Five-Dimensional Path Integrals for Six-Dimensional (S)CFTs

Based on N. Lambert, A. Lipstein, R. Mouland & P. Richmond [2109.04829]

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An outline

- **A problem**: No conventional Lagrangian for non-Abelian 6d SCFTs
- **Some models**: Construction of some interesting 5d supersymmetric non-Lorentzian gauge theories
- **A proposal**: How these models realise 6d physics, and other aspects

A problem

Six-dimensional SCFTs

- Aim: compute dynamical observables in non-Abelian six-dimensional superconformal field theories, which can have $\mathcal{N} = (2,0)$ or $\mathcal{N} = (1,0)$ SUSY
- The A_{N-1} $(2,0)$ theory describes N coincident M5-branes in M-theory
  - Study using duality with M-theory on AdS$_7 \times S^4$?
  - Large N? SUGRA does the trick ✓
  - Finite N corrections? Need M-theory corrections in the bulk ✗
- Many fruitful modern exact SCFT techniques (e.g. SUSY localisation) require a Lagrangian description
- Problem: There is no (conventional) Lagrangian description for 6d SCFTs
  - Wrap on a circle of radius $R \longrightarrow$ 5d MSYM with coupling $g^2 = 4\pi^2 R$

Some 5d gauge theories

A conformal compactification

- Take coordinates (x^+, x^-, x^i), $i = 1,\ldots 4$, with $x^+ \in (-\pi, \pi)$. Then, the metric

$$ds^2 = -2dx^+\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right) + dx^i dx^i$$

Constant, anti-self-dual $\Omega_{ij}\Omega_{jk} = -\delta_{ik}$

is conformal to the 6d Minkowski metric. So let's put our 6d CFT on it

- Let's also reduce into modes along the x^+ interval

Translations along x^+

- Conformal algebra broken as $\mathfrak{so}(2,6) \to \mathfrak{u}(1) \oplus \mathfrak{su}(1,3)$

Includes inhomogeneous scaling

$$x^- \to \lambda^2 x^-$$
$$x^i \to \lambda x^i$$

- SUSY broken as: $\mathcal{N} = (2,0)$: 32 real $Q's$ $\longrightarrow$ 24 real $Q's$

$\mathcal{N} = (1,0)$: 16 real $Q's$ $\longrightarrow$ 12 real $Q's$

An action for the modes

The basics

- We can now propose a Lagrangian describing the dynamics of these KK modes
- Focus on the $\mathcal{N} = (2{,}0)$ theories. Then, $S = \int dx^- d^4x\, \mathcal{L}$,

$$\mathcal{L} = \frac{k}{4\pi^2}\mathrm{tr}\left(\frac{1}{2}F_{-i}F_{-i} - \frac{1}{2}\hat{D}_i X^I \hat{D}_i X^I + \frac{1}{2}G_{ij}\mathcal{F}_{ij} - \frac{i}{2}\bar{\Psi}\Gamma_+ D_- \Psi + \frac{i}{2}\bar{\Psi}\Gamma_i \hat{D}_i \Psi - \frac{1}{2}\bar{\Psi}\Gamma_+\Gamma^I[X^I, \Psi]\right)$$

Coupling $g^2 = 4\pi^2/k$

- Scalars $X^I, I = 1,\ldots 5$, gauge field (A_-, A_i) with field strength (F_{-i}, F_{ij}), real 32-component spinor Ψ, self-dual spatial 2-form G_{ij}
- Ω-deformed objects: $\hat{D}_i = D_i - \frac{1}{2}\Omega_{ij}x^j D_-, \quad \mathcal{F}_{ij} = F_{ij} - \frac{1}{2}\Omega_{ik}x^k F_{-j} + \frac{1}{2}\Omega_{jk}x^k F_{-i}$
- Localises onto Ω-deformed anti-instanton moduli space: $\mathcal{F} = -\star\mathcal{F}$

An action for the modes

Some classical properties

- Has exactly the right symmetries to describe x^+ zero modes of 6d $(2,0)$ theory
  - $SU(1,3)$ spacetime symmetry [N. Lambert, A. Lipstein, **RM**, P. Richmond, 1912.02638]
  - 24 supercharges and $SO(5)$ R-symmetry
- Derived holographically [N. Lambert, A. Lipstein & P. Richmond, 1904.07547]
  - View AdS_7 as a Hopf fibration over non-compact $\mathbb{CP}^3$
  - Place M5's at fixed $\mathbb{CP}^3$ radius, reduce along fibre
  - Take M5's to boundary, get fixed point action [N. Lambert, **RM**, 1911.11504]
- Generalisation to $(1,0)$ known, with generic matter content [N. Lambert, T. Orchard, 2011.06968]

Instantons and 6d physics

Classical symmetry breaking

- Specialise to $G = SU(N_c)$, and pick a number of isolated points $\{x_a\}$, $a = 1,\ldots,N$
- Extend configuration space: allow for generic principal $SU(N_c)$ bundles over $\mathbb{R}^5 \backslash \{x_a\}$, field strength generically singular at the x_a
- Bundles classified by integers $\{n_a\}$ such that $n_a = \dfrac{1}{8\pi^2} \displaystyle\int_{S^4_a} \mathrm{tr}(F \wedge F) \in \mathbb{Z}$

  Small 4-sphere around x_a
- Solutions to $\mathscr{F} = - \star \mathscr{F}$ with $n_a \neq 0$ known [N. Lambert, A. Lipstein, **RM**, P. Richmond, `2105.02008`]
- $SU(1,3)$ spacetime symmetry broken, but action non-invariance local to the x_a

$$\delta S = \sum_{a=1}^{N} n_a f(x_a)$$

Quantum recovery

Theory has no continuous parameters!

- Consider correlators in the quantum theory, $\langle \dots \rangle = \int D\phi e^{iS[\phi]}(\dots)$
- The singular points (x_a, n_a) encoded by instanton operators $\mathcal{I}_{n_a}(x_a)$, which are disorder operators fixing the range of path integration
- Striking result: in presence of instanton operators, e^{iS} single-valued requires $k \in \frac{1}{2}\mathbb{N}$
- Despite classical symmetry breaking, find $U(1) \times SU(1,3)$ Ward-Takahashi identities, now with operators charged under the $U(1)$ factor

Some field like X^I

$$-i\, d \star \left\langle J(x) \prod_{a=1}^{N} \mathcal{I}_{n_a}(x_a) \Phi^{(a)}(x_a) \right\rangle = \star \sum_{a=1}^{N} \delta^{(5)}(x - x_a) \left\langle \delta\left(\mathcal{I}_{n_a}(x_a)\Phi^{(a)}(x_a)\right) \prod_{b \neq a} \mathcal{I}_{n_b}(x_b)\Phi^{(b)}(x_b) \right\rangle$$

- General solution for scalars known, and 2-point function entirely fixed [N. Lambert, A. Lipstein, **RM**, P. Richmond, 2012.00626]

$U(1) \times SU(1,3)$ bootstrap?

Building 6d observables

- Crucially, instanton operators $\mathcal{I}_n(x)$ carry charge kn under the $U(1)$ factor
- Takeaway: for $k \in \mathbb{N}$, we can interpret $\mathcal{I}_n(x)\Phi(x)$ as the $(kn)^{\text{th}}$ KK mode of a 6d CFT operator on the x^+ interval
  - ➡ Can interpret the theory as a 6d CFT on an orbifold $\mathbb{R}^{1,5}/\mathbb{Z}_k$. In particular, $k = 1$ gives us flat space!
- Mantra: to compute a 6d observable, decompose into Fourier sum of 5d observables, and seek to compute these instead
- Hope to offer window into the computation of dynamical (i.e. non-protected) observables in 6d SCFTs that are inaccessible by symmetries alone

Some other aspects

Further symmetry enhancement at strong coupling

- At strong coupling $k = 1$, no orbifold. Symmetry should be enhanced to full $(2{,}0)$ algebra!

$SU(1{,}3)$ spacetime $U(1)$ topological $SO(5)$ R-symmetry 24 supercharges

$SO(2{,}6)$ spacetime $SO(5)$ R-symmetry 32 supercharges

- Group-theoretically identical to ABJM at strong coupling (up to real form)

$SO(2{,}3)$ spacetime $U(1)$ topological $SU(4)$ R-symmetry 24 supercharges

$SO(2{,}3)$ spacetime $SO(8)$ R-symmetry 32 supercharges

- Clear holographically, as for ABJM $\leftrightarrow$ $\mathrm{AdS}_4 \times S^7$ it's the transverse S^7 that's fibred over $\mathbb{CP}^3$

Some other aspects

Recovering the DLCQ description

- Rescale fields and coordinates by R such that now $x^+ \in (-\pi R, \pi R)$
- Take limit $k, R \to \infty$ with $R_+ = R/k$ fixed

➡ Minkowski space with compactified null direction $x^+ \sim x^+ + 2\pi R_+$

- Action becomes $S = \frac{1}{4\pi^2 R_+}\text{tr}\int dx^- d^4x \left(\frac{1}{2}F_{-i}F_{-i} - \frac{1}{2}D_i X^I D_i X^I + \frac{1}{2}G_{ij}F_{ij} + \text{(fermions)}\right)$
- Dynamics reduce to SCQM on instanton moduli space $F = -\star F$ [RM, 1911.11504]
  - Recover original DLCQ proposal for the M5-brane [Aharony, Berkooz, Kachru, Seiberg, Silverstein, 1997]

Conclusion and outlook

- Have seen how five-dimensional non-Lorentzian gauge theories can encode the dynamics of six-dimensional CFTs
- Would be great to better understand how symmetry enhancement at strong coupling works
  - Would we see a restriction to ADE gauge groups here?
- Seek explicit results on 6d SCFTs
  - High degree of supersymmetry $\longrightarrow$ SUSY localisation. New formula for SC index?
  - Compute correlators in different regimes of k
  - Large N?
- Symmetry algebra is different real form of $\mathfrak{osp}(6\,|\,4) \oplus \mathfrak{u}(1)$ (ABJM)
  ➡ Integrability techniques applicable?

Thanks

Let's chat!