Bootstrapping Quantum Extremal Surface

Alex Belin - CERN

Based on:

2107.07516 w/ Sean Colin-Ellerin

See also:

1912.00024
1805.08782 } w/ N. Iqbal, S. Lokhande, J. Kruthoff

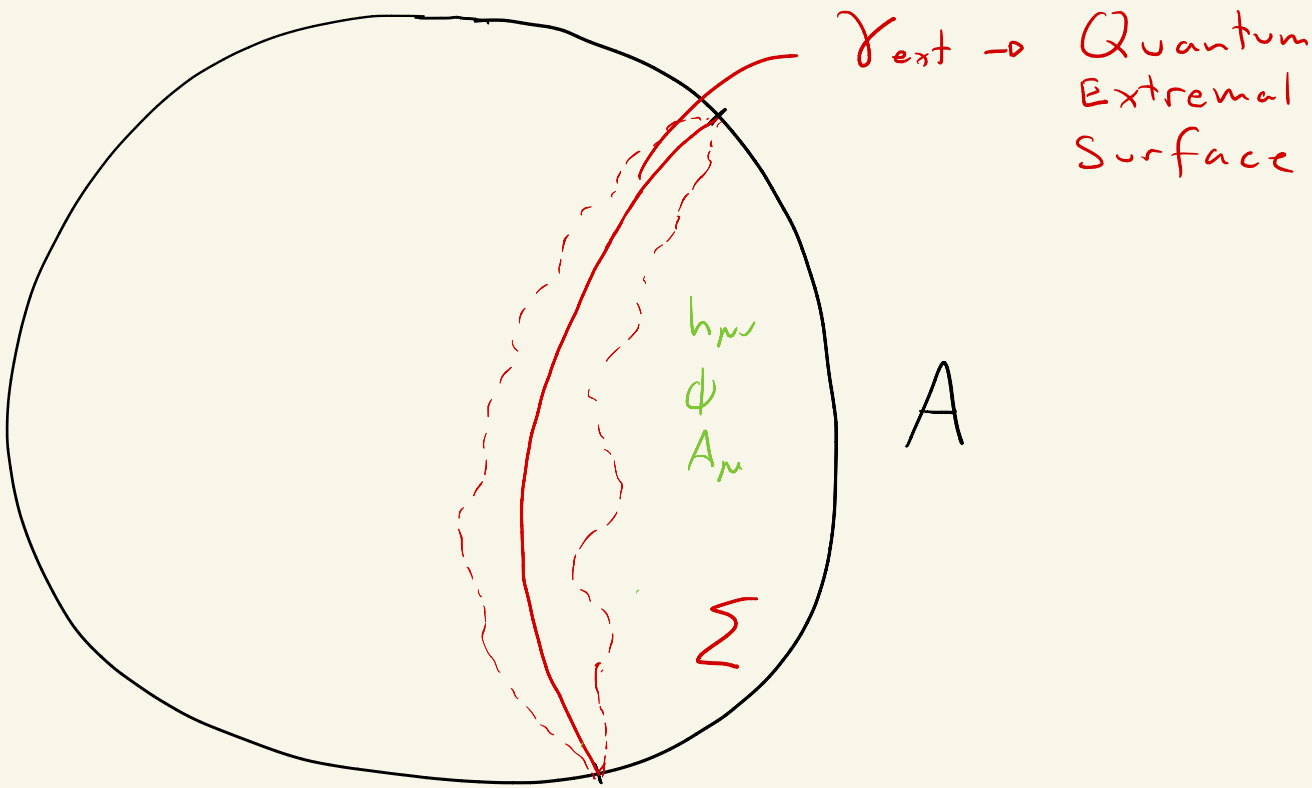
Strings, Fields & Holograms @ Ascona

Oct 15th 2021

Quantum Info. Thy $\Rightarrow$ New mindset to probe Q.G.

$$\boxed{S_{CFT}(A) = \underset{\gamma}{\mathrm{Ext}}\left[\frac{A_\gamma}{4G_N} + S_{bulk}(\Sigma)\right]}$$

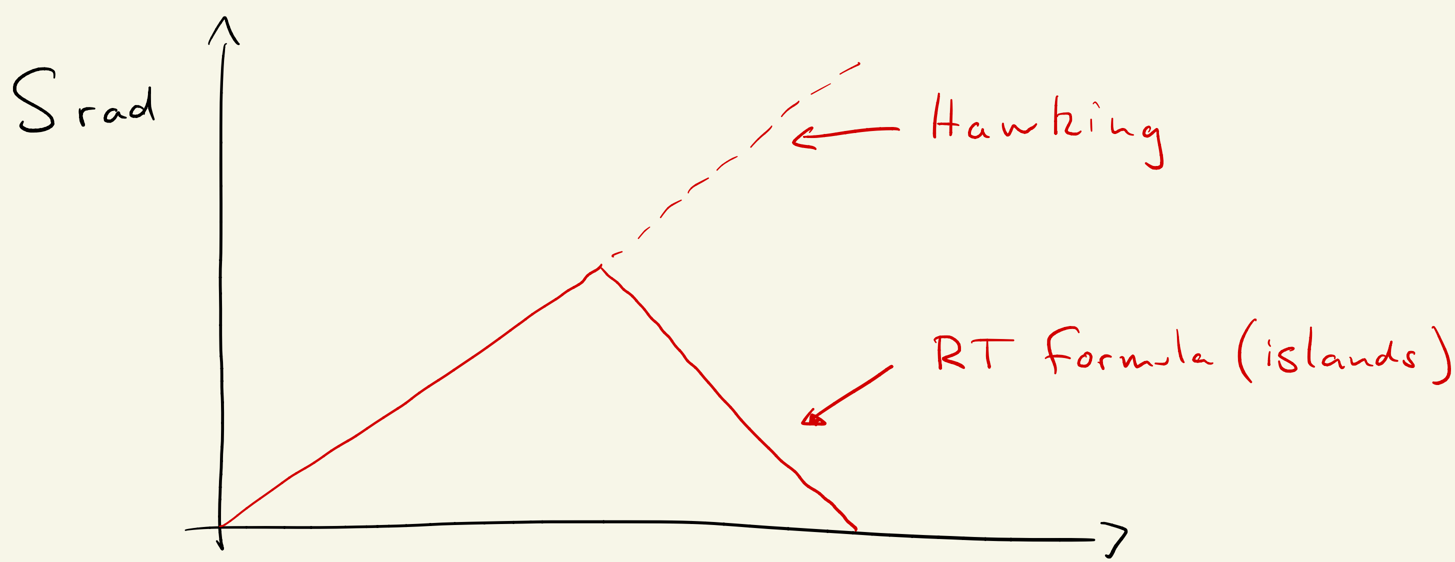
[RT, HRT, FLM, JLMS, EW, DL, ...]



This formula can be derived from the semi-classical gravitational path integral (Replica Trick)

[LM, DL, AHMST, PSSY, ...]

New insight: This formula is enough to produce a unitary Page curve



[P, AEMM, ...]

Conclusion

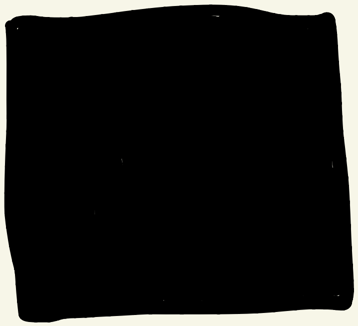
- We can see avatars of unitarity from semiclassics

- No need for the full microscopic details (no need for strings, branes, etc..)

This raises 2 questions:

1. If the full microscopics are not needed, exactly how much of the CFT do we need?

2. How does the microscopic CFT data get reorganized in S_{gen}?



Euclidean
Semi-classical
Path Integral

$\Longrightarrow$ RT formula $\Longrightarrow$ Unitarity

Goal for today

Present a dictionary between

Microscopic CFT data $\Longleftrightarrow$ Quantum Extremal Surface

$$\{\Delta_i, C_{ijk}\} \Longleftrightarrow \frac{A}{4G_N} + S_{bulk}$$

$\Rightarrow$ Restrict to low-energy sector

$\Rightarrow$ Can make the dictionary precise/sharp

Subtleties

① S_{CFT} is UV-divergent

$\Rightarrow$ not a problem, ΔS, S_{rel}, I_{AB}

② There are <u>bulk</u> UV-divergence (S_{bulk})

bulk RG

$$\frac{A}{4G_N} + S_{bulk}$$

UV-div reabsorbed in renom. of G_N

The dictionary

$$|\psi\rangle = \mathcal{O}_{\text{S.T.}}|0\rangle$$

$$\mathcal{O},\ T,\ [\mathcal{O}\mathcal{O}]_{n\ell}$$

$$\Delta_{[\mathcal{O}\mathcal{O}]_{n\ell}} = 4h_0 + 2n + \ell + \frac{\gamma_{n,\ell}}{c}$$

$$C_{\mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]_{n\ell}} = \sqrt{2} + \frac{a_{n,\ell}}{c}$$

The dictionary

$|\psi\rangle = \mathcal{O}_{S.T.}|0\rangle$

$\Delta_{[OO]_{n\ell}} = 4h_0 + 2n + \ell + \frac{\gamma_{n,\ell}}{c}$

$\mathcal{O}, T, [OO]_{n\ell}$

$C_{OO[OO]_{n\ell}} = \sqrt{2} + \frac{a_{n,\ell}}{c}$

	S_{CFT}	$S_{gen}(\gamma)$	
$\mathcal{O}(c^0)$	$\mathbb{1}$	$A[\gamma^0, g^1]$	+ cancellations
	$[OO]_{n\ell}$	$S_{bulk}[\gamma^0]$	
$\mathcal{O}(c^{-1})$	$\mathbb{1} + T$	$A[\gamma^0, g^2] + A[\gamma^1, g^1]$	
	$\gamma_{n,\ell}$	??	+ cancel ??
	$a_{n,\ell}$	??	
$1/\Delta_{gap}$	??	??	

Outline

1. Introduction

2. CFT computation

3. Gravity computation

4. Conclusion + open questions

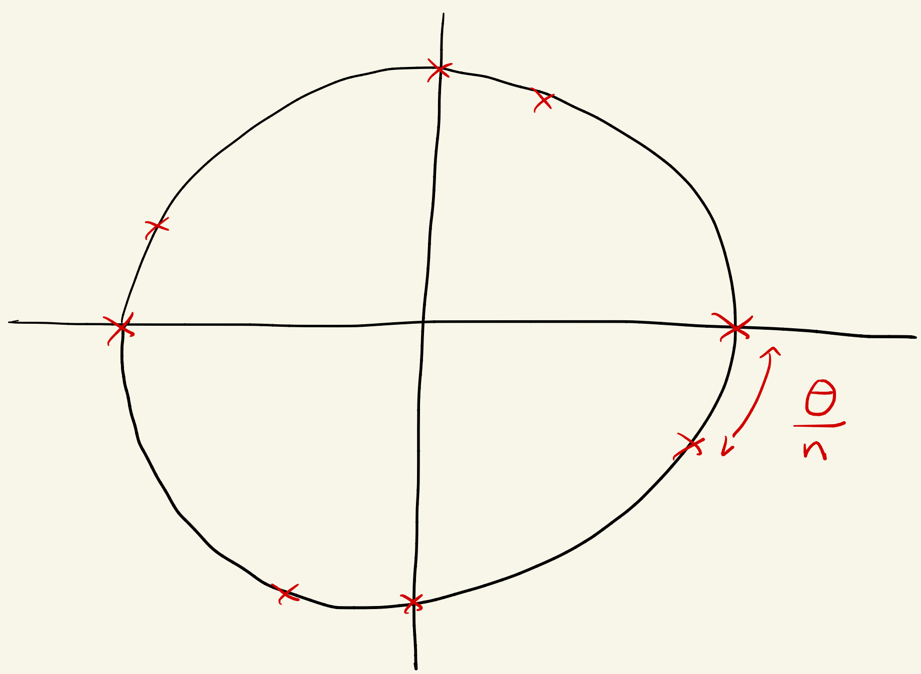
CFT calculation

$$|\psi\rangle_{CFT} = \mathcal{O}_{S.T}(0)\,|0\rangle$$

$\hookrightarrow \quad h = \bar{h}$

$$A \in [0, \theta]$$

$$S_n(|\psi\rangle) - S_n(|0\rangle) = \frac{1}{1-n}\operatorname{Log}\left[\left(\frac{2}{n}\sin\left(\frac{\theta}{2}\right)\right)^{4nh}\langle\underbrace{\mathcal{O}\cdots\cdots\mathcal{O}}_{2n}\rangle\right]$$



[Alcaraz, Berganza, Sierra]

$$S_{EE} = \lim_{n\to 1} S_n$$

doable in $\theta \to 0$ limit

$$c^{0} : \quad \Delta S = 2h\left(2 - \theta \cot\left(\frac{\theta}{2}\right)\right) - \sin\left(\frac{\theta}{2}\right)^{8h} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(4h+1)}{\Gamma\left(4h+\frac{3}{2}\right)}$$

$$\parallel \qquad\qquad :\mathcal{O}^{2}:$$

c^0 : $\Delta S = 2h\left(2 - \theta \cot\left(\frac{\theta}{2}\right)\right) - \sin\left(\frac{\theta}{2}\right)^{8h} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(4h+1)}{\Gamma\left(4h+\frac{3}{2}\right)}$

$\mathbb{1}$ $\qquad\qquad\qquad\qquad$ $:\mathcal{O}^2:$

---

c^{-1} :

1. Anom. dim. of $\mathcal{O}_{S.T.}$: $h \to h + \frac{\delta h}{c}$

2. Exchange of T $(\bar{T})$

3. $\Delta_{\mathcal{O}^2} = 4h + \frac{4\delta h + \gamma_{0,0}}{c}$

4. $C_{\mathcal{O}\mathcal{O}:\mathcal{O}^2:} = \sqrt{2} + \frac{a_{0,0}}{c}$

$\gamma_{0,0}$, $a_{0,0}$ $\longrightarrow$ fixed by crossing [Collier, Gobeil, Maxfield, Perlmutter]

δh is <u>not</u> fixed by crossing

$$\Delta S_{\mathrm{EE}}\big|_{\mathcal{O}(c^{-1})} = \frac{2\delta h}{c}\left(2-\theta\cot\left(\frac{\theta}{2}\right)\right) - \frac{16h^2}{15c}\left(\sin\frac{\theta}{2}\right)^4$$
$$+\frac{24h^2-4\delta h}{c}\left[2\log\left(\sin\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)^{8h}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(4h+1\right)}{\Gamma\left(4h+\frac{3}{2}\right)}\right.$$
$$\left.+\left(\sin\frac{\theta}{2}\right)^{8h}\Gamma\left(\frac{3}{2}\right)\frac{\Gamma\left(4h+1\right)}{\Gamma\left(4h+\frac{3}{2}\right)}\left(\psi(4h+1)-\psi\left(4h+\frac{3}{2}\right)\right)\right]$$
$$+\frac{96h^2}{c}\left(\sin\frac{\theta}{2}\right)^{8h}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(4h+1\right)}{\Gamma\left(4h+\frac{3}{2}\right)}\,.$$

$\gamma_{0,0}$ $a_{0,0}$

$$
\begin{aligned}
\Delta S_{\mathrm{EE}}\big|_{\mathcal{O}(c^{-1})} = {} & \boxed{\frac{2\delta h}{c}\left(2-\theta\cot\left(\frac{\theta}{2}\right)\right)-\frac{16h^2}{15c}\left(\sin\frac{\theta}{2}\right)^4} \\
& + \frac{24h^2-4\delta h}{c}\left[2\log\left(\sin\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)^{8h}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(4h+1\right)}{\Gamma\left(4h+\frac{3}{2}\right)}\right. \\
& \left. + \left(\sin\frac{\theta}{2}\right)^{8h}\Gamma\left(\frac{3}{2}\right)\frac{\Gamma\left(4h+1\right)}{\Gamma\left(4h+\frac{3}{2}\right)}\left(\psi(4h+1)-\psi\left(4h+\frac{3}{2}\right)\right)\right] \\
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\end{aligned}
$$

$\gamma_{0,0}$ $\qquad$ $a_{0,0}$

$\Rightarrow$ Focus on this piece today

Where does it come from in the bulk?

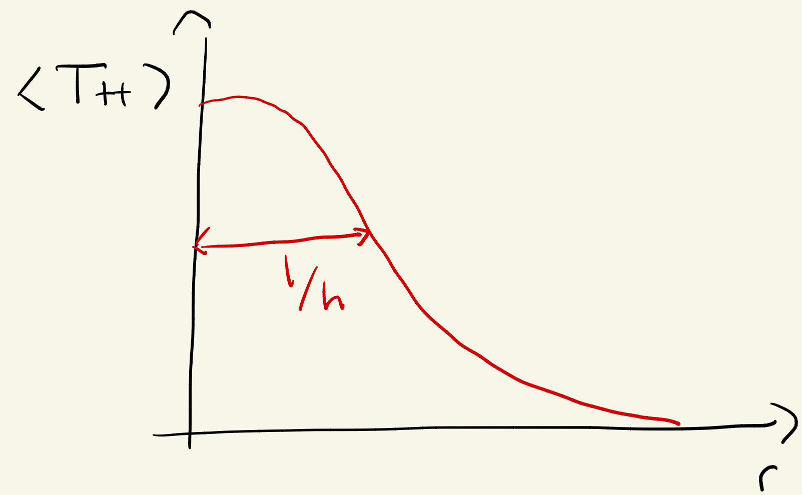
Bulk Computation

$$|\psi\rangle_{CFT} = \mathcal{O}(0)|0\rangle \Longleftrightarrow |\psi\rangle_{bulk} = a^\dagger_{0,0}|0\rangle$$

$\mathcal{O}(c^0) = \mathcal{O}(G_N^0)$: Two effects

① The geometry changes

$$\langle\psi|T^{bulk}_{\mu\nu}|\psi\rangle \neq 0$$



$$G^{(1)}_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu}\rangle$$

- $g_{\mu\nu} = g^{AdS}_{\mu\nu} + g^{(1)}_{\mu\nu}$
- No need to find $\gamma^{(1)}$ by extremality

(2) $|\Psi\rangle$ polarizes the bulk entanglement

$$a^\dagger = \sum_{k,\omega} \alpha^*_{k,\omega}\, b^\dagger_{k,\omega} + \beta_{k,\omega}\, b_{k,\omega}$$

$$|0\rangle\langle 0| \longrightarrow \mathrm{tr}_R \longrightarrow e^{-\beta H}$$

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$$S_{bulk}^{|\Psi\rangle} - S_{bulk}^{|0\rangle} \Rightarrow \underline{\text{bulk UV-finite}}$$

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$$S_{bulk}^{|\Psi\rangle} - S_{bulk}^{|0\rangle} \Rightarrow \underline{\text{bulk UV-finite}}$$

$$\frac{\Delta A}{4 G_N} = 2h\left(2 - \theta \cot\left(\frac{\theta}{2}\right)\right) - \cancel{\sin\left(\frac{\theta}{2}\right)^{4h} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(2h+1)}{\Gamma\left(2h+\frac{3}{2}\right)}}$$

$$\Delta S_{bulk} = \cancel{\sin\left(\frac{\theta}{2}\right)^{4h} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(2h+1)}{\Gamma\left(2h+\frac{3}{2}\right)}} - \sin\left(\frac{\theta}{2}\right)^{8h} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(4h+1)}{\Gamma\left(4h+\frac{3}{2}\right)}$$

[AB, Iqbal, Lokhande]

Order G_N

What do we mean by semi-classical gravity?

Order G_N

What do we mean by semi-classical gravity?

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + G_N\, g_{\mu\nu}^{(1)} + G_N^2\, g_{\mu\nu}^{(2)}$$

$$\gamma = \gamma^{AdS} + G_N\, \gamma^{(1)} + G_N^2\, \gamma^{(2)}$$

$$\frac{A}{4G_N}\Big|_{\mathcal{O}(G_N)} = \frac{A[(g^{(1)})^2, \gamma^{(0)}] + A[g^{(0)}, (\gamma^{(1)})^2] + A[g^{(1)}, \gamma^{(1)}] + A[g^{(2)}, \gamma^{(0)}]}{4G_N}$$

$\Rightarrow$ No $A(\gamma^{(2)}, g^{0})$ by extremality

$\Rightarrow$ Need $g^{(2)} + \gamma^{(1)}$

Finding $g^{(2)}$

We need to solve

$$\left(\nabla^2_{g^{AdS}+g^{(1)}} - m^2\right)\phi = 0$$

$$\phi = \sum_{n,\ell} f_{n,\ell}(r)\, e^{i\omega_{n,\ell} t}\, a_{n,\ell} + c.c.$$

$$f_{0,0} = \frac{1}{(1+r^2)^h} + G_N F^{(1)}(r)$$

$$\underset{r\to\infty}{\sim} \frac{1}{r^{2h}} + \frac{G_N}{r^{6h}}$$

To find $g^{(2)}$:

$$G_{\mu\nu}^{(2)} = 8\pi G_N \langle T_{\mu\nu}^{(1)} \rangle$$

$\rightarrow$ solve for $g^{(2)}$

This is (second-order) grav. dressed and smooth conical defect.

$\Rightarrow$ Can compute $\dfrac{A[\gamma^\circ, g^{(2)}]}{4 G_N}$

Bulk Energy and the right state?

What is the energy of our state $|\Psi\rangle_{bulk}$.

ADM mass: $E + \frac{c}{12} = 2h - \frac{24h^2}{c}\frac{2h-1}{4h-1}$

CFT: $\langle\Psi|L_0|\Psi\rangle = 2h + \frac{\delta h}{c}$

Free parameter! Bulk should not fix it!

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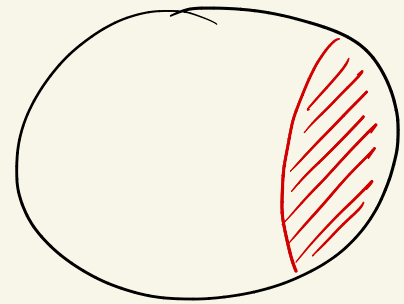
Free parameter! Bulk should not fix it!

We can also change the bulk mass!

$m \longrightarrow m + \delta m \Rightarrow$ affects $\delta h \Rightarrow$ need to agree beforehand

How to find $\gamma^{(1)}$



Go to Rindler AdS

$$ds^2 = -\sinh^2\rho\, d\tau^2 + d\rho^2 + \cosh^2\rho\, dx^2 + g_{\mu\nu}^{(1)} dx^\mu dx^\nu$$

CES $\longrightarrow$ $\rho = 0$

QES $\longrightarrow$ $\rho = 0 + G_N \rho^{(1)}(x)$

$$A[\rho^{(1)}(x)] = \int dx \sqrt{g_{xx} + g_{x\rho}\, \partial_x \rho^{(1)} + g_{\rho\rho}(\partial_x \rho^{(1)})^2}$$

For S_{bulk} : $g \to g + \delta g \to$ shape variation

$$S_{bulk}[g^{(1)}(x)] \sim \int dx \, g'(x) \underbrace{\langle T \, K \rangle}_{\text{bulk 2-pt fct}}$$

EOM for $g^{(1)}(x)$

$$\boxed{g''(x) + g(x) + V_{geo}(x) + V_{EE}(x) = 0}$$

$g^{(1)}(x)$ $\qquad$ $\sim \Theta^{4h}$

⚠ Can deal with $V_{geo} + V_{bulk}$ "independently"

Define $\delta_{geo} \mid \delta_{geo}'' + \delta_{geo} + V_{geo} = 0$

Can deal with V_{geo} + V_{bulk}

"independently"

Define g_{geo} | $g_{geo}'' + g_{geo} + V_{geo} = 0$

$$\rho^{(1)}_{\text{geo}}(x) = \theta^2 \frac{h}{3}(\cosh(2x)+3)\,\text{sech}^3 x\,(1+\mathcal{O}(\theta^2)) + \left(\frac{\theta}{2}\right)^{4h}(1+\mathcal{O}(\theta^2))\left[-8h\frac{\Gamma(2h+1)\Gamma(\frac{3}{2})}{\Gamma(2h+\frac{3}{2})}e^x - 4h\sinh x\,\text{sech}^{4h+2}x\left(\frac{1}{(2h+1)}+\frac{e^{-2x}}{(h+1)}\left(1-{}_2F_1\left(1,-2h-2,2h+1;-e^{2x}\right)\right)\right)\right].$$

Putting all the pieces together

$$\frac{1}{4G_N}\langle\Psi|A|\Psi\rangle\Big|_{\mathcal{O}(G_N)}$$

$$= -\frac{24h^2}{c}\frac{2h-1}{4h-1}\left(\frac{\theta^2}{6}+\frac{\theta^4}{360}+\cdots\right)-\frac{16h^2}{15c}\left(\frac{\theta}{2}\right)^4$$

$$+\ \mathcal{O}(\theta^{4h})$$

$$= \frac{28h}{c}\left(\frac{\theta^2}{6}+\frac{\theta^4}{360}+\cdots\right)-\frac{16h^2}{15c}\left(\frac{\theta}{2}\right)^4 = S_{CFT}$$

Summary

- I presented the first steps of a dictionary

  Microscopic CFT data $\Longleftrightarrow$ Quant. Ext. Surfaces

- Partial Check of Quantum RT formula at order where quantum extremality is important. First of a kind in $d>1$.

- To this order : $\overbrace{\mathbb{1} + T}^{\text{Vir } \mathbb{1} \text{ block}} \subset \dfrac{A}{4G_N}$

Open Questions

- Compute S_{bulk} 

- What about boundary gravitons?

  Didn't consider their contribution

  Only relevant for $L_{-n}|\psi\rangle$?

- Bulk Cancellations

  What is their meaning?

  Can we extract them from the CFT?

- Can we really distille S_{bulk} vs $\frac{A}{4G_N}$?

Thank You!