Operator spectrum of NRCFTs at large charge

Vito PELLIZZANI

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics
University of Bern

Strings, Fields and Holograms, Ascona, 14.10.21

Based on [Orlando, VP, Reffert '20], [VP '21]

[Hellerman, Orlando, VP, Reffert, Swanson, *to appear*]

UNIVERSITÄT BERN

UNIVERSITÄT BERN

Motivation

Broad question: can we understand theory space?

Motivation

Broad question: can we understand theory space?

- Ambitious $\rightarrow$ restrict to constrained theories. Here, CFTs.
- Bootstrap does an amazing job at collecting CFT data.
- Otherwise: large R-charge, large spin, etc.

Motivation

What if we keep adding symmetries?

- Natural guess: large internal charge Q (this talk: $U(1)$ symmetry)
- If this limit is tractable, is there a sense in which the predictions can be universal?

Toolbox

Effective field theory (EFT)

In the Wilsonian picture:

- Write down all possible operators.
- Wilsonian coefficients encode UV physics.
- Specify two well-separated scales Λ_{UV} and Λ_{IR}.
- Expansion parameter: $\epsilon \equiv \frac{\Lambda_{IR}}{\Lambda_{UV}} \ll 1$.
- Observables given by an asymptotic series in ϵ (or ϵ^2)

$$\langle \mathcal{O} \rangle = \# \left[\alpha_1 + \alpha_2 \epsilon^2 + \alpha_3 \epsilon^4 + \dots \right]$$

(this talk: ignore non-perturbative effects).

Toolbox

Relativistic state-operator correspondence

- Operator spectrum on $\mathbb{R}^{d+1} \leftrightarrow$ Energy spectrum on $\mathbb{R} \times S^d_R$

$$\Delta = E \cdot R.$$

- **Goal:** compute Δ_Q of lowest op. of charge Q via the GS energy E_0.

Context: the relativistic $O(2)$ model

In this case:

- Fixed-Q sector $\leftrightarrow$ superfluid phase for the Goldstone.
- Scales: $\Lambda_{IR} = \frac{1}{R}$ and $\Lambda_{UV} = \rho^{\frac{1}{d}} = \frac{Q^{\frac{1}{d}}}{R}$.
- Expansion parameter:

$$\epsilon = \frac{\Lambda_{IR}}{\Lambda_{UV}} = Q^{-\frac{1}{d}}.$$

- With $Q \gg 1$, we have $\epsilon \ll 1$ and the EFT regime is well-defined.

Context: the relativistic $O(2)$ model

Moreover,

$$\text{charge density} \sim \frac{Q}{\text{Vol}} \sim \frac{Q}{R^d}$$
$$\text{energy density} \sim \frac{E_0}{\text{Vol}} \sim \frac{\Delta_Q}{R^{d+1}}$$

are finite, even if $R \to \infty$, hence

$$\Delta_Q \sim Q^{\frac{d+1}{d}}$$

to leading order.

Context: the relativistic $O(2)$ model

We conclude that [Hellerman, Orlando, Reffert, Watanabe '15] [Cuomo '20]

$$\Delta_Q = Q^{\frac{d+1}{d}} \left[\alpha_1 + \frac{\alpha_2}{Q^{\frac{2}{d}}} + \frac{\alpha_3}{Q^{\frac{4}{d}}} + \dots\right] + Q^0 \left[\beta_0 + \frac{\beta_1}{Q^{\frac{2}{d}}} + \frac{\beta_2}{Q^{\frac{4}{d}}} + \dots\right] + \dots,$$

in $(d+1)$-dimensions.

Second line given by the Casimir energy, based on the spectrum

$$\omega_l = \sqrt{\frac{l(l+d-1)}{d}} + \mathcal{O}(Q^{-\frac{2}{d}}),$$

with multiplicity $\frac{(2l+d-1)\Gamma(l+d-1)}{\Gamma(l+1)\Gamma(d)}$ on the d-sphere.

Context: the relativistic $O(2)$ model

Typically,

$$\Delta_Q^{(d=2)} = \alpha_1 Q^{\frac{3}{2}} + \alpha_2 \sqrt{Q} - 0.0937 + \dots,$$

and

$$\Delta_Q^{(d=3)} = \alpha_1 Q^{\frac{4}{3}} + \alpha_2 Q^{\frac{2}{3}} - \frac{1}{48\sqrt{3}} \log Q + \alpha_3 + \dots$$

Nonrelativistic conformal invariance

Consider the so-called *Schrödinger* group:

- Galilean algebra
- Central extension: particle number symmetry ($U(1)$)
- Scale transformation: $(t, \vec{x}) \to (e^{2\tau} t, e^{\tau} \vec{x})$
- SCT: $(t, \vec{x}) \to \left(\frac{t}{1+\lambda t}, \frac{\vec{x}}{1+\lambda t} \right)$

Nonrelativistic state-operator correspondence

Schrödinger algebra also has an automorphism such that

- Operator sp. on $\mathbb{R}^{d+1}$ $\leftrightarrow$ Energy sp. in $A_0(\vec{x}) = \frac{m\omega^2}{2\hbar}|\vec{x}|^2$
  [Werner, Castin '05] [Nishida, Son '07] [Goldberger, Khandker, Prabhu '14]

$$\Delta = \frac{E}{\hbar\omega}$$

  (from now on: $\hbar = m = 1$).
- Defines a "turning point" region $\to$ spherical cloud/droplet.
- Focus on Δ_Q of lowest op. of charge Q via GS energy E_0.

Effective field theory

Let χ be the Goldstone associated with Q.

- Fixed-Q sector $\leftrightarrow$ superfluid phase for χ in the trap.
- IR scale: radius of the cloud $R_{cl} \sim \frac{Q^{\frac{1}{2d}}}{\sqrt{\omega}}$.
- UV scale: charge density (in the center) $\rho \sim \omega^{\frac{d}{2}}\sqrt{Q}$.
- Expansion parameter:

$$\epsilon = \frac{R_{cl}^{-1}}{\rho^{\frac{1}{d}}} \sim Q^{-\frac{1}{d}}.$$

Well-defined low-energy regime for $Q \gg 1$!

Effective field theory

Since $\mathrm{Vol} \sim \frac{\sqrt{Q}}{\omega^{\frac{d}{2}}}$ and $E_0 = \omega \Delta_Q$,

$$\text{charge density} \sim \frac{Q}{\mathrm{Vol}} \sim \omega^{\frac{d}{2}} \sqrt{Q}$$

$$\text{energy density} \sim \frac{E_0}{\mathrm{Vol}} \sim \frac{\omega^{\frac{d}{2}+1}}{\sqrt{Q}} \Delta_Q .$$

Then, the limit $\omega \to 0$ tells us that

$$\Delta_Q \sim Q^{\frac{d+1}{d}}$$

to leading order.

Intermediate conclusion

So far, it seems that

$$\Delta_Q = Q^{\frac{d+1}{d}} \left[a_1 + \frac{a_2}{Q^{\frac{2}{d}}} + \frac{a_3}{Q^{\frac{4}{d}}} + \dots \right]$$

plus quantum corrections (Casimir energy).

- Surprising? Perhaps.
- Disappointing? Somewhat...

Intermediate conclusion

But what about the boundary?

- Associated with the vanishing of the charge density ("Dirichlet").
- EFT not well-defined in its proximity.

Intermediate conclusion

In fact,

- Operators can be placed on the boundary.
- The corresponding expansion parameter is

$$\tilde{\epsilon} \sim Q^{-\frac{2}{3d}}.$$

- Contributions to Δ_Q start at $Q^{\frac{2d-1}{3d}}$.

Introduction

NRCFT at large-Q

Conclusion

Results

A careful computation leads to

[Son, Wingate '05] [Kravec, Pal '18] [Orlando, VP, Reffert, '20] [VP '21]

[Hellerman, Orlando, VP, Reffert, Swanson, *to appear*]

$$\Delta_Q = Q^{\frac{d+1}{d}}\left[a_1 + \frac{a_2}{Q^{\frac{2}{d}}} + \frac{a_3}{Q^{\frac{4}{d}}} + \dots\right] + Q^{\frac{2d-1}{3d}}\left[b_1 + \frac{b_2}{Q^{\frac{2}{3d}}} + \frac{b_3}{Q^{\frac{4}{3d}}} + \dots\right] + Q^{\frac{d-3}{3d}}\left[c_1 + \frac{c_2}{Q^{\frac{2}{3d}}} + \frac{c_3}{Q^{\frac{4}{3d}}} + \dots\right] + \dots$$

In particular

$$\Delta_Q^{(d=2)} = d_1 Q^{\frac{3}{2}} + d_2\sqrt{Q}\log Q + d_3\sqrt{Q} + d_4 Q^{\frac{1}{6}} - 0.2942 + \dots,$$

and

$$\Delta_Q^{(d=3)} = d_1 Q^{\frac{4}{3}} + d_2 Q^{\frac{2}{3}} + d_3 Q^{\frac{5}{9}} + d_4 Q^{\frac{1}{3}} + d_5 Q^{\frac{1}{9}} + \frac{1}{3\sqrt{3}}\log Q + d_6 + \dots$$

Results

What I haven't told you:

- Some of the b's contain $\log Q$-terms.
- Computation of the Casimir energy.
- Miraculous connection with experiments!

Outlook

- Include spin [Kravec, Pal '19]
- Gravity dual [Son '08] [Balasubramanian, McGreevy '08]
- BCS-BEC crossover
- Non-Abelian $Sp(N)$ at large-N

  [Veillette, Sheehy, Radzihovsky '06] [Sachdev, Nikolic '06]