

Five-Dimensional Path Integrals for Six-Dimensional (S)CFTs

Based on **N. Lambert, A. Lipstein, R. Mouland & P. Richmond [2109.04829]**

Rishi Mouland (DAMTP, Cambridge), *Strings, Fields and Holograms*, Ascona, 14th October 2021

An outline

- **A problem:** No conventional Lagrangian for non-Abelian 6d SCFTs
- **Some models:** Construction of some interesting 5d supersymmetric non-Lorentzian gauge theories
- **A proposal:** How these models realise 6d physics, and other aspects

A problem

Six-dimensional SCFTs

- **Aim:** compute dynamical observables in **non-Abelian six-dimensional superconformal field theories**, which can have $\mathcal{N} = (2,0)$ or $\mathcal{N} = (1,0)$ SUSY
- The $A_{N-1} (2,0)$ theory describes N coincident **M5-branes** in M-theory
 - Study using duality with M-theory on $\text{AdS}_7 \times S^4$?
 - Large N ? SUGRA does the trick ✓
 - Finite N corrections? Need M-theory corrections in the bulk ✗
- Many fruitful modern exact SCFT techniques (e.g. SUSY localisation) require a Lagrangian description
- **Problem:** There is no (conventional) Lagrangian description for 6d SCFTs
 - Wrap on a circle of radius $R \longrightarrow$ 5d MSYM with coupling $g^2 = 4\pi^2 R$

Some 5d gauge theories

A conformal compactification

- Take coordinates (x^+, x^-, x^i) , $i = 1, \dots, 4$, with $x^+ \in (-\pi, \pi)$. Then, the metric

$$ds^2 = -2dx^+ \left(dx^- - \frac{1}{2} \Omega_{ij} x^i dx^j \right) + dx^i dx^i$$

Constant, anti-self-dual
 $\Omega_{ij} \Omega_{jk} = -\delta_{ik}$

is conformal to the 6d Minkowski metric. So let's put our 6d CFT on it

- Let's also reduce into modes along the x^+ interval
- Conformal algebra broken as $\mathfrak{so}(2,6) \rightarrow \mathfrak{u}(1) \oplus \mathfrak{su}(1,3)$
- SUSY broken as:

$\mathcal{N} = (2,0) :$	32 real Q 's	$\longrightarrow$	24 real Q 's	$x^- \rightarrow \lambda^2 x^-$
$\mathcal{N} = (1,0) :$	16 real Q 's	$\longrightarrow$	12 real Q 's	$x^i \rightarrow \lambda x^i$

Translations along x^+

Includes inhomogeneous scaling

An action for the modes

The basics

- We can now propose a Lagrangian describing the dynamics of these KK modes

- Focus on the $\mathcal{N} = (2,0)$ theories. Then, $S = \int dx^- d^4x \mathcal{L}$,

$$\mathcal{L} = \frac{k}{4\pi^2} \text{tr} \left(\frac{1}{2} F_{-i} F_{-i} - \frac{1}{2} \hat{D}_i X^I \hat{D}_i X^I + \frac{1}{2} G_{ij} \mathcal{F}_{ij} - \frac{i}{2} \bar{\Psi} \Gamma_+ D_- \Psi + \frac{i}{2} \bar{\Psi} \Gamma_i \hat{D}_i \Psi - \frac{1}{2} \bar{\Psi} \Gamma_+ \Gamma^I [X^I, \Psi] \right)$$

 Coupling $g^2 = 4\pi^2/k$

- Scalars $X^I, I = 1, \dots, 5$, gauge field (A_-, A_i) with field strength (F_{-i}, F_{ij}) , real 32-component spinor Ψ , self-dual spatial 2-form G_{ij}
- Ω -deformed objects: $\hat{D}_i = D_i - \frac{1}{2} \Omega_{ij} x^j D_-$, $\mathcal{F}_{ij} = F_{ij} - \frac{1}{2} \Omega_{ik} x^k F_{-j} + \frac{1}{2} \Omega_{jk} x^k F_{-i}$
- Localises onto Ω -deformed anti-instanton moduli space: $\mathcal{F} = - \star \mathcal{F}$

An action for the modes

Some classical properties

- Has **exactly** the right symmetries to describe x^+ zero modes of 6d (2,0) theory
 - $SU(1,3)$ spacetime symmetry [N. Lambert, A. Lipstein, **RM**, P. Richmond, 1912.02638]
 - 24 supercharges and $SO(5)$ R-symmetry
- Derived holographically [N. Lambert, A. Lipstein & P. Richmond, 1904.07547]
 - View AdS_7 as a Hopf fibration over non-compact $\mathbb{CP}^3$
 - Place M5's at fixed $\mathbb{CP}^3$ radius, reduce along fibre
 - Take M5's to boundary, get fixed point action [N. Lambert, **RM**, 1911.11504]
- Generalisation to (1,0) known, with generic matter content
[N. Lambert, T. Orchard, 2011.06968]

Instantons and 6d physics

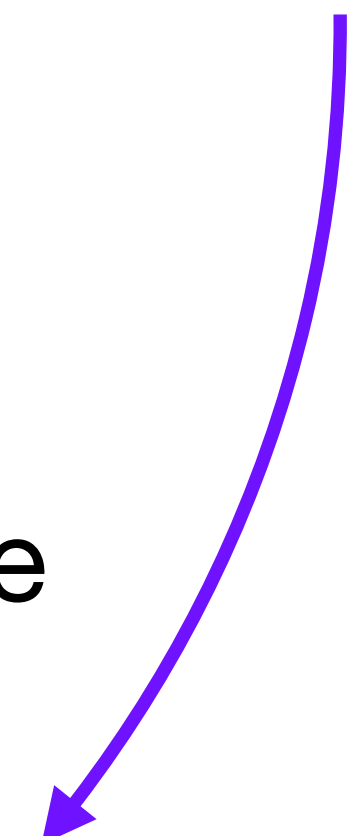
Classical symmetry breaking

- Specialise to $G = SU(N_c)$, and pick a number of isolated points $\{x_a\}$, $a = 1, \dots, N$
- **Extend configuration space**: allow for generic principal $SU(N_c)$ bundles over $\mathbb{R}^5 \setminus \{x_a\}$, field strength generically singular at the x_a
- Bundles classified by integers $\{n_a\}$ such that $n_a = \frac{1}{8\pi^2} \int_{S_a^4} \text{tr}(F \wedge F) \in \mathbb{Z}$ Small 4-sphere around x_a
- Solutions to $\mathcal{F} = - \star \mathcal{F}$ with $n_a \neq 0$ known [N. Lambert, A. Lipstein, **RM**, P. Richmond, 2105.02008]
- $SU(1,3)$ spacetime symmetry **broken**, but action non-invariance **local to the x_a**

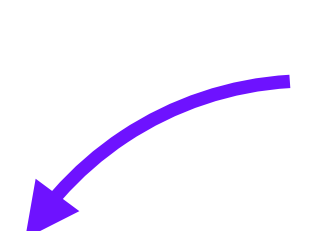
$$\delta S = \sum_{a=1}^N n_a f(x_a)$$

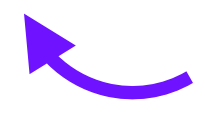
Quantum recovery

Theory has no continuous parameters!



- Consider correlators in the quantum theory, $\langle \dots \rangle = \int D\phi e^{iS[\phi]}(\dots)$
- The singular points (x_a, n_a) encoded by instanton operators $\mathcal{J}_{n_a}(x_a)$, which are disorder operators fixing the range of path integration
- **Striking result:** in presence of instanton operators, e^{iS} single-valued requires $k \in \frac{1}{2}\mathbb{N}$
- Despite classical symmetry breaking, find $U(1) \times SU(1,3)$ Ward-Takahashi identities, now with operators charged under the $U(1)$ factor

$$-i d \star \left\langle J(x) \prod_{a=1}^N \mathcal{J}_{n_a}(x_a) \Phi^{(a)}(x_a) \right\rangle = \star \sum_{a=1}^N \delta^{(5)}(x - x_a) \left\langle \delta \left(\mathcal{J}_{n_a}(x_a) \Phi^{(a)}(x_a) \right) \prod_{b \neq a} \mathcal{J}_{n_b}(x_b) \Phi^{(b)}(x_b) \right\rangle$$


- **General solution** for scalars known, and 2-point function entirely fixed
[N. Lambert, A. Lipstein, **RM**, P. Richmond, 2012.00626]
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$U(1) \times SU(1,3)$
bootstrap?

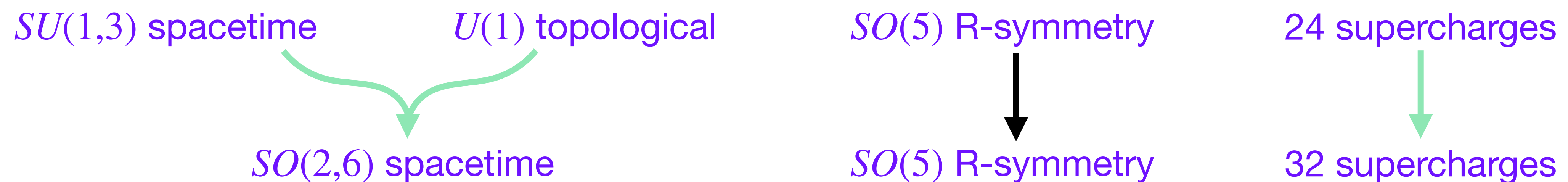
Building 6d observables

- Crucially, instanton operators $\mathcal{J}_n(x)$ carry charge kn under the $U(1)$ factor
- Takeaway: for $k \in \mathbb{N}$, we can interpret $\mathcal{J}_n(x)\Phi(x)$ as the $(kn)^{\text{th}}$ KK mode of a 6d CFT operator on the x^+ interval
 - ➔ Can interpret the theory as a 6d CFT on an orbifold $\mathbb{R}^{1,5}/\mathbb{Z}_k$. In particular, $k = 1$ gives us flat space!
- **Mantra**: to compute a 6d observable, decompose into Fourier sum of 5d observables, and seek to compute these instead
- Hope to offer window into the computation of dynamical (i.e. non-protected) observables in 6d SCFTs that are inaccessible by symmetries alone

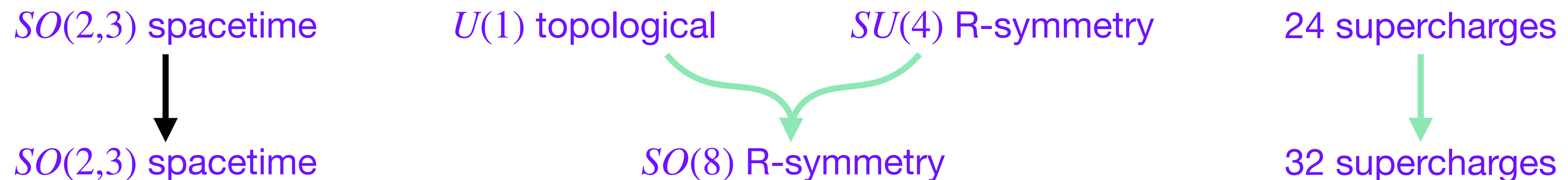
Some other aspects

Further symmetry enhancement at strong coupling

- At strong coupling $k = 1$, no orbifold. Symmetry should be **enhanced** to full (2,0) algebra!



- Group-theoretically identical to ABJM at strong coupling (up to real form)



- Clear holographically, as for ABJM $\leftrightarrow \text{AdS}_4 \times S^7$ it's the transverse S^7 that's fibred over $\mathbb{CP}^3$

Some other aspects

Recovering the DLCQ description

- Rescale fields and coordinates by R such that now $x^+ \in (-\pi R, \pi R)$
- Take limit $k, R \rightarrow \infty$ with $R_+ = R/k$ fixed
 - ➔ Minkowski space with compactified null direction $x^+ \sim x^+ + 2\pi R_+$
- Action becomes
$$S = \frac{1}{4\pi^2 R_+} \text{tr} \int dx^- d^4x \left(\frac{1}{2} F_{-i} F_{-i} - \frac{1}{2} D_i X^I D_i X^I + \frac{1}{2} G_{ij} F_{ij} + (\text{fermions}) \right)$$
- Dynamics reduce to SCQM on instanton moduli space $F = - \star F$ [RM, 1911.11504]
 - Recover original DLCQ proposal for the M5-brane [Aharony, Berkooz, Kachru, Seiberg, Silverstein, 1997]

Conclusion and outlook

- Have seen how five-dimensional non-Lorentzian gauge theories can encode the dynamics of six-dimensional CFTs
- Would be great to better understand how symmetry enhancement at strong coupling works
 - Would we see a restriction to ADE gauge groups here?
- Seek explicit results on 6d SCFTs
 - High degree of supersymmetry $\longrightarrow$ SUSY localisation. New formula for SC index?
 - Compute correlators in different regimes of k
 - Large N ?
- Symmetry algebra is different real form of $\mathfrak{osp}(6|4) \oplus \mathfrak{u}(1)$ (ABJM)
 - ➔ Integrability techniques applicable?

Thanks

Let's chat!