Structure of Holographic BCFT Correlators From Geodesics

Jani Kastikainen

Astroparticule et Cosmologie, CNRS, Université de Paris
Department of Physics, University of Helsinki

October 14, 2021

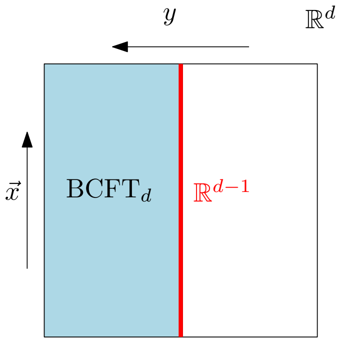
Based on 2109.00079 with Sanjit Shashi

Contents

Table of Contents

Boundary conformal field theory

- Euclidean BCFT$_d$ on a half-space $y \geq 0$ with a planar boundary $\mathbb{R}^{d-1}$ at $y = 0$
- Boundary preserved by the subgroup $SO(d, 1)$ of the global conformal group $SO(d + 1, 1)$
- The BCFT$_d$ has $SO(d, 1)$ symmetry



Scalar correlators in a BCFT$_d$

- Euclidean scalar 1- and 2-point functions are fixed by symmetry as:

$$\langle \mathcal{O}(y,\vec{x})\rangle = \frac{a_{\mathcal{O}}}{(2y)^{\Delta}}, \quad \langle \mathcal{O}(y_1,\vec{x}_1)\mathcal{O}(y_2,\vec{x}_2)\rangle = \frac{1}{(4y_1y_2)^{\Delta}}\mathcal{F}(\xi) \tag{1}$$

- Cross ratio of two points $(y_1,\vec{x}_1)$ and $(y_2,\vec{x}_2)$:

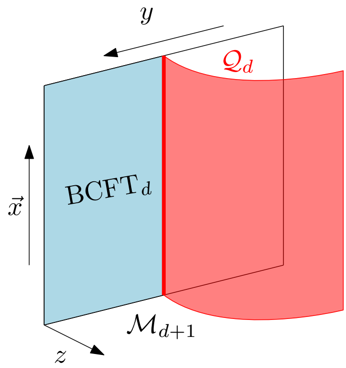
$$\xi = \frac{(y_2-y_1)^2+|\vec{x}_2-\vec{x}_1|^2}{4y_1y_2} \in [0,\infty) \tag{2}$$

Holographic dual of the BCFT boundary

- BCFT boundary modeled as an end-of-the-world (EOW) brane $\mathcal{Q}_d \subset \mathcal{M}_{d+1}$ in the bulk

[Takayanagi 11]

- EOW condition: $\partial\mathcal{M}_{d+1} = \mathrm{BCFT}_d \cup \mathcal{Q}_d$



Holographic bottom-up model of a BCFT

- $I_{\text{bulk}} = I_{\text{gravity}} + T_s I_{\text{scalar}}$
- Gravity part contains a brane of tension T:

$$I_{\text{gravity}} = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^{d+1}X \sqrt{g} \left[R + \frac{d(d-1)}{\ell^2} \right] + \frac{1}{8\pi G_N} \int_{\mathcal{Q}} d^d\hat{x} \sqrt{h} \, (K - T) \tag{3}$$

- Scalar field Φ with a brane localized potential $V(\Phi)$:

$$I_{\text{scalar}} = \frac{1}{2} \int_{\mathcal{M}} d^{d+1}X \sqrt{g} \left(\nabla^a \Phi \nabla_a \Phi + m^2 \Phi^2 \right) - \int_{\mathcal{Q}} d^d\hat{x} \sqrt{h} \, V(\Phi). \tag{4}$$

- Classical limit with scalar and brane in the probe limit:

$$\ell^{d-1} / G_{\text{N}} \gg \ell T \gg T_s \gg 1. \tag{5}$$

Holographic bottom-up model of a BCFT

- Brane embedding determined by

$$\delta I_{\text{gravity}} = 0 \quad \Rightarrow \quad K_{ij} = (K - T)h_{ij}$$

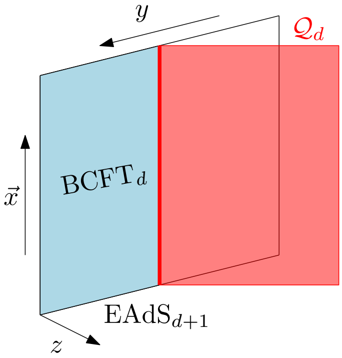
[Takayanagi 11]

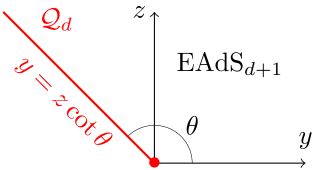
- BCFT in the vacuum state (set $\ell = 1$):

$$ds^2_{\mathcal{M}_{d+1}} = \frac{1}{z^2}(dz^2 + dy^2 + d\vec{x}^2) \qquad (6)$$

$\Rightarrow$ embedding $y(z) = z\cot\theta$

- $T = 0 \Leftrightarrow \theta = \frac{\pi}{2}$





Holographic bottom-up model of a BCFT

- Probe scalar Φ in the wedge $y \geq z \cot\theta$

  [Fujita–Takayanagi–Tonni 11]

- Boundary condition at the brane:

$$[n^a \partial_a \Phi + V'(\Phi)]|_{\mathcal{Q}} = 0. \tag{7}$$

- We consider quadratic potentials

$$V(\Phi) = \lambda_1 \Phi + \frac{\lambda_2}{2}\,\Phi^2 \tag{8}$$

and modified Robin boundary conditions

$$(n^a \partial_a + \lambda_2)\Phi|_{\mathcal{Q}} = -\lambda_1. \tag{9}$$

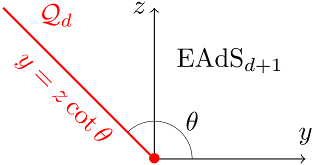


Table of Contents

Holographic dictionary with an EOW brane

- Split $\Phi = \phi_B + \phi$ with an on-shell background field ϕ_B:

$$(n^a\partial_a + \lambda_2)\phi_B|_{\mathcal{Q}} = -\lambda_1, \quad (n^a\partial_a + \lambda_2)\phi|_{\mathcal{Q}} = 0. \tag{10}$$

[McAvity–Osborn 93]

- Boundary conditions in the UV ($\Delta > d/2$)

$$\phi_B(z,y,\vec{x}) = z^{\Delta}[f_B(y,\vec{x}) + \dots], \quad \phi(z,y,\vec{x}) = z^{d-\Delta}[J(y,\vec{x}) + \dots] + \dots, \quad z \to 0. \tag{11}$$

- Correlators obtained by varying the renormalized on-shell action w.r.t $J(y,\vec{x})$

Holographic dictionary with an EOW brane

- Holographic renormalization by adding counterterms in ϕ
- 1-point function determined by the background field:

$$\langle \mathcal{O}(y,\vec{x}) \rangle = \Delta\, f_B(y,\vec{x}). \tag{12}$$

[Almheiri–Mousatov–Shyani 18]

- Background field gives a non-zero 1-point function even when $J = 0$

Holographic dictionary with an EOW brane

- 1-point function is an integral over the brane:

$$\langle \mathcal{O}(\zeta) \rangle = \Delta\, \lambda_1 \lim_{z \to 0} z^{-\Delta} \int_{\mathcal{Q}} d^d \hat{x} \sqrt{h}\, G(z, \zeta; Q(\hat{x})) \tag{13}$$

where G is the bulk-to-bulk propagator obeying Robin boundary condition at the brane

- 2-point function (connected) given by the extrapolate dictionary:

$$\langle \mathcal{O}(\zeta) \mathcal{O}(\zeta') \rangle = (2\Delta - d)^2 \lim_{z, z' \to 0} (z z')^{-\Delta}\, G(z, \zeta; z', \zeta'), \tag{14}$$

[JK–Shashi 21]

Table of Contents

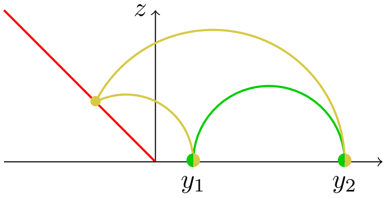
Geodesic approximation

- Large-Δ limit of $G(X, X')$ controlled by the geodesic approximation
- EOW brane $\Rightarrow$ Have to sum over reflecting geodesics obeying law of reflection

$$G(X, X') \sim \sum_{n=0}^{\infty} e^{-\Delta L_n(X,X')}, \quad \Delta \to \infty \tag{15}$$

- L_n length of the geodesic reflecting n times

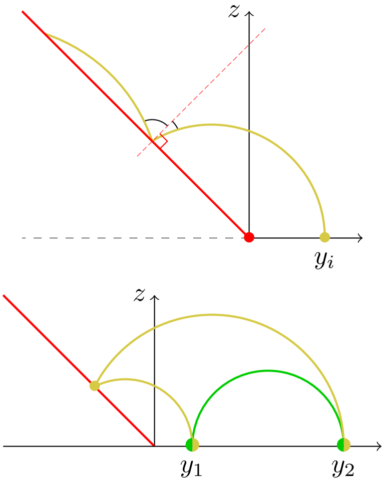
[McAvity–Osborn 91, Ludewig 17]



Constraint on reflections

- Geodesics in EAdS_{d+1} are circular arcs
- Take one end-point to the conformal boundary
- No-Go theorem: $n \geq 2$ reflections not possible

[JK–Shashi 21]

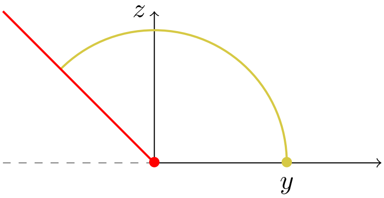


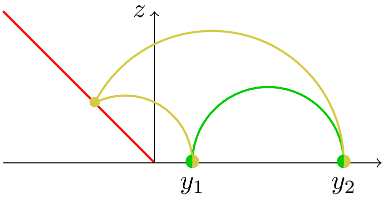
Correlators from geodesics

- After taking bulk points to the conformal boundary:

$$\langle \mathcal{O}(y, \vec{x}) \rangle \sim \lambda_1 e^{-\Delta L}$$

$$\langle \mathcal{O}(y_1, \vec{x}_1) \mathcal{O}(y_2, \vec{x}_2) \rangle \sim e^{-\Delta L_0} + e^{-\Delta L_1}$$





L denotes renormalized geodesic length

The 1-point function

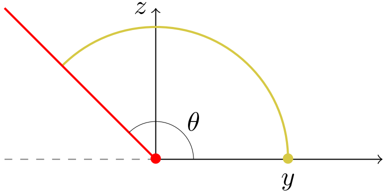
- Minimal geodesic intersects the brane orthogonally with the result

$$\langle \mathcal{O}(y,\vec{x})\rangle \sim \frac{\lambda_1 \cot^{\Delta}\frac{\theta}{2}}{(2y)^{\Delta}}, \quad \Delta \to \infty \tag{16}$$

- Matches the large-Δ expansion of the exact finite-Δ solution for f_B
- In $d=2$, the coefficient can be interpreted as boundary entropy:

$$a_{\mathcal{O}} \sim e^{-(6\Delta/c)S_{\mathrm{bdy}}} \tag{17}$$

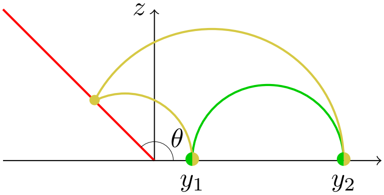
[JK–Shashi 21]



The 2-point function

- For $\theta > \arctan\sqrt{\xi}$:

$$\langle \mathcal{O}(y_1, \vec{x}_1)\mathcal{O}(y_2, \vec{x}_2)\rangle \sim \frac{1}{(4y_1y_2)^{\Delta}}\left[\xi^{-\Delta} + \left(\frac{\sin\theta}{\sqrt{1+\xi} - \cos\theta}\right)^{2\Delta}\right], \quad \Delta \to \infty.$$



At zero brane tension

- At zero brane tension $\theta = \frac{\pi}{2}$:

$$\langle \mathcal{O}(y_1, \vec{x}_1) \mathcal{O}(y_2, \vec{x}_2) \rangle \sim \frac{1}{[(y_1 - y_2)^2 + |\vec{x}_1 - \vec{x}_2|^2]^\Delta} + \frac{1}{[(y_1 + y_2)^2 + |\vec{x}_1 - \vec{x}_2|^2]^\Delta}, \quad \Delta \to \infty.$$

- Matches with method of images given the $\mathbb{Z}_2$-isometry $y \to -y$ in the bulk
- Actually exact to all orders in $1/\Delta$ due to this enhanced symmetry
- Zero tension brane studied in many papers

[DeWolfe–Freedman–Ooguri 02]
[Aharony–DeWolfe–Freedman–Karch 03]
[Liendo–Rastelli–van Rees 13]
[Mazáč–Rastelli–Zhou 19]

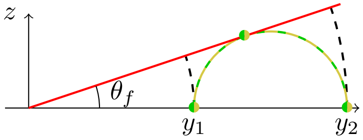
Factorization phase transition

- For $\theta < \arctan\sqrt{\xi}$:

$$\langle \mathcal{O}(y_1, \vec{x}_1)\mathcal{O}(y_2, \vec{x}_2)\rangle \sim 0, \quad \Delta \to \infty$$

- Factorization phase transition at $\theta = \theta_f$ with $\theta_f = \arctan\sqrt{\xi}$

- Occurs only for $T < 0$ and it is a large-Δ effect



Additional Lorentzian singularity

$$\langle \mathcal{O}(y_1, \vec{x}_1)\mathcal{O}(y_2, \vec{x}_2)\rangle \sim \frac{1}{(4y_1y_2)^{\Delta}}\left[\xi^{-\Delta} + \left(\frac{\sin\theta}{\sqrt{1+\xi} - \cos\theta}\right)^{2\Delta}\right].$$

- The reflection term has a Lorentzian singularity at

$$\xi = -\sin^2\theta. \tag{18}$$

- This is a light-ray singularity associated to a reflecting *bulk* null geodesic

[Reeves et. al. 21]

- Such bulk-point singularities should not exist at finite-c

[Maldacena–Simmons-Duffin–Zhiboedov 15]

Summary and future directions

- BCFT correlators in a holographic model with an EOW brane with scalar coupling
- Geodesic approximation involves geodesics reflecting off the brane

- Understand the boundary OPE and the spectrum of boundary operators at $\theta \neq \frac{\pi}{2}$
- Improve the bottom-up model by allowing increasing levels of backreaction and interactions

Relation to top-down models

- In top-down models usually no EOW branes
- Instead, a cycle of the internal space shrinks to zero size

[Chiodaroli–D'Hoker–Gutperle 12]

- Bulk contains an "EOW brane region"

[Uhlemann 21, Raamsdonk–Waddell 21]