The string dual of free N=4 SYM

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Strings, Fields and Holograms
Monte Verita, Ascona
15.10.2021

Based mainly on work with Rajesh Gopakumar

AdS/CFT correspondence

The relation between the parameters of string theory on AdS and the dual CFT is

$$g_s \sim \frac{1}{N} \qquad\qquad \frac{R}{l_s} \sim g_{\mathrm{YM}}^2 N = \lambda$$

string coupling constant

AdS radius in string units

‘t Hooft parameter

AdS/CFT correspondence

In particular, weakly coupled (planar) gauge theory corresponds to the tensionless regime of string theory

$$g_s \sim \frac{1}{N}$$

small

$$\frac{R}{l_s} \sim g_{\mathrm{YM}}^2 N = \lambda$$

small

$$\boxed{l_{\mathrm{s}} \to \infty \qquad \text{`tensionless strings'}}$$

[Sundborg '01] [Witten '01]
[Sezgin,Sundell '01]

Tensionless limit

This is the regime where AdS/CFT becomes perturbative:

tensionless strings on AdS ↔ weakly coupled/free SYM theory

- very stringy (far from sugra)
- higher spin symmetry
- maximally symmetric phase of string theory

Tensionless limit

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Could it have a free worldsheet description?

AdS3 review

For example, in the 3d case, the AdS/CFT duality relates string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

to a CFT that is on the same moduli space of CFTs as the symmetric orbifold theory

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^N / S_N$$

[Maldacena ’97], …. see e.g. [David et.al. ’02]

AdS3 review

The analogue of free SYM is the symmetric orbifold theory itself. It has a tensionless (k=1) string dual with $\mathrm{AdS}_3 \times \mathrm{S}^3$ worldsheet theory described by

4 symplectic bosons & 4 free fermions

↕

free field realisation of $\mathfrak{psu}(1,1|2)_1$

hybrid formalism of [Berkovits, Vafa, Witten '99]

Physical degrees of freedom come from spectrally flowed representations: matches precisely with single particle spectrum of dual symmetric orbifold.

[Eberhardt, MRG, Gopakumar '18]

AdS5 proposal

Similarly, free N=4 SYM in 4d should be dual to tensionless strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$: we **propose** \`twistorial’ worldsheet description via

8 symplectic bosons & 8 free fermions

$\updownarrow$

free field realisation of $\mathfrak{psu}(2,2|4)_1$

similar to twistor string of [Berkovits ’04]

Key ingredient: spectrally flowed representations.

Natural quantisation leads to a \`reduced model’ whose spectrum matches exactly that of free N=4 SYM.

[MRG, Gopakumar ’21]

Plan of talk

1. **Introduction and Motivation**
2. Review of AdS3
3. Generalisation to AdS5
4. Conclusions and Outlook

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Hybrid formalism

[Berkovits, Vafa, Witten ’99]

AdS3 theory at k=1 best described in hybrid formalism: for pure NS-NS flux, hybrid string consists of WZW model based on

$$\mathfrak{psu}(1,1|2)_k$$

together with the (topologically twisted) sigma model for T4. For generic k, this description agrees with the NS-R description a la Maldacena-Ooguri.

[Troost ’11], [MRG, Gerigk ’11]
[Gerigk ’12]

Free field realisation

The level k=1 theory has a free field realisation

$$\mathfrak{u}(1,1|2)_1 \;\cong\; \left\{ \begin{array}{l} 4 \;\textbf{symplectic bosons}\; \xi^{\pm}\,,\;\eta^{\pm} \\ 4 \;\textbf{real fermions}\; \psi^{\pm}\,,\;\chi^{\pm} \end{array} \right.$$

with

$$\boxed{\{\psi^{\alpha}_{r}\,,\chi^{\beta}_{s}\} = \epsilon^{\alpha\beta}\,\delta_{r,-s}} \qquad \boxed{[\xi^{\alpha}_{r}\,,\eta^{\beta}_{s}] = \epsilon^{\alpha\beta}\,\delta_{r,-s}}$$

Generators of $\mathfrak{u}(1,1|2)_1$ are bilinears in these free fields.

In order to reduce this to $\mathfrak{psu}(1,1|2)_1$ one has to gauge by the `diagonal' u(1) field

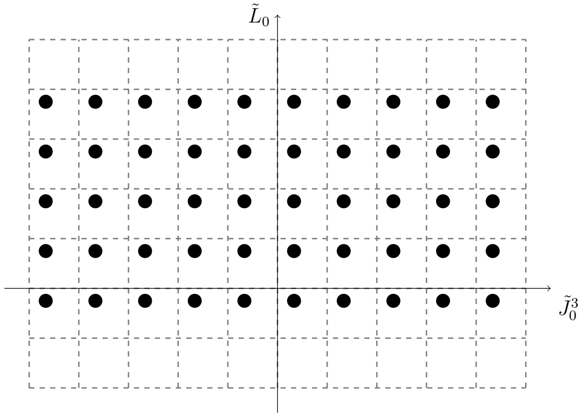
$$Z = \tfrac{1}{2}\left(\eta^{-}\xi^{+} - \eta^{+}\xi^{-} + \chi^{-}\psi^{+} - \chi^{+}\psi^{-}\right)\,.$$

Free field realisation

The only highest weight representations are:

- NS sector: all fields half-integer moded
- R sector: all fields integer moded

Here positive modes annihilate ground state.



In R sector, ground states form representation of zero modes: `singleton’ representation of $\mathfrak{psu}(1,1|2)$.

Spectral flow

The worldsheet spectrum consists of R-sector rep, together with spectrally flowed images.

[Henningson et.al. '91]
[Maldacena, Ooguri '00]

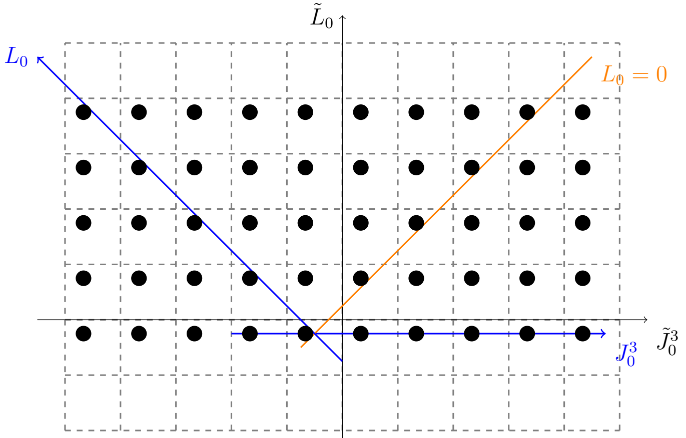
Spectral flow:

$$\tilde{\xi}^{\pm}_{n} = \xi^{\pm}_{n\pm\frac{w}{2}} \ , \qquad \tilde{\eta}^{\pm}_{n} = \eta^{\pm}_{n\pm\frac{w}{2}} \ , \qquad \tilde{\psi}^{\pm}_{n} = \psi^{\pm}_{n\mp\frac{w}{2}} \ , \qquad \tilde{\chi}^{\pm}_{n} = \chi^{\pm}_{n\mp\frac{w}{2}} \ ,$$

consider R sector rep, for tilde modes → interpret in terms of `untilde' modes

$$J^3_0 = \tilde{J}^3_0 + \frac{w}{2}$$
$$L_0 = \tilde{L}_0 - w\tilde{J}^3_0 - \frac{w^2}{4}$$

Not highest weight rep



(w=1; only bosons)

Physical spectrum

Since $\mathfrak{psu}(1,1|2)_1$ has many (!) null-vectors, it has effectively only 2 bosonic + fermionic oscillator degrees of freedom.

Thus after imposing the physical state conditions, only the degrees of freedom of $\mathbb{T}^4$ survive, and we get exactly the (single-particle) spectrum of

$$\mathrm{Sym}_N(\mathbb{T}^4)$$

in the large N limit, where w-cycle twisted sector comes from w spectrally flowed sector.

[Eberhardt, MRG, Gopakumar ’18]

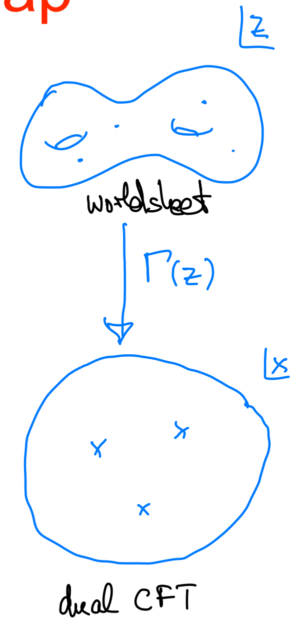
Correlators

The worldsheet correlators localise to those configurations that admit holomorphic covering map

worldsheet
correlator

$$\Big\langle \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i;z_i)\Big\rangle = \sum_{\Gamma} c_\Gamma\, \delta\big(z_i \text{ compatible with cov. map } \Gamma\big)$$

$$\Gamma(z) = x_i + a_i(z-z_i)^{w_i} + \cdots \qquad (z\sim z_i)$$



After integral over worldsheet moduli recover

$$\int_{\mathcal{M}} d\mu(z_i)\,\Big\langle \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i;z_i)\Big\rangle = \sum_{\Gamma} \tilde{c}_\Gamma \cong \Big\langle \prod_{i=1}^{n} \mathcal{O}_{h_i}^{w_i}(x_i)\Big\rangle$$

symmetric orbifold correlators

[Lunin, Mathur '00]
[Pakman, Rastelli, Razamat '09]

[Eberhardt, MRG, Gopakumar '19]
[Dei, MRG, Gopakumar, Knighton '20]

Correlation functions

In the above free field realisation this localisation can be deduced from the identity

$$\left\langle \left(\xi^-(z) + \underset{\substack{\uparrow \\ \text{covering map}}}{\Gamma(z)}\, \xi^+(z)\right) \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i; z_i) \right\rangle_{\text{phys}} = 0 \ .$$

[Dei, MRG, Gopakumar, Knighton ’20]

Very reminiscent of incidence relation in twistor space…

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Ansatz for worldsheet

Given the structure of the free field realisation for the case of $\mathrm{AdS}_3 \times \mathrm{S}^3$, we have proposed that the dual to free N=4 SYM in 4d should be described by a worldsheet theory consisting of

[MRG, Gopakumar ’21]

8 symplectic bosons
8 real fermions

They generate $\mathfrak{u}(2,2|4)_1$. After removing again an overall u(1), we get $\mathfrak{psu}(2,2|4)$: guarantees that dual spacetime theory has the correct symmetry.

Ansatz for worldsheet

More concretely, the worldsheet theory consists of what can be interpreted as components of ambitwistor fields

see also [Berkovits '04]

$$\begin{array}{rcl} Y_I &=& (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a) \qquad\qquad \alpha, \dot\alpha \in \{1,2\} \\ Z^I &=& (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a) \qquad\qquad a \in \{1,2,3,4\} \end{array}$$

with defining relations

$$\boxed{\begin{array}{c} [\lambda^\alpha_r, (\mu^\dagger_\beta)_s] = \delta^\alpha_\beta\, \delta_{r,-s} \ , \qquad [\mu^{\dot\alpha}_r, (\lambda^\dagger_{\dot\beta})_s] = \delta^{\dot\alpha}_{\dot\beta}\, \delta_{r,-s} \ , \\ \{\psi^a_r, (\psi^\dagger_b)_s\} = \delta^a_b\, \delta_{r,-s} \ . \end{array}}$$

Free fields on worldsheet

The bilinears

$$\mathcal{J}^I_J =: Y_J Z^I \ :$$

$$\begin{array}{rcl} Y_I & = & (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a) \\ Z^I & = & (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a) \end{array}$$

generate $\mathfrak{u}(2,2|4)_1$, and in order to obtain $\mathfrak{psu}(2,2|4)_1$ we need to gauge by the overall u(1) field

$$\mathcal{C} = \frac{1}{2} Y_I Z^I = \frac{1}{2}\left(\mu^\dagger_\gamma\, \lambda^\gamma + \lambda^\dagger_{\dot\gamma}\, \mu^{\dot\gamma} + \psi^\dagger_c \psi^c\right) \ .$$

[MRG, Gopakumar ’21]

This is the current algebra version of oscillator construction of $\mathfrak{psu}(2,2|4)$ which enters into spin chain discussion.

see e.g. [Beisert thesis], [Alday, David, Gava, Narain ’06]

Spectral flow

As in the case for AdS_3, all non-trivial aspects come from spectral flow where now

$$\begin{array}{|l|}\hline \mathrm{AdS}_3 : J_0^3 - K_0^3 \\ \mathrm{AdS}_5 : \mathcal{D}_0 - \mathcal{R}_0 \\ \hline \end{array}$$

$$\begin{aligned}
(\tilde{\lambda}^{\alpha})_r &= (\lambda^{\alpha})_{r-w/2} \ , & (\tilde{\lambda}^{\dagger}_{\dot{\alpha}})_r &= (\lambda^{\dagger}_{\dot{\alpha}})_{r-w/2} \ , & & \\
(\tilde{\mu}^{\dot{\alpha}})_r &= (\mu^{\dot{\alpha}})_{r+w/2} \ , & (\tilde{\mu}^{\dagger}_{\alpha})_r &= (\mu^{\dagger}_{\alpha})_{r+w/2} \ , & & \\
(\tilde{\psi}^{a}_r) &= \psi^{a}_{r-w/2} \ , & (\tilde{\psi}^{\dagger}_{a})_r &= (\psi^{\dagger}_{a})_{r+w/2} & & (a=1,2) \ , \\
(\tilde{\psi}^{b}_r) &= \psi^{b}_{r+w/2} \ , & (\tilde{\psi}^{\dagger}_{b})_r &= (\psi^{\dagger}_{b})_{r-w/2} & & (b=3,4) \ .
\end{aligned}$$

Starting from the usual NS-sector representation, the `untilde' modes act as

$$\begin{aligned}
\mu^{\dot{\alpha}}_r \, |0\rangle_w = (\mu^{\dagger}_{\alpha})_r \, |0\rangle_w = (\psi^{\dagger}_{1,2})_r \, |0\rangle_w = \ \psi^{3,4}_r \, |0\rangle_w = 0 \ , \qquad & (r \geq \tfrac{w+1}{2}) \\
\lambda^{\alpha}_r \, |0\rangle_w = (\lambda^{\dagger}_{\dot{\alpha}})_r \, |0\rangle_w = (\psi^{1,2})_r \, |0\rangle_w = (\psi^{\dagger}_{3,4})_r \, |0\rangle_w = 0 \ , \qquad & (r \geq -\tfrac{w-1}{2})
\end{aligned}$$

Wedge modes

Since

$$\mu_r^{\dot{\alpha}}\,|0\rangle_w = (\mu_\alpha^\dagger)_r\,|0\rangle_w = (\psi_{1,2}^\dagger)_r\,|0\rangle_w = \;\psi_r^{3,4}\,|0\rangle_w = 0\ , \qquad (r \geq \tfrac{w+1}{2})$$
$$\lambda_r^{\alpha}\,|0\rangle_w = (\lambda_{\dot{\alpha}}^\dagger)_r\,|0\rangle_w = (\psi^{1,2})_r\,|0\rangle_w = (\psi_{3,4}^\dagger)_r\,|0\rangle_w = 0\ , \qquad (r \geq -\tfrac{w-1}{2})$$

the non-zero modes acting on $|0\rangle_w$ are the wedge modes

$$\mu_r^{\dot{\alpha}}\ ,\ (\mu_\alpha^\dagger)_r\ ,\ (\psi_{1,2}^\dagger)_r\ ,\ \psi_r^{3,4}\ ,\quad (-\tfrac{w-1}{2} \leq r \leq \tfrac{w-1}{2})$$

as well as the `out-of-the-wedge' modes

$$Z_r^I \text{ and } (Y_J)_r \qquad \text{with } r \leq -\tfrac{w+1}{2}$$

Wedge modes

the non-zero modes acting on $|0\rangle_w$ are the wedge modes

$$\boxed{\mu_r^{\dot\alpha}\ ,\ (\mu_\alpha^\dagger)_r\ ,\ (\psi_{1,2}^\dagger)_r\ ,\ \psi_r^{3,4}\ ,\qquad (-\tfrac{w-1}{2} \le r \le \tfrac{w-1}{2})}$$

as well as the `out-of-the-wedge' modes

$$Z_r^I \text{ and } (Y_J)_r \qquad \text{with } r \le -\tfrac{w+1}{2}$$

<u>Postulate</u>: physical state conditions (N=4 critical string) remove all out-of-the-wedge modes. [MRG, Gopakumar '21]

Retain only:

generalised zero modes = (one copy of) wedge modes.

cf. [Dolan, Goddard '07], [Nair '08]

Wedge modes

On the resulting (wedge) Fock space, we furthermore need to impose the residual gauge conditions

[MRG, Gopakumar '21]

$$\mathcal{C}_n \phi = 0 \ \ (n \geq 0) \qquad \quad (L_0 + pw)\phi = 0 \ \ (p \in \mathbb{Z}) \ .$$

similar to Virasoro condition
in light-cone gauge

$$L_0 \sim -2\, p^- p^+$$

with

$$2\, p^- = \sum_{n=-\infty}^{\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} \sim N_{\text{tot}} \qquad \text{and} \qquad p^+ \cong w$$

[Berenstein, Maldacena, Nastase '02]

ground state $|0\rangle_w$
gives rise to
$\big(0, 0; [0, w, 0]\big)_w$
BMN vacuum

Key ingredients

Resulting spectrum reproduces exactly that of free N=4 SYM in 4d in planar limit.

More specifically, wedge modes can be thought of as momentum modes of w position space generators

$$\hat{Z}^I{}_j = \frac{1}{\sqrt{w}} \sum_{r=-(w-1)/2}^{(w-1)/2} Z^I_r \, e^{-2\pi i \frac{rj}{w}} \qquad (j = 1, \ldots, w) \ ,$$

and similarly for $(\hat{Y}_I)_j$. These position modes then satisfy

$$[\hat{Z}^I_{j_1}, (\hat{Y}^\dagger_J)_{j_2}]_\pm = \delta^I_J \, \delta_{j_1,j_2} \ .$$

[MRG, Gopakumar '21]

Key ingredients

The residual gauge conditions imply

$$\mathcal{C}_n \phi = 0 \;\; (n \geq 0) \qquad (L_0 + pw)\phi = 0 \;\; (p \in \mathbb{Z}) \ .$$

at each site j: $\hat{\mathcal{C}}_j = 0$

singleton rep

cyclic invariance

Get w-fold tensor product of singleton rep of $\mathfrak{psu}(2,2|4)$, subject to cyclicity condition: spectrum of free N=4 SYM.

[MRG, Gopakumar ’21]

Key ingredients

Get w-fold tensor product of singleton rep of $\mathfrak{psu}(2,2|4)$, subject to cyclicity condition: spectrum of free N=4 SYM.

w-spectrally
flowed sector ⟷ $\mathrm{Tr}\big(\underbrace{S_1\cdots S_w}_{w\ \text{letters}}\big)$

$$S_l=\{\partial^s\phi^i,\partial^s\Psi^a_\alpha,\partial^s\Psi^{\dot\alpha}_a,\partial^s\mathcal{F}_{\alpha\beta},\partial^s\mathcal{F}^{\dot\alpha\dot\beta}\}$$

String bit picture!

$$\hat{Y}=(\hat{\mu}^\dagger_\alpha,\hat{\lambda}^\dagger_{\dot\alpha},\hat{\psi}^\dagger_a)\ ,\quad \hat{Z}=(\hat{\lambda}^\alpha,\hat{\mu}^{\dot\alpha},\hat{\psi}^a)$$

twistor-valued string bits

[MRG, Gopakumar ’21]

Explicit states

From this worldsheet perspective, the physical states are all generated by DDF-like operators

[MRG, Gopakumar ’21]
[Sriprachyakul, unpub.]

$$S^{\mathbf{a}}_m \equiv (S_I{}^J)_m = \sum_{r=m-\frac{w-1}{2}}^{\frac{w-1}{2}} (Y_I)_r \, (Z^J)_{m-r}$$

interesting
algebraic structure
similar to Yangian

In particular, zero modes generate $\mathfrak{u}(2,2|4)$: physical states fall into representations of $\mathfrak{psu}(2,2|4)$.

Acting on ground state $|0\rangle_w$ generate full BPS multiplet

$$L_0 = 0 : \qquad \Big(\underbrace{0,0}_{\mathfrak{su}(2)\oplus\mathfrak{su}(2)} ; \underbrace{[0,w,0]}_{\mathfrak{su}(4)} \Big)_w$$

$\mathcal{D}_0$ eigenvalue

Explicit states

[MRG, Gopakumar ’21]

- w=0: only the vacuum state survives — $\mathbf{1}$ in SYM
- w=1: wedge modes = zero modes: BPS singleton representation — absent in su(N).
- w=2: $L_0 = 0:$ BPS rep. $\big(0,0;[0,2,0]\big)_2$

  $L_0 = -2:$ Konishi multiplet $\big(0,0;[0,0,0]\big)_2$

  generated from hwv $|\mathrm{K}\rangle = (\psi^{\dagger}_1)_{\frac{1}{2}}(\psi^{\dagger}_2)_{\frac{1}{2}}\psi^3_{\frac{1}{2}}\psi^4_{\frac{1}{2}}|0\rangle_2 \sim S^{\mathbf{a}}_1 S^{\mathbf{b}}_1|0\rangle_2$

  $L_0 = -2p:$ hs multiplet $\big(p-1,p-1;[0,0,0]\big)_{2p}$

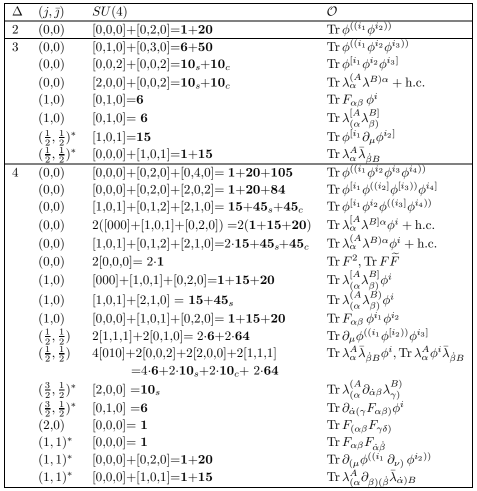
  generated from hwv $\prod_{i=1}^{2p-2}(\mu^{\dagger}_{\alpha_i})_{\frac{1}{2}}\,\mu^{\dot{\alpha}_i}_{\frac{1}{2}}\,|\mathrm{K}\rangle = \prod_{i=1}^{2p-2} S^{\alpha_i\dot{\alpha}_i}_1\,|\mathrm{K}\rangle$

Explicit states

[MRG, Gopakumar ’21]

- w=3: structure is quite complicated…

but we have enumerated the low-lying states and compared to the N=4 SYM spectrum (for $\mathcal{D}_0 \leq 4$)



Δ	$(j,\bar{j})$	$SU(4)$	$\mathcal{O}$
2	(0,0)	$[0,0,0]+[0,2,0]=\mathbf{1}+\mathbf{20}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2))}$
3	(0,0)	$[0,1,0]+[0,3,0]=\mathbf{6}+\mathbf{50}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2}\phi^{i_3))}$
	(0,0)	$[0,0,2]+[0,0,2]=\mathbf{10}_s+\mathbf{10}_c$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{i_2}\phi^{i_3]}$
	(0,0)	$[2,0,0]+[0,0,2]=\mathbf{10}_s+\mathbf{10}_c$	$\mathrm{Tr}\,\lambda_\alpha^{(A}\lambda^{B)\alpha}+\mathrm{h.c.}$
	(1,0)	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,F_{\alpha\beta}\,\phi^i$
	(1,0)	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,\lambda^{[A}_{(\alpha}\lambda^{B]}_{\beta)}$
	$(\frac{1}{2},\frac{1}{2})^*$	$[1,0,1]=\mathbf{15}$	$\mathrm{Tr}\,\phi^{[i_1}\partial_\mu\phi^{i_2]}$
	$(\frac{1}{2},\frac{1}{2})^*$	$[0,0,0]+[1,0,1]=\mathbf{1}+\mathbf{15}$	$\mathrm{Tr}\,\lambda^A_\alpha\bar{\lambda}_{\dot{\beta}B}$
4	(0,0)	$[0,0,0]+[0,2,0]+[0,4,0]=\mathbf{1}+\mathbf{20}+\mathbf{105}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2}\phi^{i_3}\phi^{i_4))}$
	(0,0)	$[0,0,0]+[0,2,0]+[2,0,2]=\mathbf{1}+\mathbf{20}+\mathbf{84}$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{((i_2]}\phi^{[i_3))}\phi^{i_4]}$
	(0,0)	$[1,0,1]+[0,1,2]+[2,1,0]=\mathbf{15}+\mathbf{45}_s+\mathbf{45}_c$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{i_2}\phi^{((i_3]}\phi^{i_4))}$
	(0,0)	$2([000]+[1,0,1]+[0,2,0])=2(\mathbf{1}+\mathbf{15}+\mathbf{20})$	$\mathrm{Tr}\,\lambda^{[A}_\alpha\lambda^{B]\alpha}\phi^i+\mathrm{h.c.}$
	(0,0)	$[1,0,1]+[0,1,2]+[2,1,0]=2\cdot\mathbf{15}+\mathbf{45}_s+\mathbf{45}_c$	$\mathrm{Tr}\,\lambda^{(A}_\alpha\lambda^{B)\alpha}\phi^i+\mathrm{h.c.}$
	(0,0)	$2[0,0,0]=2\cdot\mathbf{1}$	$\mathrm{Tr}\,F^2, \mathrm{Tr}\,F\widetilde{F}$
	(1,0)	$[000]+[1,0,1]+[0,2,0]=\mathbf{1}+\mathbf{15}+\mathbf{20}$	$\mathrm{Tr}\,\lambda^{[A}_{(\alpha}\lambda^{B]}_{\beta)}\phi^i$
	(1,0)	$[1,0,1]+[2,1,0]=\mathbf{15}+\mathbf{45}_s$	$\mathrm{Tr}\,\lambda^{(A}_{(\alpha}\lambda^{B)}_{\beta)}\phi^i$
	(1,0)	$[0,0,0]+[1,0,1]+[0,2,0]=\mathbf{1}+\mathbf{15}+\mathbf{20}$	$\mathrm{Tr}\,F_{\alpha\beta}\,\phi^{i_1}\phi^{i_2}$
	$(\frac{1}{2},\frac{1}{2})$	$2[1,1,1]+2[0,1,0]=2\cdot\mathbf{6}+2\cdot\mathbf{64}$	$\mathrm{Tr}\,\partial_\mu\phi^{((i_1}\phi^{[i_2))}\phi^{i_3]}$
	$(\frac{1}{2},\frac{1}{2})$	$4[010]+2[0,0,2]+2[2,0,0]+2[1,1,1]$ $=4\cdot\mathbf{6}+2\cdot\mathbf{10}_s+2\cdot\mathbf{10}_c+2\cdot\mathbf{64}$	$\mathrm{Tr}\,\lambda^A_\alpha\bar{\lambda}_{\dot{\beta}B}\phi^i, \mathrm{Tr}\,\lambda^A_\alpha\phi^i\bar{\lambda}_{\dot{\beta}B}$
	$(\frac{3}{2},\frac{1}{2})^*$	$[2,0,0]=\mathbf{10}_s$	$\mathrm{Tr}\,\lambda^{(A}_{(\alpha}\partial_{\dot{\alpha}\beta}\lambda^{B)}_{\gamma)}$
	$(\frac{3}{2},\frac{1}{2})^*$	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,\partial_{\dot{\alpha}(\gamma}F_{\alpha\beta)}\phi^i$
	(2,0)	$[0,0,0]=\mathbf{1}$	$\mathrm{Tr}\,F_{(\alpha\beta}F_{\gamma\delta)}$
	$(1,1)^*$	$[0,0,0]=\mathbf{1}$	$\mathrm{Tr}\,F_{\alpha\beta}F_{\dot{\alpha}\dot{\beta}}$
	$(1,1)^*$	$[0,0,0]+[0,2,0]=\mathbf{1}+\mathbf{20}$	$\mathrm{Tr}\,\partial_{(\mu}\phi^{((i_1}\partial_{\nu)}\phi^{i_2))}$
	$(1,1)^*$	$[0,0,0]+[1,0,1]=\mathbf{1}+\mathbf{15}$	$\mathrm{Tr}\,\lambda^A_{(\alpha}\partial_{\beta)(\dot{\beta}}\bar{\lambda}_{\dot{\alpha})B}$

Table 3: N=4 SYM at $\lambda = 0$. Brackets denote antisymmetrization. Parentheses denote complete symmetrization when traces cannot appear. Double parentheses denote complete symmetrization not excluding traces.

and answer reproduces intricate spectrum of BPS & non-BPS $\mathfrak{psu}(2,2|4)$ multiplets.

from [Bianchi, Morales, Samtleben, ’03]

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Conclusions and Outlook

The free field realisation of the $\mathrm{AdS}_3 \times \mathrm{S}^3$ worldsheet theory dual to the symmetric orbifold suggests a natural generalisation to $\mathrm{AdS}_5 \times \mathrm{S}^5$.

With some assumptions about the structure of the physical state conditions, we have managed to reproduce the exact single-trace spectrum of free SYM in 4d from our worldsheet model.

This **opens the door for a proof of the AdS/CFT correspondence** for this most relevant case.

Future directions

- Understand physical state condition from first principles. [MRG, Gopakumar, Naderi, Sriprachyakul, in progress]
- Study structure of correlation functions for AdS_5 . [MRG, Gopakumar, Knighton, Maity, in progress]
- Analyse perturbation away from free case.
- D-branes and non-perturbative effects [MRG, Knighton, Vosmera '21]
- Study $\mathfrak{hs}(2,2|4)$ higher spin & Yangian symmetry from worldsheet perspective. cf [Beisert, Bianchi, Morales, Samtleben '04]
- Explore novel (BMN-like) perspective on N=4 spectrum.
- Relation of correlators of twistor-like variables to hexagon approach. cf [Basso, Komatsu, Vieira '15]
- …

Thank you!

Physical states for AdS3

As a consistency check we have also imposed this wedge construction in the case of AdS_3.

reproduces a subset of `compactification independent' states, e.g. the BPS state is the `upper' BPS state in the (w-1)-cycle twisted sector $[w_{\mathrm{AdS}_3} = w_{\mathrm{AdS}_5} - 1]$

$$\bar{\psi}^{\,1}_{-\frac{1}{2}}\bar{\psi}^{\,2}_{-\frac{1}{2}}\underset{\underset{h=j=\frac{w-2}{2}}{\uparrow}}{|\mathrm{BPS}_{\mathrm{lower}}\rangle}^{(w-1)} \quad \text{with} \quad h = \underset{\uparrow}{j} = \frac{w}{2}$$

zero'th cohomology of 4d manifold

[MRG, Gopakumar '21]

see e.g. [David et.al. '02]

Spinchain vs sym orbifold

While the above wedge mode description has a natural spin-chain interpretation, the associated cyclic symmetry does not seem to be naturally realised in the symmetric orbifold theory since it requires

$$\mathbb{Z}_w \text{ symmetry on } (w-1)\text{-cycle twisted sector}$$

$$[w_{\mathrm{AdS}_3} = w_{\mathrm{AdS}_5} - 1]$$

This could be the reason why it has been so difficult to relate symmetric orbifold and spinchain descriptions for this case.

[Babichenko, Borsato, David, Ohlsson Sax, Sahoo, Sfondrini, Stefanski, Torielli, Zarembo…]