In nature: ^4He.
- Simplest example of spontaneous symmetry breaking.
- Not accessible in perturbation theory. Not accessible in $4 - \varepsilon$. Not accessible in large N.
- Lattice. Bootstrap.

Charge fixing

We consider a subsector of fixed charge Q.
Generically, the classical solution at fixed charge breaks spontaneously $U(1) \to \emptyset$.

We have one Goldstone boson χ.

An action for χ

Start with two derivatives:

$$L[\chi] = \frac{f_\pi}{2} \partial_\mu \chi \, \partial_\mu \chi - C^3$$

(χ is a Goldstone so it is dimensionless.)

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We want to describe a CFT: we can **dress with a dilaton**

$$L[\sigma, \chi] = \frac{f_\pi e^{-2f\sigma}}{2} \partial_\mu \chi \, \partial_\mu \chi - e^{-6f\sigma} C^3 + \frac{e^{-2f\sigma}}{2} \left(\partial_\mu \sigma \, \partial_\mu \sigma - \frac{\xi R}{f^2} \right)$$

The fluctuations of χ give the Goldstone for the broken $U(1)$, the fluctuations of σ give the (massive) Goldstone for the broken conformal invariance.

Linear sigma model

We can put together the two fields as

$$\Sigma = \sigma + i f_\pi \chi$$

and rewrite the action in terms of a complex scalar

$$\varphi = \frac{1}{\sqrt{2f}} e^{-f\Sigma}$$

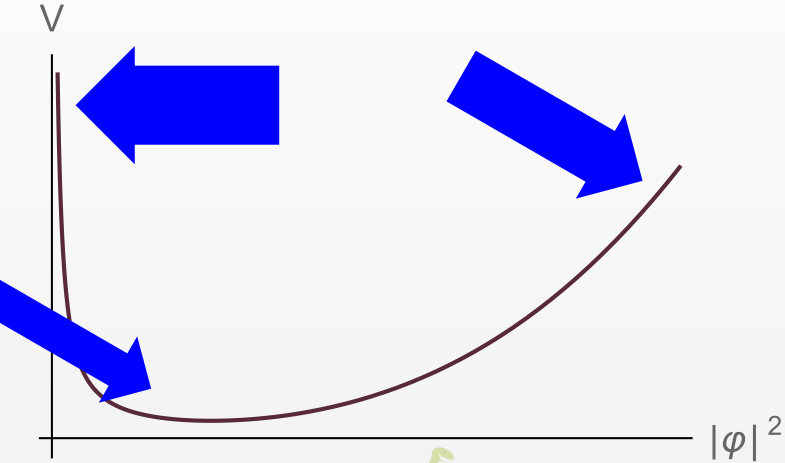
We get

$$L[\varphi] = \partial_\mu \varphi^* \partial^\mu \varphi - \xi R \varphi^* \varphi - u(\varphi^* \varphi)^3$$

Only depends on dimensionless quantities $b = f^2 f_\pi$ and $u = 3(Cf^2)^3$.
Scale invariance is manifest.
The field φ is some complicated function of the original ϕ.

Centrifugal barrier

The $O(2)$ symmetry acts as a shift on χ.
Fixing the charge is the same as adding a centrifugal term $\propto \frac{1}{|\varphi|^2}$.



Ground state

We can find a fixed-charge solution of the type

$$\chi(t,x) = \mu t \qquad \sigma(t,x) = \frac{1}{f}\log(v) = \text{const.},$$

where

$$\mu \propto Q^{1/2} + \dots \qquad v \propto \frac{1}{Q^{1/2}}$$

The classical energy is

$$E = c_{3/2} V Q^{3/2} + c_{1/2} R V Q^{1/2} + \mathcal{O}\left(Q^{-1/2}\right)$$

Fluctuations

The fluctuations over this ground state are described by two modes.

- A universal "conformal Goldstone". It comes from the breaking of the $U(1)$.

$$\omega = \frac{1}{\sqrt{2}} p$$

- The massive dilaton. It controls the magnitude of the quantum fluctuations. All quantum effects are controled by $1/Q$.

$$\omega = 2\mu + \frac{p^2}{2\mu}$$

(This is a heavy fluctuation around the semiclassical state. It has nothing to do with a light dilaton in the full theory)

Non-linear sigma model

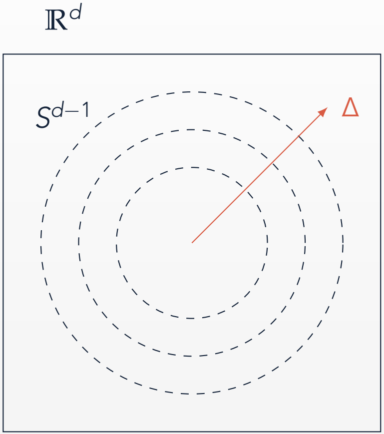
Since σ is heavy we can integrate it out and write a non-linear sigma model (NLSM) for χ alone.

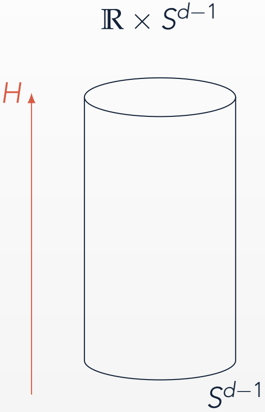
$$L[\chi] = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \, \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$.
All other terms are suppressed by powers of $1/Q$.

State-operator correspondence

The anomalous dimension on $\mathbb{R}^d$ is the energy in the cylinder frame.





Protected by conformal invariance: a well-defined quantity.

Conformal dimensions

We know the energy of the ground state.
The leading quantum effect is the Casimir energy of the conformal Goldstone.

$$E_G = \frac{1}{2\sqrt{2}} \zeta\left(-\tfrac{1}{2}|S^2\right) = -0.0937\ldots$$

This is the unique contribution of order Q^0.

Final result: the conformal dimension of the lowest operator of charge Q in the $O(2)$ model has the form

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

What happened?

We started from a CFT.
There is no mass gap, there are no particles, there is no Lagrangian.

We picked a sector.

In this sector the physics is described by a semiclassical configuration plus massless fluctuations.

The full theory has no small parameters but we can study this sector with a simple EFT.
We are in a strongly coupled regime but we can compute physical observables using perturbation theory.

would you like to know more?

Large N vs. Large Charge

The model

ϕ^4 model on $\mathbb{R} \times \Sigma$ for N complex fields

$$S_\theta[\varphi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\, \mathrm{d}\Sigma \left[g^{\mu\nu} (\partial_\mu \varphi_i)^* (\partial_\nu \varphi_i) + r \varphi_i^* \varphi_i + \frac{u}{2} (\varphi_i^* \varphi_i)^2 \right]$$

It flows to the WF in the IR limit $u \to \infty$ when r is fine-tuned to $R/8$.
We compute the partition function at fixed charge

$$Z(Q_1, \dots, Q_N) = \operatorname{Tr}\left[e^{-\beta H} \prod_{i=1}^{N} \delta(\hat{Q}_i - Q_i) \right]$$

where

$$\hat{Q}_i = \int \mathrm{d}\Sigma\, j_i^0 = i \int \mathrm{d}\Sigma\, [\dot{\varphi}_i^* \varphi_i - \varphi_i^* \dot{\varphi}_i].$$

Dimensions of operators of fixed charge Q on $\mathbb{R}^3$ (state/operator):

$$\Delta(Q) = -\frac{1}{\beta} \log Z_{S^2}(Q).$$

Fix the charge

Explicitly

$$Z = \int_{-\pi}^{\pi} \prod_{i=1}^{N} \frac{d\theta_i}{2\pi} \prod_{i=1}^{N} e^{i\theta_i Q_i} \operatorname{Tr}\left[e^{-\beta H} \prod_{i=1}^{N} e^{-i\theta_i \hat{Q}_i} \right].$$

Since $\hat{Q}$ depends on the momenta, the integration is not trivial but well understood.

$$\begin{aligned} Z_\Sigma(Q) &= \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} \int_{\varphi(2\pi\beta)=e^{i\theta}\varphi(0)} D\varphi_i \, e^{-S[\varphi]} \\ &= \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} \int_{\varphi(2\pi\beta)=\varphi(0)} D\varphi_i \, e^{-S^\theta[\varphi]} \end{aligned}$$

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$$= \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} \int_{\varphi(2\pi\beta)=\varphi(0)} D\varphi_i\, e^{-S^\theta[\varphi]}$$

Effective actions

The covariant derivative approach:

$$S^{\theta}[\varphi] = \sum_{i=1}^{N} \int \mathrm{d}t\,\mathrm{d}\Sigma \left((D_{\mu}\varphi_i)^* (D^{\mu}\varphi_i) + \frac{R}{8}\varphi_i^*\varphi_i + 2u(\varphi_i^*\varphi_i)^2 \right)$$

where

$$\begin{cases} D_0\varphi &= \partial_0\varphi + i\frac{\theta}{\beta}\varphi \\ D_i\varphi &= \partial_i\varphi \end{cases}$$

Stratonovich transformation: introduce Lagrange multiplier λ and rewrite the action as

$$S_Q = \sum_{i=1}^{N} \left[-i\theta_i Q_i + \int \mathrm{d}t\,\mathrm{d}\Sigma \left[\left(D^i_{\mu}\varphi_i\right)^* \left(D^i_{\mu}\varphi_i\right) + \left(\frac{R}{8} + \lambda\right)\varphi_i^*\varphi_i \right] \right]$$

Expand around the VEV

$$\varphi_i = \frac{1}{\sqrt{2}}A_i + u_i, \qquad \lambda = \left(\mu^2 - \frac{R}{8}\right) + \hat{\lambda}$$

Saddle point equations

The integral over the φ is Gaussian.
We can perform it, *e.g.* in terms of zeta functions.

$$\zeta(s|\Sigma,\mu) = \mathrm{Tr}\left((\nabla^2_\Sigma - \mu^2)^{-s}\right)$$

With some massaging, we find the final equations

$$\begin{cases} F_\Sigma(Q) = \mu Q + N\zeta(-\tfrac{1}{2}|\Sigma,\mu) = \mu Q - \omega(\mu), \\ \mu\,\zeta(\tfrac{1}{2}|\Sigma,\mu) = -\frac{Q}{N}. \end{cases}$$

The control parameter is actually Q/N.
The free energy $F(q)$ is the Legendre transform of the grand potential $\omega(\mu)$.

Large Q/N

If $Q/N \gg 1$ we can use Weyl's asymptotic expansion.

$$\mathrm{Tr}\left(e^{\Delta_\Sigma t}\right) = \sum_{n=0}^{\infty} K_n t^{n/2-1}.$$

The zeta function is written in terms of the geometry of Σ (heat kernel coefficients)

$$\mu_\Sigma = \sqrt{\frac{4\pi}{V}}\left(\frac{Q}{2N}\right)^{1/2} + \frac{R}{24}\sqrt{\frac{V}{4\pi}}\left(\frac{Q}{2N}\right)^{-1/2} + \dots$$

$$\frac{F_\Sigma}{2N} = \frac{2}{3}\sqrt{\frac{4\pi}{V}}\left(\frac{Q}{2N}\right)^{3/2} + \frac{R}{12}\sqrt{\frac{V}{4\pi}}\left(\frac{Q}{2N}\right)^{1/2} + \dots$$

Order N

$$
\begin{aligned}
F_{S^2}(Q) = {} & \frac{4N}{3}\left(\frac{Q}{2N}\right)^{3/2} + \frac{N}{3}\left(\frac{Q}{2N}\right)^{1/2} \\
& - \frac{7N}{360}\left(\frac{Q}{2N}\right)^{-1/2} - \frac{71N}{90720}\left(\frac{Q}{2N}\right)^{-3/2} + \mathcal{O}\left(e^{-\sqrt{Q/(2N)}}\right)
\end{aligned}
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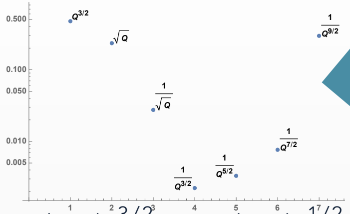

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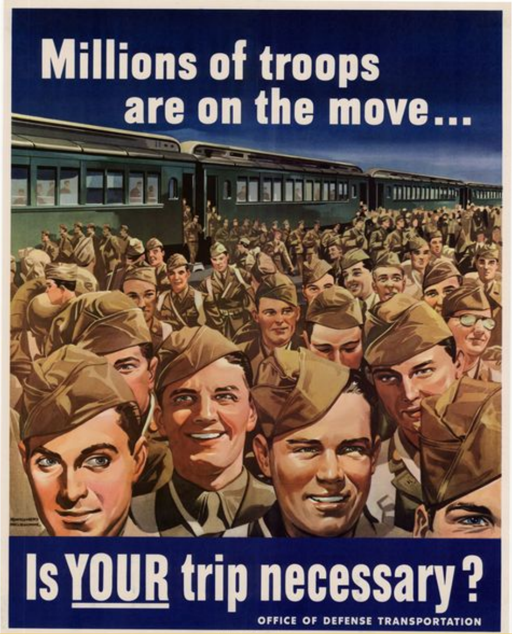
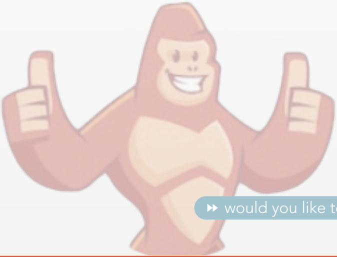
Was it worth it?



Final result

$$\Delta(Q) = \left(\frac{4N}{3} + \mathcal{O}(1)\right)\left(\frac{Q}{2N}\right)^{3/2} + \left(\frac{N}{3} + \mathcal{O}(1)\right)\left(\frac{Q}{2N}\right)^{1/2} + \dots - 0.0937\dots$$



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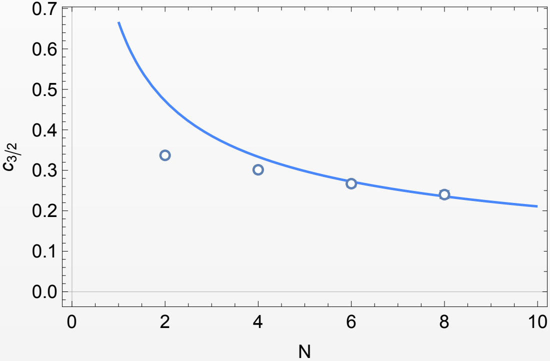

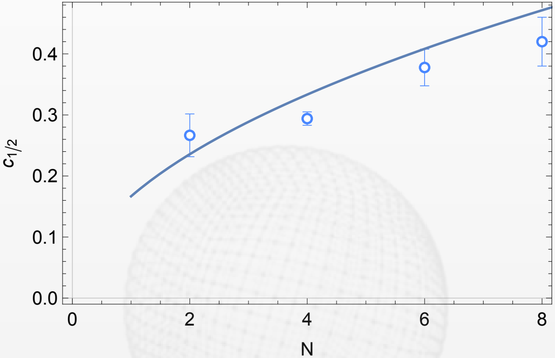

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The Effective Potential at Large N

The grand potential

The large-N computation can be (formally) generalized to any dimension.
We can also keep track of the quadratic and quartic term explicitly

$$r|\phi|^2 + \frac{u}{2N}|\phi|^4.$$

Grand potential:

$$\omega(\mu) = -N\zeta(-1/2 \mid \Sigma, \mu) + \frac{(\mu^2 - r)^2}{4u}$$

This function contains all the information about the large-N limit of the vector model.

Effective potential and large charge

Assume that the large-N physics is described by an effective action

$$\Gamma[\phi_c] = \frac{1}{2}\partial_\mu \phi_c^* \partial_\mu \phi_c - V(\phi_c).$$

This must describe in particular the fixed-charge sector.

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By now we know that the fixed-charge ground state takes the form $\phi_c = \Phi e^{i\mu t}$.
Solve the problem in two steps: first eliminate the radial model Φ and get an effective action for μ alone:

$$\begin{cases} \frac{d}{d(\Phi^2)}\left[\mu^2\Phi^2 - V(\Phi)\right] = \mu^2 - \frac{dV}{d(\Phi^2)} = 0, \\ \omega(\mu) = \Phi^2\mu^2 - V(\Phi)\big|_{\Phi=\Phi(\mu)} . \end{cases}$$

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Crucial: The grand potential $\omega(\mu^2)$ is the the Legendre transform of the effective potential $V(\phi_c)$ as function of ϕ_c^2.

Effective potential as a Legendre transform

For convex functions the Legendre transform is an involution.
The effective potential is the Legendre transform of the grand potential

$$V(\phi_c^2) = \omega^*(\phi_c^2)$$

In a unitary theory $V(\phi_c)$ is convex: we have a necessary condition on $\omega(\mu)$.

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Convexity suggests a generalization:

$$V(\phi_c^2) = \omega^*(\phi_c^2) = \sup_{\mu^2}(\mu^2 \phi_c^2 - \omega(\mu^2))$$

which agrees with the naive definition as long as $\phi_c^2 = \omega'(\mu^2)$ admits real solutions.

Effective potential in three dimensions

In $D = 3$ in flat space

$$\omega(\mu) = (2N)\left(\frac{\mu^3}{12\pi} + \frac{(\mu^2 - r)^2}{4u}\right)$$

and we derive directly

$$V(\phi_c) = \frac{Nu^3}{3 \times 2^{10}\pi^4}\left(1 + 96\pi^2\left(\frac{\phi_c^2}{Nu} + \frac{r}{u^2}\right) + 1536\pi^4\left(\frac{\phi_c^2}{Nu} + \frac{r}{u^2}\right)^2 - \left(1 + 64\pi^2\left(\frac{\phi_c^2}{Nu} + \frac{r}{u^2}\right)\right)^{3/2}\right).$$

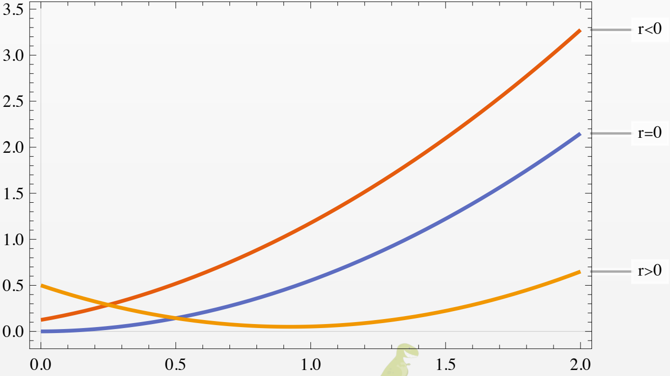
Computed a long time ago at the critical point $r = 0$ [Appelquist and Heinz].

Maxwell's rule

Convexity is guaranteed if we use the convex conjugate:

$$V(\phi_c^2) = \sup_{\mu^2}(\phi_c^2\mu^2 - \omega(\mu^2))$$

If the derivative of the grand potential has a minimum greater than zero, there are regions where the transform is a constant. This is known as Maxwell rule.



Maxwell's rule

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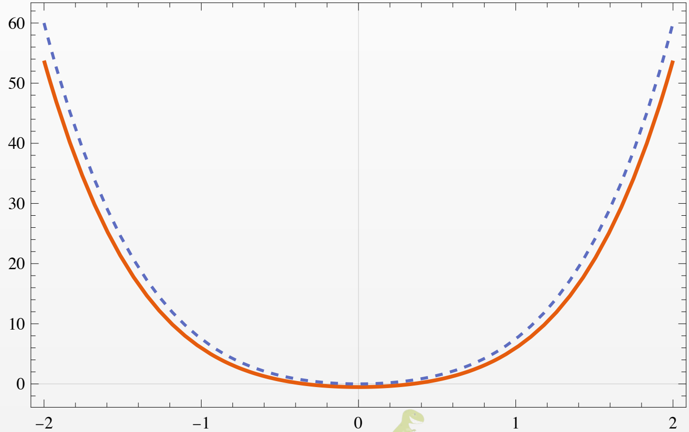
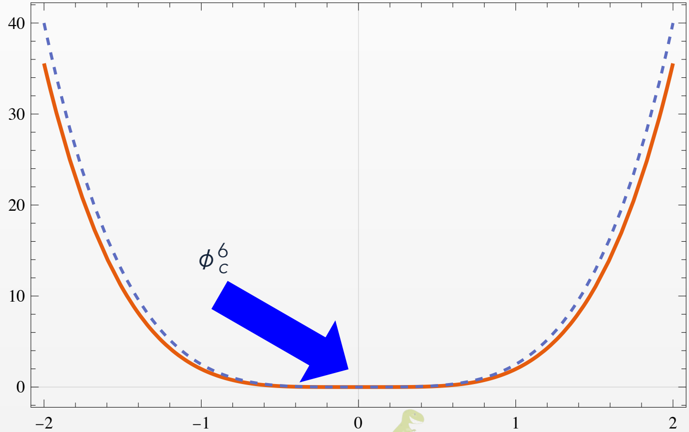


Unbroken phase $r > 0$

Maxwell's rule

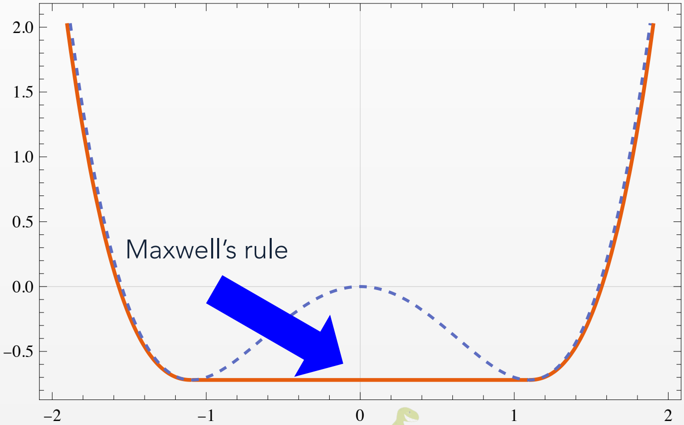
$$V(\phi_c^2) = \sup_{\mu^2}(\phi_c^2\mu^2 - \omega(\mu^2))$$



Critical phase $r = 0$

Maxwell's rule

$$V(\phi_c^2) = \sup_{\mu^2}(\phi_c^2\mu^2 - \omega(\mu^2))$$



Broken phase $r < 0$

D=5

In $D = 5$ the sign of the leading term in the grand potential is negative

$$\omega(\mu) = (2N)\left(-\frac{\mu^5}{120\pi^2} + \frac{(\mu^2 - r)^2}{4u}\right)$$

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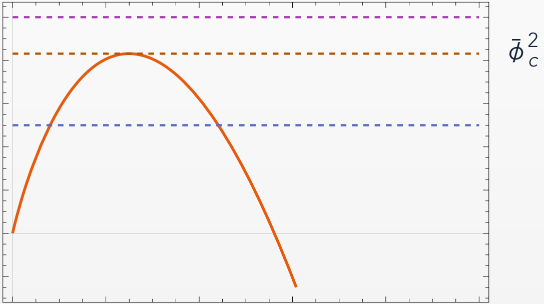
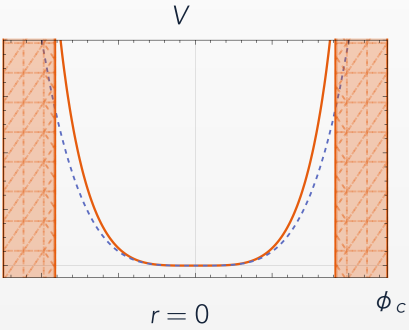


A problem in the UV

If we use the supremum definition, the effective potential is literally infinite for

$$\phi_c > \bar{\phi}_c = \sqrt{\frac{N}{3}\frac{(4\pi)^2}{u^{3/2}}}\sqrt{1-\frac{3ru^2}{(4\pi)^4}},$$



In the $u \to \infty$ limit the box vanishes. The theory is not UV complete.

A complex completion

We can pick a complex solution for the maximization condition $\phi_c^2 = \omega'(\mu^2)$.
The value $\phi_c = \bar{\phi}_c$ is a branch point and we have five branches.
For example

$$V(\phi_c) = \frac{12}{5}\left(\frac{3\pi^2}{n}\right)^{2/3} e^{-2\pi i/3}\phi^{10/3}\Bigg(1 + 10 e^{\pi i/3}\left(\frac{n\pi^4}{3}\right)^{1/3}\frac{1}{\phi^{2/3}u} + 80 e^{2\pi i/3}\left(\frac{n\pi^4}{3}\right)^{2/3}\frac{1}{\phi^{4/3}u^2} - \frac{1600 n\pi^4}{9\phi^2 u^3} + \mathcal{O}\left(u^{-4}\right)\Bigg).$$

From here we can compute the CFT data, which is in agreement with the literature.

$$\Delta(Q) = r_0 F_{S^4}(Q) = 2N\left[e^{i\pi/4}\frac{4\sqrt{3}}{5}\left(\frac{Q}{2N}\right)^{\frac{5}{4}} - \frac{e^{3i\pi/4}}{\sqrt{3}}\left(\frac{Q}{2N}\right)^{\frac{3}{4}}\right],$$

What happened?

- We have written the large-N grand potential for finite values of the couplings to ϕ^2 and ϕ^4.
- We have derived the effective potential as a Legendre transform. No Feynman diagrams.
- In $2 < D < 4$ convexity leads to Maxwell's rule.
- In $4 < D < 6$ we find a UV completion in terms of a complex CFT.

Conclusions

Conclusions

- With the large-charge approach we can study strongly-coupled systems perturbatively.
- Select a sector and we write a controllable effective theory.
- The strongly-coupled physics is (for the most part) subsumed in a semiclassical state.
- Qual(nt)itative control of the non-pertubative effects.
- Compute the CFT data.
- Very good agreement with lattice (supersymmetry, large N).
- Precise and testable predictions.
- We have started the exploration of the whole phase diagram.