Non-perturbative Cosmological Bootstrap

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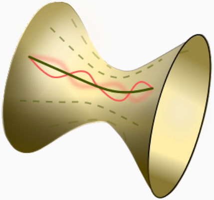
October 14, 2021

Based on ArXiv:2107.13871 with Matthijs Hogervorst and João Penedones



EPFL

Motivation

- dS is the simplest model of expanding universe.
  A good approximation of our universe in early times and now!
- First step: to study QFT in fixed (non-dynamical) background
- Several studies of this subject including recent developments:
  - Cosmological bootstrap: perturbative
    [Arkani-Hamed, Baumann, Lee, Pimentel] [Pajer, Stefanyszyn, Supel] [Goodhew, Jazayeri, Pajer]
  - Dictionary between dS and EAdS diagrams
    [Sleight, Taronna] [Di Pietro, Gorbenko, Komatsu]

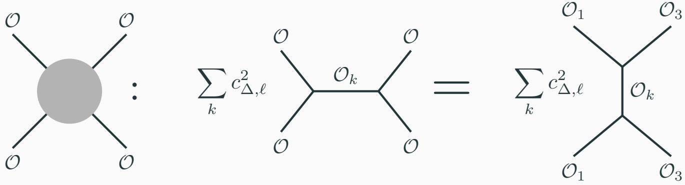
Goal

- Bootstrap approach: Study the implications of general properties such as symmetry and unitarity

- Conformal bootstrap:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(x_1,\ldots,x_4)$$

  with $G_{\Delta,\ell}$ known and $c^2_{\Delta,\ell} \geq 0$.



Crossing symmetry: expanding the four point function in different channels leads to bounds on CFT data (e.g. determination of 3d Ising model critical exponents)

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with $G_{\Delta,\ell}$ known and $c^2_{\Delta,\ell} \geq 0$.

- Non-perturbative cosmological bootstrap:
$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \sum_{\ell} \int d\nu \; I_{\nu,\ell} \Psi_{\nu,\ell}(x_1,\ldots,x_4)$$
with $\Psi_{\nu,\ell}$ known and $I_{\nu,\ell} \geq 0$.

dS$_{d+1}$ spacetime

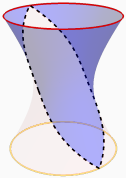
- A hyperboloid in $\mathbb{M}^{d+2}$:

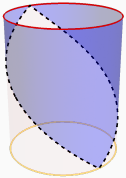
$$-{X^0}^2 + {X^1}^2 + \cdots + {X^{d+1}}^2 = R^2$$

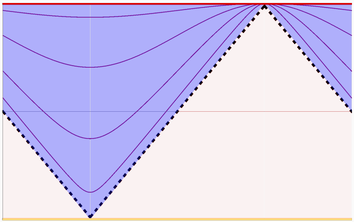
  Maximally symmetric with constant positive curvature

- Conformal coordinates (flat slicing):

$$ds^2 = R^2 \frac{-d\eta^2 + d\vec{x}^2}{\eta^2} \ , \quad \eta \in (-\infty, 0) \ , \quad \vec{x} \in \mathbb{R}^n$$







- Symmetry group of dS$_{d+1}$: $SO(d+1,1)$; Conformal algebra

Representation theory

- Unitary irrep of dS isometry group: infinite dimensional labelled by quantum numbers ℓ (spin) and Δ (scaling dimension):
  - *Principal series:* $\Delta = \frac{d}{2} + i\nu, \ \nu \in \mathbb{R}$
    (in massive scalar: $mR \geq \frac{d}{2}$ - heavy field)
  - *Complementary series:* $\Delta = \frac{d}{2} + c$
    (in massive scalar: $mR \leq \frac{d}{2}$ - light field)
  - *Discrete series:* $\Delta \in$ (half-)Integer
- In flat slicing, states within a multiplet take the form: $|\Delta, \vec{x}\rangle_A$ and transform like primary operators under dS symmetry generators $(\ell = 0)$:

$$P_\mu |\Delta, \vec{x}\rangle = i\partial_\mu |\Delta, \vec{x}\rangle \ , \quad D|\Delta, \vec{x}\rangle = (x.\partial + \Delta)|\Delta, \vec{x}\rangle \ , \quad \cdots$$

- Resolution of identity-completeness:

$$\mathbb{1} = \sum_\ell \int_\Delta \int_x |\Delta, \vec{x}\rangle \langle \Delta, \vec{x}|$$

Boundary operators

- Boundary operators:
$$\phi(\eta,\vec{x}) = \sum_k b_{\phi k}(-\eta)^{\Delta_k}(\mathcal{O}_k(\vec{x}) + \text{des})$$
where $\{\Delta_k\}$ is unrelated to unitary irreps of dS.

- The action of dS isometries on $\mathcal{O}_k$ at $\eta = 0 \Longrightarrow$ conformal theory

- Hermiticity implies existence of both Δ_k and Δ_k^*

- No state-operator correspondence unlike AdS

Källén–Lehmann decomposition

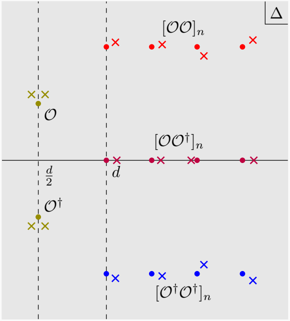
$$G_2(\xi) = \langle \phi(\eta_1, \vec{x}_1)\phi(\eta_2, \vec{x}_2)\rangle = \int_\Delta \int_x \langle \Omega|\phi(\eta_1, \vec{x}_1)|\Delta, \vec{x}\rangle\langle \Delta, \vec{x}|\phi(\eta_2, \vec{x}_2)|\Omega\rangle$$

$$= \int_\Delta \underbrace{|c_\phi(\Delta)|^2}_{\rho_\Delta} G_{\rm f}(\xi, \Delta)$$

- $\langle \Omega|\phi(\eta_1, \vec{x}_1)|\Delta, \vec{x}\rangle$ are fixed by dS ismoetries up to a normalization factor c_ϕ.
- Unitarity of the bulk theory $\Longrightarrow \rho_\Delta \geq 0$
- Using analytic continuation from S^{d+1}, we found a Froissart-Gribov type inversion formula for ρ_Δ

$$\rho_\Delta = C(\Delta). \int_1^\infty d\xi \ {}_2F_1(1-\Delta, 1-d+\Delta, (3-d)/2, (1-\xi)/2)\ \mathrm{Disc}\left[G(\xi)\right]$$

Spectral density



- Deforming the contour away from principal series and take the late-time limit $\eta \to 0^-$ $\implies$ Residues of ρ correspond to boundary operators:

$$(b_{\phi k})^2 \sim \text{Res}\rho_{\Delta_k}$$

- Examples:
  - free massive $\langle \phi(\eta_1, \vec{x}_1)\phi(\eta_2, \vec{x}_2)\rangle$ and $\langle \phi^2(\eta_1, \vec{x}_1)\phi^2(\eta_2, \vec{x}_2)\rangle$
  - bulk CFT $\langle \phi(\eta_1, \vec{x}_1)\phi(\eta_2, \vec{x}_2)\rangle \sim \frac{1}{(1-\xi)^{2\delta}}$

Positivity in boundary 4ptf

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_\ell \int_\Delta \int_x \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)|\Delta,\vec{x}\rangle\langle\Delta,\vec{x}|\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$
$$= \sum_\ell \int_\Delta I_{\Delta,\ell}\Psi_{\Delta,\ell}(x_1,\dots,x_4)$$

- Like in two-point function, each factor is fixed by conformal symmetry $\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)|\Delta,\vec{x}\rangle = \mathcal{F}^{12}_{\Delta,\ell}\,\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)O(x)\rangle$
- Positivity condition: for example $\langle \mathcal{O}\mathcal{O}^\dagger\mathcal{O}^\dagger\mathcal{O}\rangle \implies I_{\Delta,\ell} \geq 0$
- Few implications of positivity:
  - Necessity of ultra local terms in Generalized Free Field: $\langle \mathcal{O}\mathcal{O}^\dagger\mathcal{O}^\dagger\mathcal{O}\rangle \ni \delta(x_1 - x_3)\delta(x_2 - x_4)$
  - Bounds for breakdown of perturbative unitarity for $\lambda\phi^4$

Bootstrap

- Crossing equation + $I_{\Delta,\ell} > 0$. Let's Bootstrap!

- We focused on $d = 1$ and $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$ with a regulated crossing equation [an integral over cross ratio $\int_z \Longrightarrow (\gamma, \sigma)$]:

$$\int_0^\infty \frac{d\nu}{2\pi} I_{\frac{1}{2}+i\nu} \tilde{F}_{\frac{1}{2}+i\nu}(\gamma,\sigma) + \sum_{n\in 2\mathbb{N}} I_n \tilde{F}_n(\gamma,\sigma) + D(\gamma,\sigma) = 0$$

- Non trivial bounds: assuming gaps (restricting support in ν) one obtains e.g. an upper bound on I_2.

Future directions

- What type of irreps actually appear for generic interacting QFTs?
  Free field Fock space decomposition.
  CFT (decompose conformal multiplets of $SO(d+1,2)$ into irreps of $SO(d+1,1)$)

- Källén-Lehmann for spinning two-point functions

- What are the interesting questions—where/what to look at/for?
  + Technical obstacles

- Better regularised crossing equations!

- Position-less crossing equation

$$I^t_{\Delta,\ell} = \frac{1}{n_{\Delta,\ell}} \sum_{\ell'} \int \frac{d\Delta'}{2\pi i}\, I^s_{\Delta',\ell'}\, \mathcal{J}_d(\tilde{\Delta}', \ell', \tilde{\Delta}, \ell | \tilde{\Delta}_1, \tilde{\Delta}_2, \tilde{\Delta}_3, \tilde{\Delta}_4) + \mathcal{D}^{st}_{\Delta,\ell}$$

- How about gravity?

Thank You!